

Name Marking scheme Paper 1 ADM No. _____

Date _____

Candidate's signature _____

121/1

MATHEMATICS ALT A

PAPER 1

FORM 3

TIME: 2½ HOURS

FORM 3 END OF YEAR EXAMINATION

INSTRUCTIONS TO CANDIDATES

1. Write your name and index number in the spaces provided above.
2. Sign and write the date of examination in the space provided above.
3. This paper consists of **TWO** sections: **Section I** and **Section II**.
4. Answer **ALL** the questions in **section I** and only **FIVE** questions from **Section II**.
5. All answers and working must be written on the question paper in the space provided below each question.
6. Show all the steps in your calculations, giving your answers at each stage in the spaces below each question.
7. Marks may be given for correct working even if the answer is wrong.
8. Non-programmable silent calculators and KNEC mathematical tables may be used except where stated otherwise.
9. This paper consists of **15** printed papers.
10. Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

FOR EXAMINER'S USE ONLY

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

SECTION II

17	18	19	20	21	22	23	24	TOTAL

GRAND TOTAL	

SECTION I (50 MARKS)

Answer ALL questions in the spaces provided.

1. Simplify:

$$\frac{3/5 \text{ of } 60 - 2^2/3 \times 1 \frac{1}{2}}{5^{5/8} \times 1^{7/9} - 5/4 \times 4^{4/5} + 2^{4/5} \div 7/10}$$

(3 marks)

Num: $\frac{3 \times 12}{5} - \left(\frac{2^4}{3} \times \frac{3}{2}\right)$ Den: $\frac{5^2}{8} - \left(\frac{5}{4} \times \frac{6}{5}\right) + \frac{2^4}{5} \div \frac{7}{10}$

$\frac{36}{5} - \left(\frac{8}{3} \times \frac{3}{2}\right)$ $\frac{25}{8} - \left(\frac{5}{4} \times \frac{6}{5}\right) + \frac{16}{5} \div \frac{7}{10}$

$36 - 4$ $10 - 6 + 4$

$32 \checkmark M_1$ $\frac{2 \times 74 \times 10}{5 \times 71}$

$8 \checkmark M_1$

$$\frac{32}{8} = 4 \checkmark A_1$$

2. The LCM of two numbers is 180 and their GCD is 12. If the two numbers are 36 and y, find the value of y. (2 marks)

$\frac{36 \times y}{12} = 180$ $36y = 180 \times 12$

$y = \frac{180 \times 12}{36}$

$y = 60 \checkmark A_1$

3. Solve for x in the equation.

$$27^x \times 3^{(2x-2)} = 9^{(x+2)}$$

(3 marks)

$3^x \times 3^{2x-2} = 3^{2(x+2)}$ $x = 2 \checkmark A_1$

$3^{3x+2x-2} = 3^{2x+4}$ $3 \checkmark M_1$

$5x - 2 = 2x + 4$ $3x = 6$

$3x = 6$

4. The cost of a tablet is US \$1000. Kaloki intends to buy one tablet through an agent who deals in Japanese Yen. The agent charges him a commission of 5% on the price of the tablet and further 1260 Yen as import tax. How many Ksh. Will he need to sent to the agent to obtain the tablet given that

$$1 \text{ US \$} = 105.00 \text{ Yen}$$

$$1 \text{ US \$} = \text{Ksh. } 85.$$

(3 marks)

$$\begin{aligned} 1 \text{ US \$} &\Rightarrow 105 \text{ yen} \\ 1000 \text{ US} &\Rightarrow ? \\ &\quad \underline{105 \times 1000} \\ &\quad 1 \checkmark M_1 \\ &= 105000 \text{ yen} \end{aligned}$$

$$\frac{105}{100} \times 105000 = 110,250$$

Total amount paid

$$\begin{array}{r} 110250 \\ + 1260 \\ \hline 111510 \end{array}$$

$$\begin{aligned} 105 &\rightarrow 100 \\ 111510 &\rightarrow ? \\ &\quad \underline{\frac{111510}{105}} \end{aligned}$$

$$106245 \checkmark M_1$$

$$\begin{aligned} 100 &\rightarrow 85 \\ 1062 &\rightarrow ? \end{aligned}$$

$$\begin{aligned} 85 \times 1062 \\ = \underline{90,270} \checkmark A_1 \end{aligned}$$

5. The size of each interior angle of a regular polygon is five times the size of the exterior angle. Find the number of sides of the polygon.

(2 marks)

$$\begin{aligned} &\text{Diagram showing an interior angle } 5x \text{ and an exterior angle } x. \\ 5x + x &= 180^\circ \\ 6x &= 180^\circ \\ x &= 30^\circ \end{aligned}$$

$$\begin{array}{r} 360 \\ \div 30 \\ \hline \end{array}$$

$$= 12 \text{ sides } \checkmark A_1$$

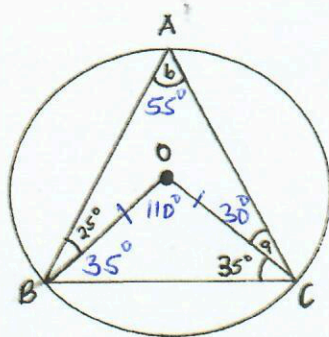
6. Francis who runs a clothing shop bought a shirt at ksh. 500 and marked it at ksh. 600. A customer bought it at ksh. 550 after engaging Francis in a lengthy negotiations process. What was the customer's percentage discount.

(3 marks)

$$\begin{array}{r} \text{Discount} = 600 \\ - 550 \\ \hline 50 \checkmark M_1 \end{array}$$

$$\begin{aligned} &\text{Percentage discount.} \\ &\quad \underline{\frac{50}{600} \times 100} \checkmark M_1 \\ &= 8.333\% \checkmark A_1 \end{aligned}$$

7. In the following figure, O is the Centre. Find a and b



a 30° B_1
b 55° B_1

8. The table below shows height of 50 students

Height (cm)	Frequency	
140 – 144	3	3
145 – 149	16	19
150 – 154	20	39
155 – 159	10	49
160 – 164	1	50

- (a) State the modal class

(1 mark)

150-154 $\checkmark B_1$

- (b) Calculate the median height.

(3 marks)

$$\frac{50}{2} = 25$$

$$\left(\frac{25 - 19}{20} \times 5 \right) + 149.5$$

$$1.5 + 149.5$$

$$= 151 A_1$$

9. A line passes through (1, 3) and is perpendicular to the line $y = 5 - 2x$. What is the equation of the line?

(3 marks)

$$G_1 = -2$$

$$G_2 = \frac{1}{2}$$

$$\frac{y-3}{x-1} = \frac{1}{2}$$

$$2(y-3) = x-1$$

$$2y-6 = x-1$$

$$2y = x + 5$$

$$y = \frac{x}{2} + \frac{5}{2} A_1$$

10. Mulwa bought 4 pencils and 6 biro pens for Ksh. 64 and Mwanja bought 2 pencils and 5 biro pens for sh. 50.

(a) Find the price for each item.

(3 marks)

$$\begin{aligned} 4P + 6b &= 64 \\ 2(2P + 5b) &= 50 \end{aligned} \quad \left. \begin{array}{l} M_1, \text{ at least one.} \end{array} \right\}$$

$$\begin{aligned} 4P + 6b &= 64 \\ -4P + 10b &= 50 \quad \checkmark M_1 \\ \hline -4b &= -36 \\ b &= 9 \end{aligned}$$

$$4P + 5b = 50$$

$$4P = 10$$

$$P = 2.5$$

$$\begin{aligned} \text{Biro} &= 9/- \\ \text{Pencil} &= 2.5/- \end{aligned} \quad \left. \begin{array}{l} A_1, \text{ for both.} \end{array} \right\}$$

- (b) Mutinda spend Ksh. 220 to buy the same type of pencils and biro pens. If the number of biro pens he bought were 4 more than the number of pencils, find the number of pencils he bought.

(2 marks)

$$\begin{aligned} \text{Biro} & \quad \text{Pencil} \\ (4+x) & \quad x \\ (4+x)9 + x(2.5) &= 220 \\ 36 + 9x + 2.5x &= 220 \\ 11.5x &= 184 \end{aligned}$$

$$x = \frac{184}{11.5} \quad \checkmark M_1$$

$$x = 16 \text{ pencils} \quad \checkmark A_1$$

11. Two similar containers have capacities 256 litres and 108 litres. If the surface area of the smaller container is 810cm^2 , what is the surface area of the larger container in m^2 ? (3 marks)

$$V \cdot S \cdot f = \frac{256}{108} = \frac{128}{54} = \frac{64}{27}$$

$$(L \cdot S \cdot f)^2 = A \cdot S \cdot f$$

$$\left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

$$9 \rightarrow 810$$

$$16 \rightarrow ?$$

$$\frac{810 \times 16}{9} \quad \checkmark M_1$$

$$= 1440\text{m}^2 \quad \checkmark A_1$$

$$\begin{aligned} \sqrt[3]{V \cdot S \cdot f} &= L \cdot S \cdot f \\ \sqrt[3]{\frac{64}{27}} &= \frac{4}{3} \quad \checkmark M_1 \end{aligned}$$

12. A stool is made by shaping a tree stump into a conical frustum of vertical height 60cm. If the top radius is 12cm and the bottom is 24cm. calculate the surface area of the stool $\pi=3.142$.



$$S.A = \pi RL - \pi rl$$

$$\frac{24}{12} = \frac{60+l}{l}$$

$$24l = 720 + 12l$$

$$12l = 720$$

$$l = 60 \quad \checkmark B_1$$

$$L = \sqrt{120^2 + 24^2}$$

$$= 122.38$$

$$L = \sqrt{60^2 + 12^2}$$

$$= 61.19$$

$$SA = (3.142 \times 24 \times 122.38) - 3.142 \times 12 \times 61.19$$

$$= 6921.32 \text{ cm}^2$$

Area of the circle

$$3.142 \times 12 \times 12 = 452.448 \quad \checkmark M_1$$

$$3.142 \times 24 \times 24 = 1809.792$$

$$6921.32 + 452.448 + 1809.792$$

$$= 9183.56 \quad \checkmark A_1$$

(4 marks)

13. Simplify the expression

$$\frac{3a^2 + 4ab + b^2}{4a^2 + 3ab - b^2}$$

Num:

$$3a^2 + 3ab + ab + b^2$$

$$3a(a+b) + b(a+b)$$

$$(3a+b)(a+b) \quad \checkmark M_1$$

Den

$$4a^2 + 4ab - ab - b^2$$

$$4a(a+b) - b(a+b)$$

$$(4a-b)(a+b) \quad \checkmark M_1$$

$$\frac{(3a+b)(a+b)}{(4a-b)(a+b)}$$

$$\frac{(3a+b)}{(4a-b)}$$

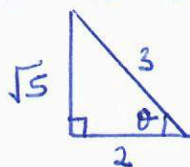
$$\frac{3a+b}{4a-b} \quad \checkmark A_1$$

(3 marks)

14. Given that $\cos \theta = \frac{2}{3}$ and θ is an acute angle, find:

(a) $\tan \theta$, giving your answer in surd form.

(2 marks)



$$\cos \theta = \frac{A}{H} = \frac{2}{3}$$

$$\tan \theta = \frac{O}{A} = \frac{\sqrt{5}}{2}$$

(b) $1 - \sin^2 \theta$

(1 mark)

$$\sin \theta = \frac{O}{H} = \frac{\sqrt{5}}{3}$$

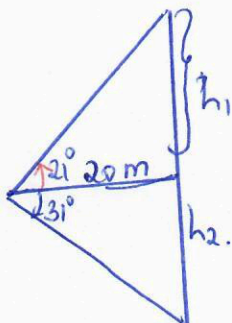
$$\sin^2 \theta = \frac{\sqrt{5}}{3} \times \frac{\sqrt{5}}{3}$$

$$= \frac{5}{9}$$

$$1 - \frac{5}{9}$$

$$= \frac{4}{9}$$

15. From the roof of a house, a boy can see an avocado tree which is 20m away from the house. He measures the angle of elevation of the top of the tree as 21° and the angle of depression of the bottom of tree as 31° . Find the height of the avocado tree. (3 marks)



$$\tan 21^\circ = \frac{h_1}{20}$$

$$h_1 = 20 \times \tan 21^\circ = 7.677 \text{ m} \quad \checkmark M_1$$

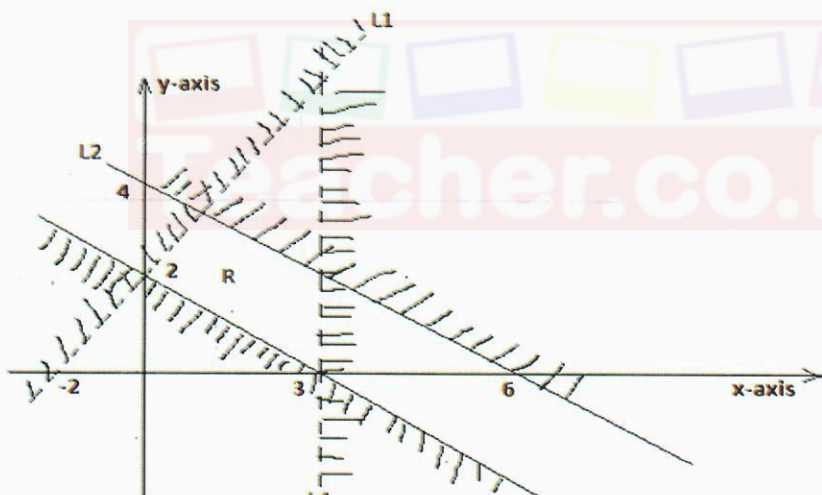
$$\tan 31^\circ = \frac{h_2}{20 \text{ m}}$$

$$h_2 = 20 \times \tan 31^\circ = 12.02 \text{ m} \quad \checkmark M_1$$

height of the avocado

$$\begin{array}{r} 7.677 \\ + 12.02 \\ \hline 19.697 \quad \checkmark A_1 \end{array}$$

16. Write down four inequalities which fully describe the unshaded region R in the figure below (4marks)



$$L_1 = (0, 2), (-2, 0)$$

$$G = \frac{2 - 0}{0 - -2} = \frac{2}{2} = 1$$

$$\text{Eqn} = \frac{y - 2}{x - 0} = 1$$

$$= y - 2 = x$$

$$y = x + 2$$

$$= y - x < 2 \quad \checkmark B_1$$

$$L_2 = (0, 4), (6, 0)$$

$$G = \frac{4 - 0}{0 - 6} = \frac{4}{-6} = -\frac{2}{3}$$

$$\text{Eqn} = \frac{y - 4}{x - 0} = -\frac{2}{3}$$

$$= 3y - 12 = -2x$$

$$3y + 2x = 12$$

$$3y + 2x \leq 12 \quad \checkmark B_1$$

$$L_3 = (3, 0), (0, 2)$$

$$G = \frac{0 - 2}{3 - 0} = -\frac{2}{3}$$

$$\text{Eqn} = \frac{y - 2}{x - 0} = -\frac{2}{3}$$

$$= 3(y - 2) = -2x$$

$$= 3y - 6 = -2x$$

$$= 3y + 2x = 6$$

$$3y + 2x \leq 6 \quad \checkmark B_1$$

$$L_4 = (3, 0), (3, 2)$$

$$x = 3$$

$$x < 3 \quad \checkmark B_1$$

SECTION II (SHORT ANSWERS)

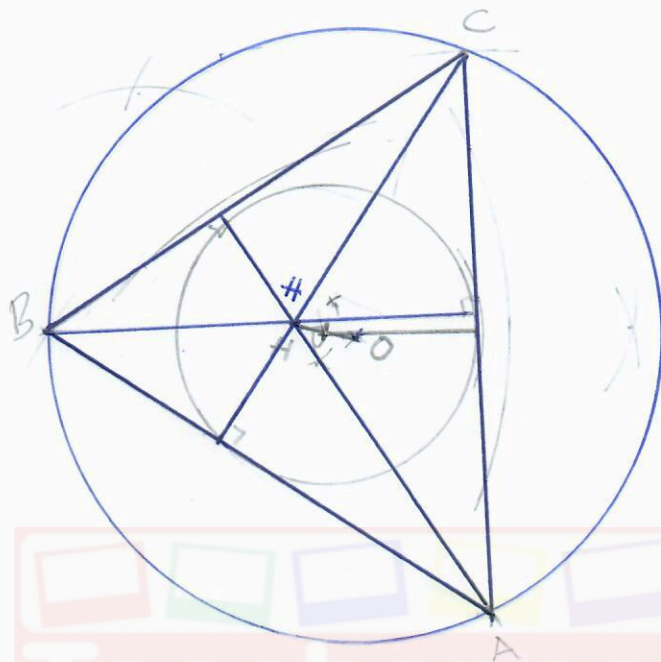
Answer Any five questions from this section.

18. Draw a circle Centre O and radius 4 cm.

(1 mark)

(a) From any point A on the circumference, draw two chords AB = 7cm and AC = 7.5cm on the opposite sides of the Centre.

(2 marks)



(b) Join B to C and locate the orthocenter (H) of triangle ABC. Join OH.

(4 marks)

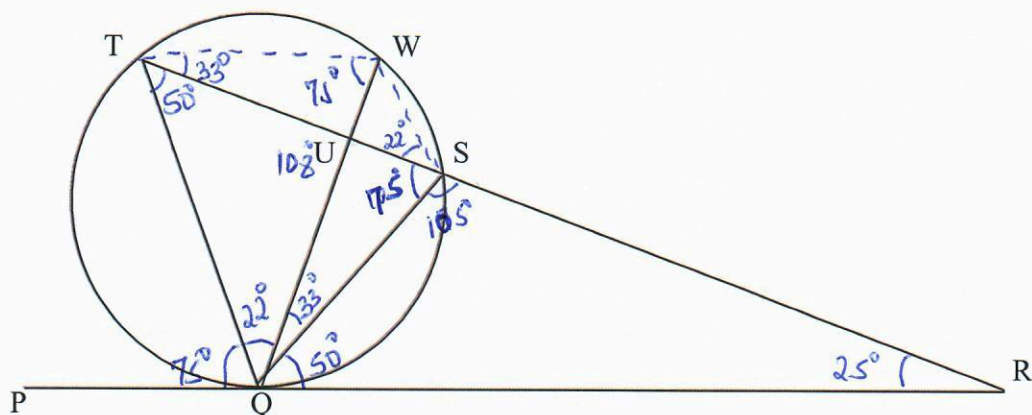
(c) Use the midpoint of OH as the Centre to inscribe a circle to triangle ABC and measure its radius.

(3 marks)

Centre ✓ B1
inscribed circle ✓ B1
radius = 2 cm ✓ B1
± 0.1

10

18.



In the above figure, PQR is a tangent at Q and RST is a straight line. $\angle QUT = 50^\circ$, $\angle UTW = 33^\circ$ and $\angle TRQ = 25^\circ$

Giving reasons in each case, calculate

(i) Angle TSQ

(3 marks)

75°
Angles in a triangle TSQ are equal to 180°
 $180 - (50^\circ + 22^\circ + 33^\circ) = 75^\circ$

(ii) Angle TSW

(3 marks)

22°
Angles subtended by the same chord in the same segment are equal. Chord TW subtends $\angle TSW = \angle TQW$.

(iii) Angle TUQ

(2 marks)

108°
Angles in a $\Delta TUQ = 180^\circ$ $(180 - (50 + 22) = 108)$

(iv) Angle PQW

(2 marks)

97°
Angles in alternate segment are equal.

19. Matrix P is given by $\begin{bmatrix} 4 & 7 \\ 5 & 8 \end{bmatrix}$

(a) Find P^{-1}

(2 marks)

$$\begin{aligned} \det (4 \times 8) - (7 \times 5) \\ = 32 - 35 \\ = -3 \end{aligned}$$

$$P^{-1} = \frac{1}{-3} \begin{bmatrix} 8 & -7 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} \frac{8}{3} & \frac{7}{3} \\ \frac{5}{3} & -\frac{4}{3} \end{bmatrix}$$

(b) Two schools, Utaati sec. school and Ngosini sec. school purchased beans at Ksh. B per bag and maize at Ksh. M per bag. Utaati purchases 8 bags of beans and 14 bags of maize for Ksh. 99,600. Ngosini purchases 10 bags of beans and 16 bags of maize at Ksh. 120,000

(i) Form a matrix equation to represent above information.

(2 marks)

$$\begin{aligned} 8b + 14m &= 99600 \\ 10b + 16m &= 120000 \end{aligned} \quad \begin{pmatrix} 4 & 7 \\ 5 & 8 \end{pmatrix} \begin{pmatrix} b \\ m \end{pmatrix} = \begin{pmatrix} 49800 \\ 60000 \end{pmatrix}$$

$$\begin{pmatrix} 8 & 14 \\ 10 & 16 \end{pmatrix} \begin{pmatrix} b \\ m \end{pmatrix} = \begin{pmatrix} 99600 \\ 120000 \end{pmatrix}$$

(ii) Using P^{-1} in (a) above, determine the cost of a bag of beans and a bag of maize.

(3 marks)

$$\begin{pmatrix} 4 & 7 \\ 5 & 8 \end{pmatrix} \begin{pmatrix} \frac{8}{3} & \frac{7}{3} \\ \frac{5}{3} & -\frac{4}{3} \end{pmatrix} \begin{pmatrix} b \\ m \end{pmatrix} = \begin{pmatrix} \frac{8}{3} & \frac{7}{3} \\ \frac{5}{3} & -\frac{4}{3} \end{pmatrix} \begin{pmatrix} 49800 \\ 60000 \end{pmatrix}$$

$$\begin{aligned} 1 \text{ bag bean} &= 7200 \\ 1 \text{ bag Maize} &= 3000 \end{aligned}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} b \\ m \end{pmatrix} = \begin{pmatrix} 7200 \\ 3000 \end{pmatrix}$$

(c) The price of beans later went up by 5% and that of maize remained constant. Utaati bought the same quantity of beans but spent the same total amount of money as before on the two items. State the new ratio of beans to maize.

(3 marks)

$$\frac{105}{100} \times 7200 = 7560$$

$$7560(8) + 3000(x) = 99600$$

$$x = 13.04$$

$$60480 + 3000x = 99600$$

$$= 13$$

$$3000x = 39120$$

$$\text{ratio } 8:13$$

20. The table below shows marks obtained by 100 candidates at Mithila Secondary in a mathematics examination.

Marks	15-24	25-34	35-44	45-54	55-64	65-74	75-84
Frequency	6	14	24	14	x	10	10

- (i) Determine the value of x (2 marks)

$$x = 100 - 78 \checkmark M_1$$

$$x = 22 \checkmark A_1$$

- (ii) State the modal class (1 mark)

35-44 B₁

- (iii) Calculate the mean (3 marks)

	f	X	fx	c.f
15-24	6	19.5	117	4
25-34	14	29.5	413	20
35-44	24	39.5	948	44
45-54	14	49.5	693	58
55-64	22	59.5	1309	80
65-74	10	69.5	695	90
75-84	10	79.5	795	100
			$\Sigma fx = 4970$	

$$\bar{X} = \frac{\Sigma fx}{f}$$

$$= \frac{4970}{100} \checkmark M_1$$

$$= 49.7 \checkmark A_1$$

- (iv) Determine the median mark. (4 marks)

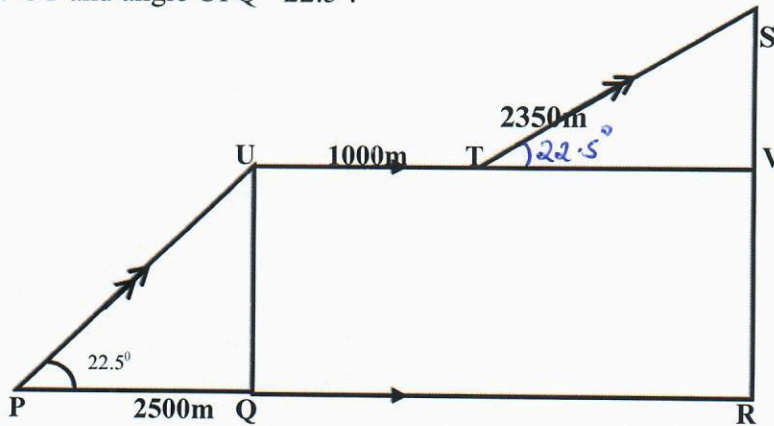
$$\frac{100}{2} = 50^{th} \text{ c.f.} \quad (B_1 - B_2)$$

$$\left(\frac{50 - 44}{14} \times 10 \right) + 44.5 \checkmark M_1$$

$$\frac{6}{14} \times 10 + 44.5 \checkmark M_1$$

$$48.79 \checkmark A_1$$

21. In the figure below $PQ = 2500\text{m}$, $UT = 1000\text{m}$ and $TS = 2350\text{m}$. PQR is a straight line. Parallel to UT and angle $UPQ = 22.5^\circ$.



Calculate to the nearest meter

(a) (i) UQ

$$\tan 22.5^\circ = \frac{UQ}{2500} \quad M_1 \quad \left| \quad UQ = 2500 \tan 22.5^\circ \right. \quad (2\text{marks})$$

$$= 1036\text{m} \quad A_1$$

(ii) TV

$$\sin 22.5^\circ = \frac{TV}{2350} \quad M_1 \quad \left| \quad TV = \sin 22.5^\circ \times 2350 \right. \quad (2\text{marks})$$

$$= 2171\text{m} \quad A_1$$

(iii) VS

$$\cos 22.5^\circ = \frac{VS}{2350} \quad M_1 \quad \left| \quad VS = 2350 \cos 22.5^\circ \right. \quad (2\text{marks})$$

$$= 899\text{m} \quad A_1$$

(iv) PU

$$\cos 22.5^\circ = \frac{2500}{PU} \quad M_1 \quad \left| \quad PU = \frac{2500}{\cos 22.5^\circ} \right. \quad (2\text{marks})$$

$$= 2706\text{m} \quad A_1$$

(b) Find the perimeter of the figure.

$$P = 2706 + 1000 + 2350 + 899 + 1036 + 2171 + 1000 + 2500 \quad M_1 \quad (2\text{marks})$$

$$= 13662\text{m} \quad A_1$$

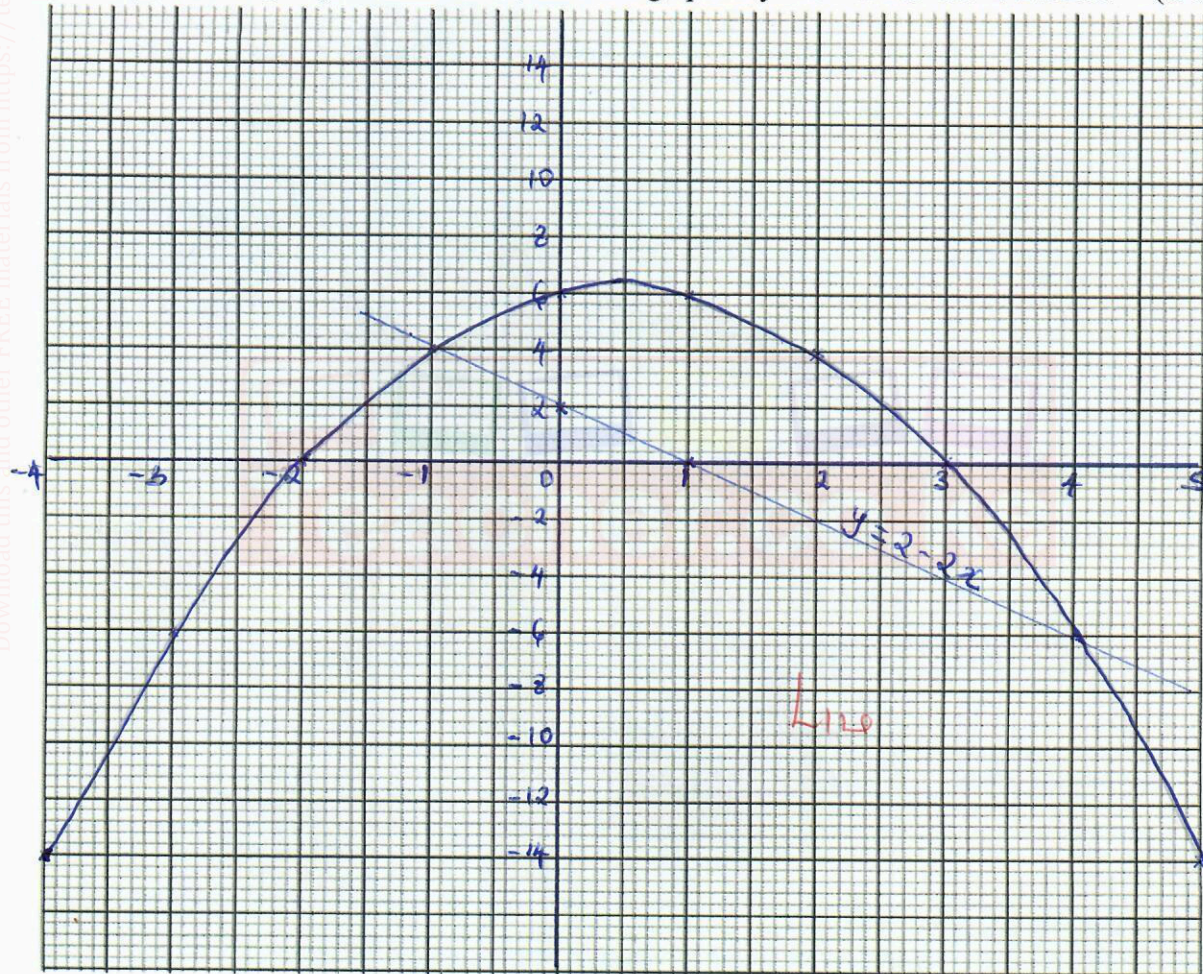
22. (a) Complete the table below for the function $y = 6 + x - x^2$.

(2 marks)

x	-4	-3	-2	-1	0	1	2	3	4	5
y	-14	-6	0	4	6	6	4	0	-6	-14

B_2 All values
If B_1 S values

(b) On the grid provided below, draw the graph of $y = 6 + x - x^2$ for $-4 \leq x \leq 5$. (3 marks)



S_1
 P_1
 C_1

(c) On the same axes draw the graph of $y = 2 - 2x$.

Line P_1

(2 marks)

x	0	1
y	2	0

$(0, 2)$ $(1, 0)$

(d) From the graphs, find the values of x which satisfy the simultaneous equations.

$$y = 6 + x - x^2$$

$$y = 2 - 2x.$$

$$x = -1, 4$$

B_1

(1 marks)

(e) Write down and simplify a quadratic equation in the form $ax^2 + bx + c = 0$ which is satisfied by the values of x where the two graphs intersect.

(2 marks)

$$2 - 2x = 6 + x - x^2 \quad | \quad 6 + x + 2x - x^2 - 2 \quad | \quad -x^2 + 3x + 4 = 0$$

23. (a) In a stadium, the cost per seat during a match is Ksh. 200. The stadium is built in a such a way that the first row has five seats more than the previous one and there are thirty-one rows. If during the match between AFC Leopards and GorMahia the stadium is 75% full, how much money is collected if every fan pays. (4 marks)

No of seats

$$5 + 30 \times 5 \checkmark M_1$$

$$= 155 \checkmark A_1$$

$$\frac{75}{100} \times 155 \times 200 \checkmark M_1$$

$$= 23,250 \checkmark A_1$$

- (b) The 3rd term of a geometric sequence is 20 and 6th term is -160.

Calculate:

- (i) The common ratio

(2 marks)

$$ar^2 = 20$$

$$a = \frac{20}{r^2}$$

$$20r^3 = -160$$

$$r^3 = -8$$

$$ar^5 = -160$$

$$\frac{20}{r^2}(r^5) = -160 \checkmark M_1$$

$$r^3 = \frac{-160}{20}$$

$$r = \sqrt[3]{-8}$$

$$= -2 \checkmark A_1$$

- (ii) The first term

(2 marks)

$$a(-2^2) = 20 \checkmark M_1$$

$$a(4) = 20$$

$$a = 5 \checkmark A_1$$

- (iii) The 8th term

(2 marks)

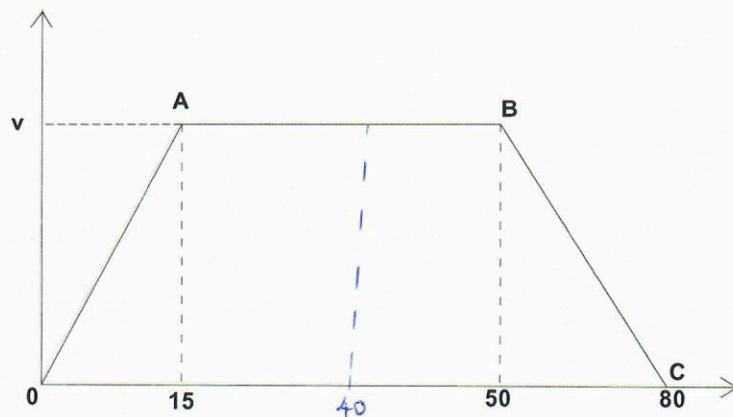
$$ar^7$$

$$= 5(-2^7) \checkmark M_1$$

$$= 5(-128)$$

$$= -640 \checkmark A_1$$

24. The figure below is the speed – time graph of a journey



If the total distance travelled in the 80 seconds is 920m, calculate:

a) The value of V

(4 marks)

$$\left(\frac{1}{2} \times 15 \times v\right) + (35 \times v) + \frac{1}{2} \times 30 \times v = 920$$

$$\frac{15v}{2} + \frac{35v}{1} + \frac{15v}{1} = 920$$

$$\frac{15v + 70v + 30v}{2} = 920$$

$$115v = 1840$$

$$v = \frac{1840}{115}$$

$$v = 16 \text{ m/s}$$

b) The acceleration during the first 15 seconds

(3 marks)

$$a = \frac{16 - 0}{15}$$

$$= 1.067 \text{ m/s}^2$$

c) The distance travelled in the final 40 seconds.

(3 marks)

$$s = (L \times W) + \frac{1}{2} \times 30 \times 16$$

$$= (10 \times 16) + \frac{1}{2} \times 30 \times 16$$

$$160 + 240$$

$$400 \text{ m}$$