MARKING GUIDE FOR EXAMINERS

121/1
MATHEMATICS

OPENER EXAMINATION TERM TWO 2024

FORM THREE MATHEMATICS

INSTRUCTIONS TO CANDIDATES

- Write your name and Admission number in the spaces provided at the top of this page.
- This paper consists of two sections: Section A and Section B.
- Answer ALL questions in section A and ONLY FIVE questions from section B.
- All answers and workings must be written on the question paper in the spaces provided below each question.
- Show all the steps in your calculation, giving your answer at each stage in the spaces below each question.
- Non – Programmable silent electronic calculators and KNEC mathematical tables may be used, except where stated otherwise.

FOR EXAMINERS USE ONLY

SECTION A

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

SECTION B

17	18	19	20	21	22	23	24	TOTAL

GRAND TOTAL

This paper consists of 14 printed pages. Candidates should check the question paper to ascertain that all pages are printed as indicated and that no pages are missing.

SECTION A (50 MARKS) (*Answer all questions in this section*)

1. Evaluate: $\dfrac{\frac{1}{2}+\frac{1}{4}\div\frac{1}{2}\text{ of }\frac{1}{3}}{\frac{1}{2}\text{ of }\frac{4}{5}-\frac{3}{4}+\frac{1}{2}}$ (3 marks)

$\frac{1}{2}+\frac{1}{4}\times\frac{6}{1}$

$\frac{1}{2}+\frac{3}{2}=2$ ✓

$\frac{2}{5}-\frac{3}{4}+\frac{1}{2}=\frac{3}{20}$ ✓

$2\div\frac{3}{20}$

$2\times\frac{20}{3}$

$=13\frac{1}{3}$ ✓

2. Use square and square root tables to evaluate: $(61.70)^2+\sqrt{0.007932}$ (4 marks)

$(6.170\times10^{1})^2$

$=38.07\times10^{2}$ or 3.807×10^{3} ✓

$\sqrt{79.32\times10^{-4}}$

$=8.9062\times10^{-2}$ ✓

$=3\,807+0.089062$ ✓

$=3\,807.089062$ ✓

3. Solve for the value of x: $9^{4x-1}\times3^{2x+1}=2\,187$ (3 marks)

$3^{8x-2}\times3^{2x+1}=3^{7}$ ✓

$3^{10x-1}=3^{7}$

$3^{10x}=3^{8}$ ✓

$10x=8$

$x=0.8$ or $\frac{4}{5}$ ✓

4. A certain material of weight **12 N** was used to make a sphere of radius 3.2 cm. Calculate the density of the material used in kgm^{-3}. (3 marks)

$\text{Volume}=\frac{4}{3}\pi r^3$

$=\frac{4}{7}\times\frac{22}{7}\times(0.032)^3$

$=1.373\times10^{-4}\,m^3$ ✓

$\text{mass}=\frac{12}{10}$

$=1.2\,kg$

$\text{Density}=\frac{1.2}{1.373\times10^{-4}}$ ✓

$=8\,739.96\,kgm^{3}$ ✓

5. Given that $Sin\ \theta = \frac{3}{5}$ and **θ** is an acute angle, calculate the value of $\tan\theta + \cos\theta$. (3 marks)

$\text{opp} = 3 \qquad \text{hyp} = 5$

$\text{adj} = \sqrt{5^2 - 3^2} = 4$ ✓

$tan\ \theta = \frac{3}{4} \qquad cos\ \theta = \frac{4}{5}$

$= \frac{3}{4} + \frac{4}{5}$ ✓

$= 1\frac{11}{20}$ ✓

6. A map has a scale of **1:25000** on this map; a square piece of land is represented by an area of **2cm²**. Calculate the actual area, in hectares of the plot. (3 marks)

$ASF = \left(\frac{1}{250}\right)^2$

$= \frac{1}{62\,500}$ ✓

$\text{Area} = 2 \times 62\,500 = 125\,000\ m^2$ ✓

$= \frac{125\,000}{10\,000}$

$= 12.5\ \text{ha}$ ✓

7. The size of an interior angle of a regular polygon is $3x°$ while exterior is $(x - 60)°$. Find the number of sides of the polygon. (3 marks)

$4x - 60 = 180$ ✓

$4x = 240$

$x = 60°$ ✓

$n = \frac{360}{60}$

$= 6\ \text{sides}$ ✓

8. Mr Onyango's piece of land is in a form of triangle whose dimensions are 1200m, 1800m and 1500m respectively. Find the largest angle between two sides of this land. (Give your answer to the nearest degree). (3 marks)

Largest angle is between two shorter sides.

$a = 1800, b = 1200, c = 1500$

$a^2 = b^2 + c^2 - 2cbCos\ A$

$1800^2 = 1200^2 + 1500^2 - 2 \times 1200 \times 1500 \times Cos\ A$ ✓

$Cos\ A = 0.125$ ✓

$A = 82.82°$ ✓

9. Simplify the expression $\frac{x^2 + 3x + 2}{x^2 - 1}$ (3 marks)

$\frac{(x+2)(x+1)}{(x+1)(x-1)}$ ✓ ✓

$= \frac{x+2}{x-1}$ ✓

10. Eight years ago the age of a father was six times the age of his son and after eight years from today the age of the father would be only twice the age of his son. Find their present ages. (3 marks)

8 years ago

$S = x$

$F = 6x$

In 8 years time

$6x + 16 = 2x + 32$ ✓

$4x = 16$

$x = 4$

Son = 12 ✓

Father = 32 ✓

Alt: let present ages be $F = x$ and $S = y$

$(x - 8) = 6(y - 8)$

$x - 6y = -40$ ..... ① ✓

$(x + 8) = 2(y + 8)$

$x - 2y = 8$ ..... ② ✓

subtracting ② from ①

$-4y = -48$

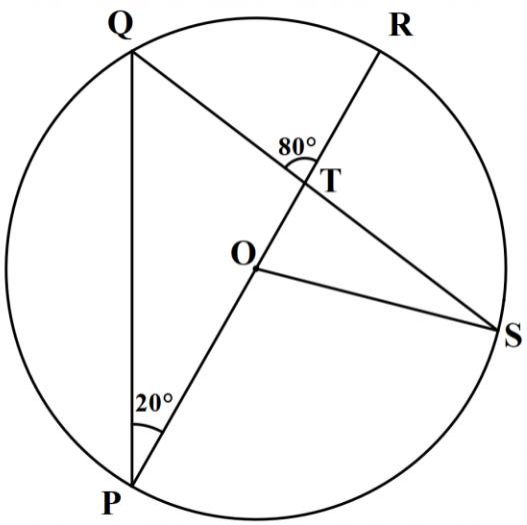
$y = 12$

$x = 8 + 2(12) = 32$

Son = 12 years

Father = 32 years ✓

11. In the figure below PQR is the diameter of the circle centre O. Angle QPR = 20° and angle QTR = 80°.



Determine the size of:

a. Reflex angle POS (2 marks)

$= 360 - 2(180 - 20 - 100)$ ✓

$= 240°$ ✓

b. Angle OSQ (1 mark)

$120 - 80 = 40°$ ✓

or

$180 - (60 + 80) = 40°$ ✓

12. A Kenyan bank buys and sells foreign currencies at the exchange rates shown below.

Currency	Buying (Ksh)	Selling (Ksh)
1 Euro	147.56	148.00
1 US dollar	94.22	94.50

A tourist arrived in Kenya with 11,155 Euros. He converted all the Euros to Kenya shillings at the bank. He spent Ksh. 1,130,200.50 while in Kenya and converted the remaining Kenya shillings into US dollars at the bank. Find the amount in dollars that he received correct to 2 decimal places. (4 marks)

$11\,155 \times 147.46$

$= 1\,646\,031.80$ ✓

$1\,646\,031.80$

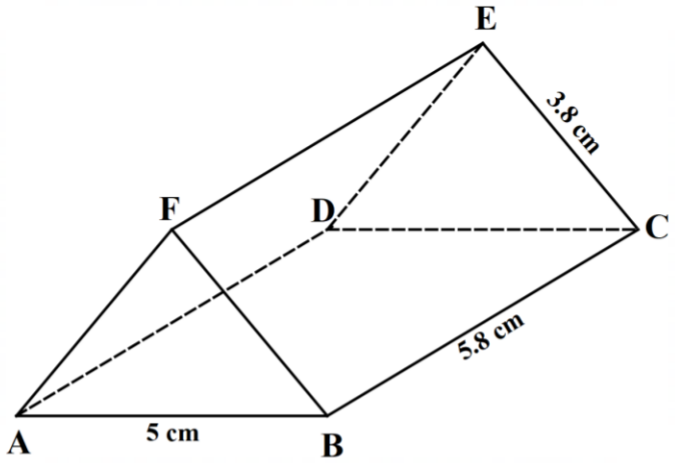
$-1\,130\,200.50$

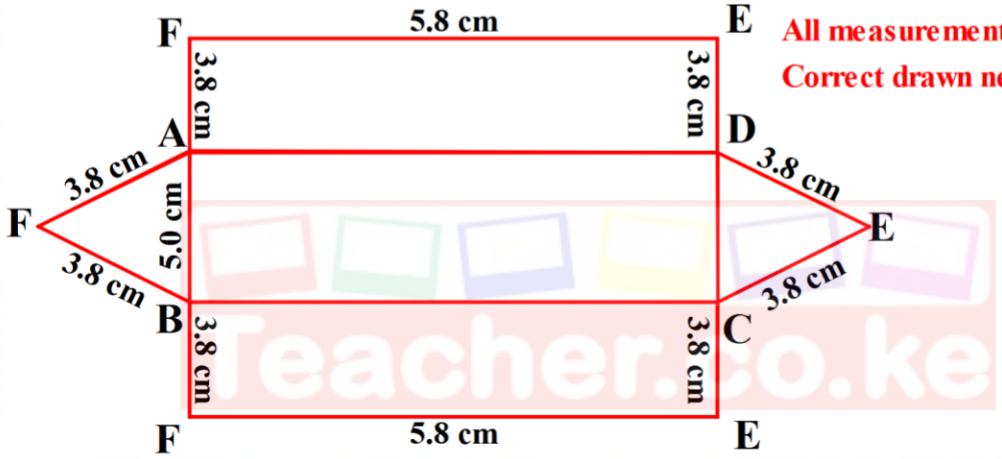
$515\,831.30$ ✓

$\frac{515\,831.30}{94.50}$ ✓

$= 5\,458.53$ *US Dollars* ✓

13. Draw a net of the solid below. (3 marks)





All vertices labelled ✓
All measurements labelled ✓
Correct drawn net ✓

14. Two men each working for 8 hours a day can cultivate an acre of land in 4 days. How long would 6 men, each working 4 hours a day take to cultivate 4 acres? (3 marks)

M	R	D	L
2	8	4	1
6	4	?	4 ✓

$= 4 \times \frac{2}{6} \times \frac{8}{4} \times \frac{4}{1}$ ✓

$= 10\frac{2}{3}$ ✓

15. Given the column vectors. $a = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, b = \begin{pmatrix} -3 \\ 9 \end{pmatrix}$ and $c = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$ and that $p = a + \frac{1}{3}b - c$ express p as a column vector and hence calculate its magnitude to 3 significant figures. (3 marks)

$$p = \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} -3 \\ 9 \end{pmatrix} - \begin{pmatrix} 3 \\ -3 \end{pmatrix} \checkmark$$

$$= \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} -3 \\ 4 \end{pmatrix} \checkmark$$

$$= \sqrt{(-3)^2 + (4)^2}$$

$$= 5 \text{ units} \checkmark$$

16. Gloria, the school time keeper of Anestar Victory Girls High School, noted that the school electric bell auto timer gains 2 seconds every one hour. On Monday of week five the auto timer was correctly set at 8:00 am. What will be the reading of auto timer on Friday of week six at 8:00 am? (3 marks)

$$\text{gained time} = \frac{11 \times 24 \times 2}{60} \checkmark$$

$$= 8.8 \text{ minutes}$$

$$= 8min \text{ and } 48s \checkmark$$

$$08.00.00 + 00.08.48$$

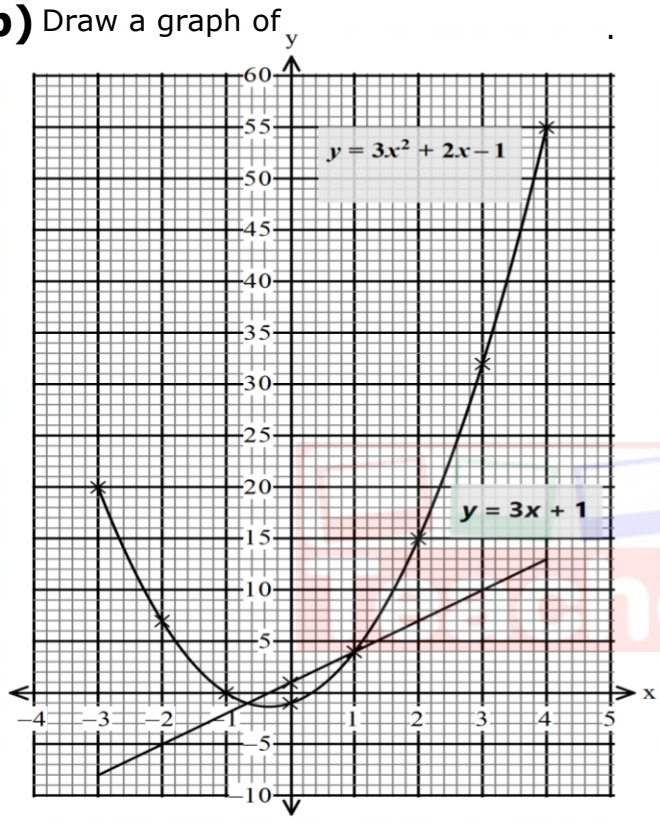
$$= 8.08.48 \text{ am} \checkmark$$

SECTION B (50 MARKS) (*Answer only five questions in this section.*)

17. a) Complete the table below for the function $y = 3x^2 + 2x - 1$ for $-3 \leq x \leq 4$. (2 marks)

x	-3	-2	-1	0	1	2	3	4
$y = 3x^2 + 2x - 1$	20	7	0	-1	4	15	32	55

b) Draw a graph of $y = 3x^2 + 2x - 1$. (3 marks)



$P = 1$ ✓
$S = 1$ ✓
$C = 1$ ✓
$L = 1$ ✓

c) Draw the line $y = 3x + 1$ on the same axis hence find the values of x for which $y = 3x + 1$ and $y = 3x^2 + 2x - 1$ are equal. (3 marks)

$x = -0.7$ or 1 ± 0.1 ✓

d) Write down the simplified quadratic equation whose roots are the solutions of the simultaneous equation in (c) above. (2 marks)

$3x^2 + 2x - 1 = 3x + 1$ ✓
$3x^2 - x - 2 = 0$ ✓

18. Kisumu and Nakuru are two towns 320 kilometres apart. A bus left Kisumu at 8.00 am travelling at 60km/h for Nakuru. After forty minutes, a saloon car left Kisumu travelling in the same direction as the bus at a speed of 80km/h.

a. How far from Nakuru did the saloon car catch up with the bus? (4 marks)

distance between the vehicles: $\frac{2}{3} \times 60 = 40\ km$ ✓

time to overtake the bus: $= \frac{40}{80 - 60} = 2\ \text{hrs}$ ✓

distance from Nakuru $= 320 - (80 \times 2)$ ✓

$= 160\ km$ ✓

b. At what time did it catch up with the bus? (2 marks)

$8.40\ \text{am} + 2\ \text{hrs}$ ✓

$= 10.40\ \text{am}$ ✓

c. When the saloon caught up with the bus it got a break – down and had to be repaired before proceeding to Nakuru at the same speed. If they both reached at Nakuru at the same time, find how long it took to repair the saloon? (4 marks)

let the time be t

distance already covered by bus: $= 60t\ km$ ✓

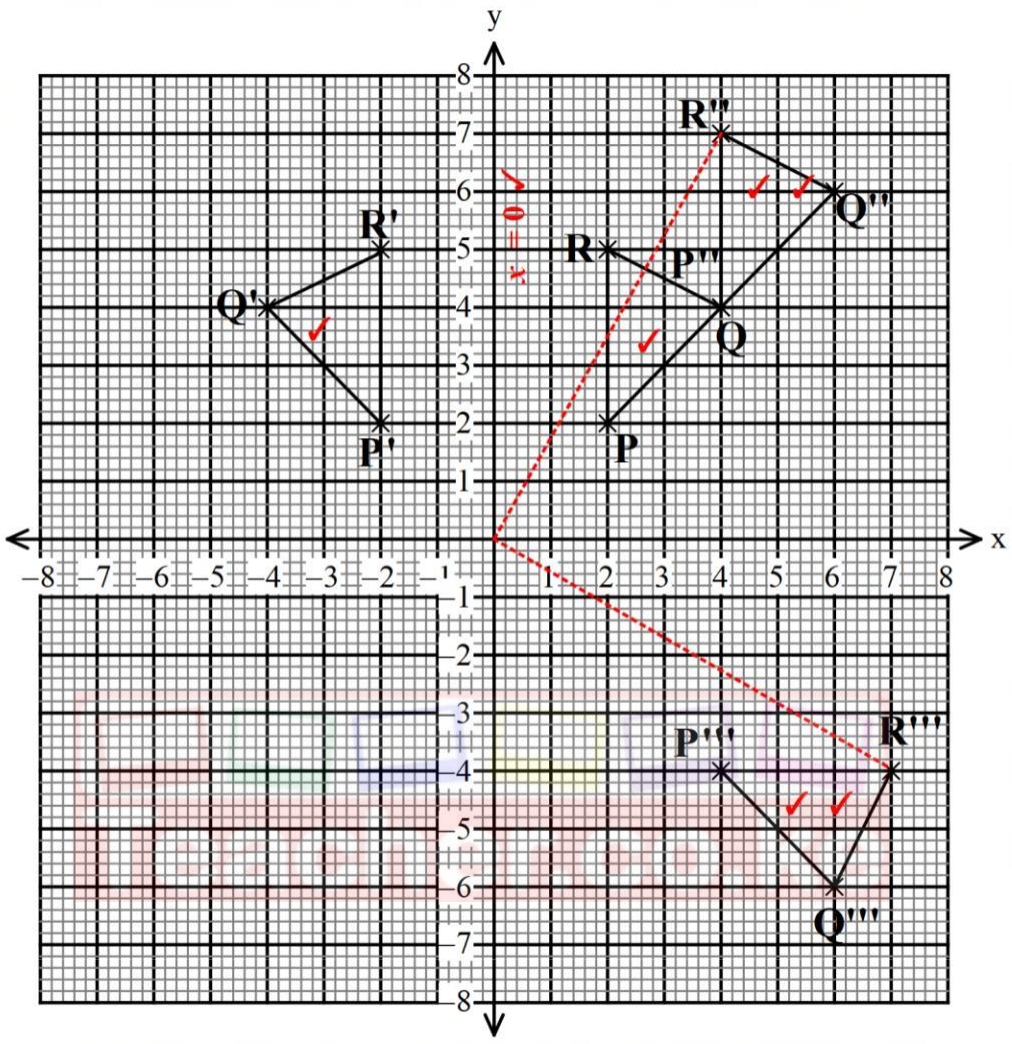
$\frac{160}{80} = \frac{160 - 60t}{60}$ ✓

$60t = 40$ ✓

$t = \frac{2}{3}\ \text{hrs}$

$t = 40\ \text{minutes}$ ✓

19. a) On the graph paper provided plot the point **P (2,2)**, **Q (2,5)** and **R (4,4)**. Join them to form a triangle **PQR**. (1 mark)



b) Reflect the triangle **PQR** in the line $x = 0$ and label the image as $P'Q'R'$. (2 marks)

c) Triangle PQR is given a translation by vector. $T\begin{pmatrix}2\\2\end{pmatrix}$ to $P''Q''R''$. Plot the triangle $P''Q''R''$. (3 marks)

$P''(4,4)\ ,\ Q''(4,7)\ ,\ R''(6,6)$ ✓

d) Rotate triangle $P''Q''R''$ about the origin through **-90°**. State the coordinates of $P'''Q'''R'''$. (3 marks)

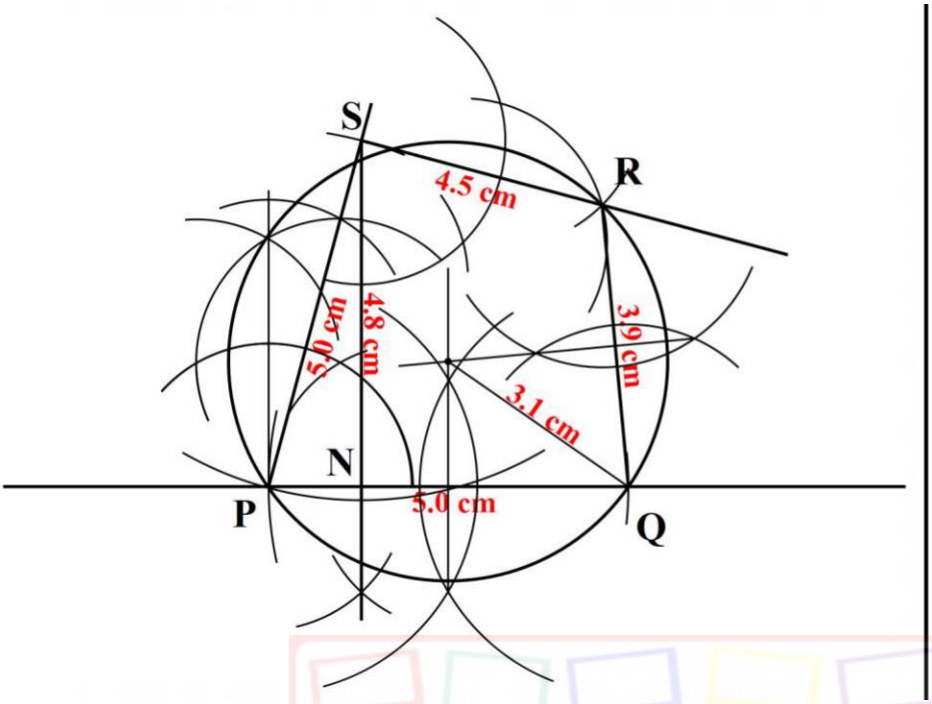
$P'''(4,-3)\ ,\ Q'''(6,-6)\ ,\ R'''(7,-4)$ ✓

e) Identify two pair of triangles that are direct congruence. (1 mark)

ΔPQR and $\Delta P''Q''R''$, $\Delta P''Q''R''$ and $\Delta P'''Q'''R'''$ ✓

20. Use a ruler and a pair of compasses only for all construction in this question.

a) Construct quadrilateral PQRS such that PQ = 5cm, PS = 5cm and SR = 4.5cm, angle SPQ = 75° and angle PSR = 90°. (4 marks)



Construction of 75° ✓
locating point S ✓
Construction of 90° ✓
Shape of $PQRS$ ✓
Correct line NS ✓
Location of circumcetre ✓
Correct drawn circle ✓
Radius ✓

b) Drop a perpendicular from S to meet line PQ at N. Measure SN and calculate the area of the triangle SPN. (3 marks)

$$= 90 - 75 = 15°$$

$$= \frac{1}{2} \times 5 \times 4.8 \times sin\ 15 \checkmark$$

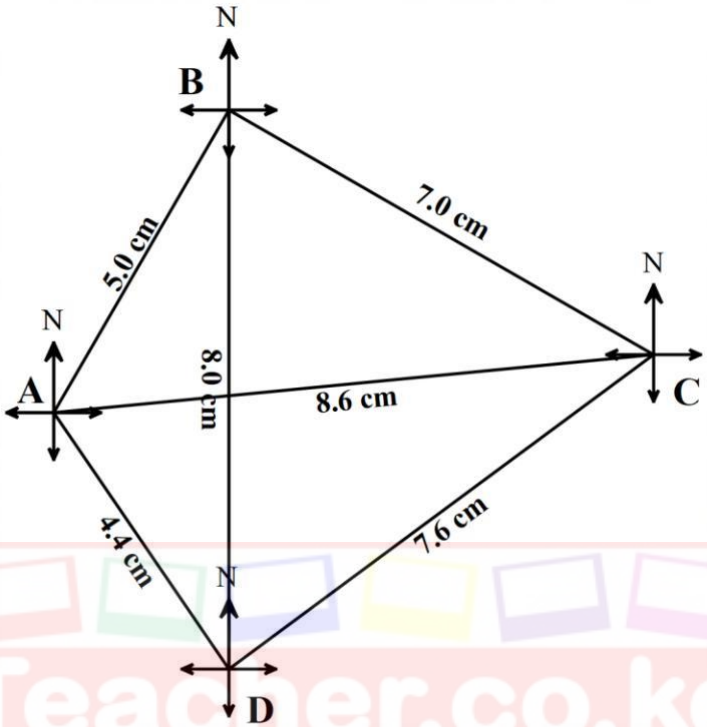
$$= 3.106\ cm^2 \checkmark$$

c) Construct a circle passing through vertices P, Q and R of quadrilateral PQRS. Measure the radius of the circle. (3 marks)

$$\text{Radius} = 3.1\ cm \checkmark$$

21. Three warships A, B and C are at sea such that ship B is 500km on a bearing 030° from ship A. Ship C is 700km from ship B on a bearing of 120°. An enemy ship D is sighted 800km due south of ship B.

a) Taking a scale of 1cm to represent 100km, locate the positions of ships A, B, C and D. (4 marks)



b) Find the bearing of:

Ship A from D. (1 mark)

$326° \pm 1°$ ✓

Ship D from C. (1 mark)

$233° \pm 1°$ ✓

c) Use scale drawing to determine the distance between D and A. (1 mark)

$4.4 \times 100 = 440 \pm 10\ km$ ✓

C and D. (1 mark)

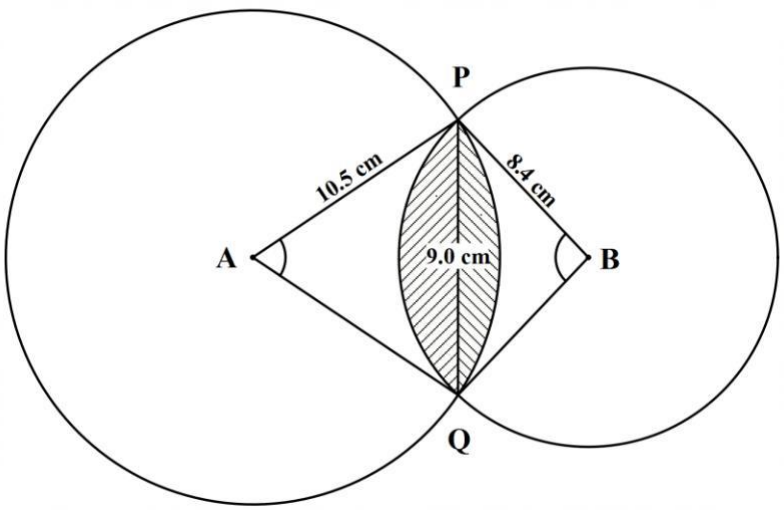
$7.6 \times 100 = 760 \pm 10\ km$ ✓

d) Measure angle DAC and angle BCD. (2 marks)

$\angle DAC = 62° \pm 1°$ ✓

$\angle BCD = 67° \pm 1°$ ✓

22. The figure below shows two circles of radii 10.5cm and 8.4cm and with centres A and B respectively. The common chord PQ is 9cm.



a) Calculate angle PAQ. (2 marks)

$$= 2 \times Sin^{-1}\left(\frac{4.5}{10.5}\right) \checkmark$$

$$= 50.75° \checkmark$$

b) Calculate angle PBQ. (2 marks)

$$= 2 \times Sin^{-1}\left(\frac{4.5}{8.4}\right) \checkmark$$

$$= 64.78° \checkmark$$

c) Calculate the area of the shaded part. (6 marks)

$$= \frac{50.75}{360} \times \frac{22}{7} \times (10.5)^2 - \frac{1}{2} \times (10.5)^2 \times Sin\,(50.75) \checkmark$$

$$= 6.158\ cm^2 \checkmark$$

$$= \frac{64.78}{360} \times \frac{22}{7} \times (8.4)^2 - \frac{1}{2} \times (8.4)^2 \times Sin\,(64.78) \checkmark$$

$$= 7.987\ cm^2 \checkmark$$

$$= 6.158 + 7.987 \checkmark$$

$$= 14.145\ cm^2 \checkmark$$

23. In a botanical experiment, the length of 60 leaves of a certain type of a tree were measured correct to the nearest 0.1cm.

Length (cm)	3.0 - 3.4	3.5 - 3.9	4.0 - 4.4	4.5 - 4.9	5.0 - 5.4	5.5 - 5.9	6.0 - 6.4	6.5 - 6.9	7.0 - 7.4
No: of leaves	1	4	9	14	12	10	6	3	1

a) State the:

I. modal frequency (1 mark)

= 14 ✓

II. modal class. (1 mark)

4.5 – 4.9 ✓

b) Calculate the:

I. mean length (4 marks)

CLASS	f	x	fx	Cf
3.0-3.4	1	3.2	3.2	1
3.5-3.9	4	3.7	14.8	5
4.0-4.4	9	4.2	37.8	14
4.5-4.9	14	4.7	65.8	28
5.0-5.4	12	5.2	62.4	40
5.5-5.9	10	5.7	57	50
6.0-6.4	6	6.2	37.2	56
6.5-6.9	3	6.7	20.1	59
7.0-7.4	1	7.2	7.2	60
	Σf=60		Σfx = 305.5	

$$\overline{X} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{305.5}{60} \checkmark$$

$$= 5.092\ cm \checkmark$$

correct x column ✓

correct fx column ✓

II. median length. (4 marks)

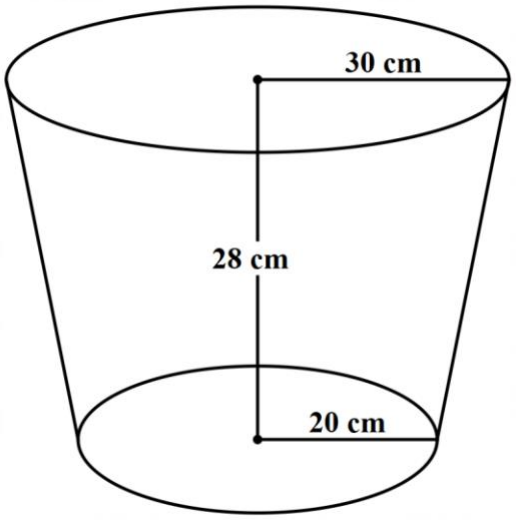
correct cf column ✓

$$= L + \left(\frac{\frac{N}{2} - C}{f}\right) \times i$$

$$= 4.95 + \frac{30 - 28}{12} \times 0.5 \checkmark \checkmark$$

$$= 5.033 \checkmark$$

24. The diagram below shows a bucket with top diameter 30cm and bottom diameter 20cm. The height of the bucket is 28cm.



Find;

a) The height of the solid cone from which the bucket was made. (2 marks)

$$\frac{x+28}{x} = \frac{30}{20} \checkmark$$

$$10x = 560$$

$$x = 56$$

$$\text{Height} = 56 + 28 = 84cm \checkmark$$

b) The volume of the bucket. (3 marks)

$$V = \frac{1}{3}\pi \times A \times h$$

$$= \frac{1}{3} \times \frac{22}{7} \times (30^2 \times 84 - 20^2 \times 56) \checkmark \checkmark$$

$$= 55\,733.33 \checkmark$$

c) The surface area of the bucket. (5 marks)

Note: the bucket is open at the top

$$L = \sqrt{30^2 + 84^2} = 89.196\ cm \checkmark$$

$$l = \sqrt{20^2 + 56^2} = 59.464\ cm \checkmark$$

$$S.A = \pi(r^2 + RL - rl)$$

$$S.A = \frac{22}{7} \times (20^2 + 30 \times 89.196 - 20 \times 59.464) \checkmark \checkmark$$

$$= 5\,929.31\ cm \checkmark$$