**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTURIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS**

**1st YEAR 1st SEMESTER 2016/2017 ACADEMIC YEAR**

**MAIN REGULAR**

**COURSE CODE: 801**

**COURSE TITLE: ABSTRACT INTEGRATION I**

**EXAM VENUE: STREAM: (Msc. Pure Mathematics)**

DATE: EXAM SESSION: ONE

TIME:

**Instructions:**

1. **Answer any THREE questions only**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTION ONE [20 MARKS]**

(a). Describe the terms: Vitali’s Cover, Outer measure, Upper Riemann integral and Measurable

 function. (8 marks)

(b). State and prove Vitali’s Covering Theorem. (6 marks)

(b). Show that any non-degenerate interval of **R** is uncountable. (6 marks)

**QUESTION TWO [20 MARKS]**

 (a). Prove that a measure is countably additive. (7 marks)

 (b). State and prove Fatou’s lemma. Moreover, describe its consequences. (7 marks)

 (c). State and prove the complex form of Lebesgue’s Dominated Convergence Theorem.

 (6 marks)

**QUESTION THREE [20 MARKS]**

(a). Deﬁne the following terms giving relevant examples.

 (i). Sigma-finite Complete measure space. (4 marks)

 (ii). Counting measure. (4 marks)

(b). Show that the length of an interval is equal to its outer measure. (12 marks)

**QUESTION FOUR [20 MARKS]**

State and prove the following theorems and hence give their applications in other fields:

(a). Monotone Convergence Theorem. (10 marks)

(b). Cantor’s intersection Theorem. (10 marks)

**QUESTION FIVE [20 MARKS]**

(a) Describe Young’s inequality and show its applications in integration theory. (7 marks)

(b). Show that outer measure is monotone. (6 marks)

(c). Describe the relevance of integration theory to other fields of mathematics. (7 marks)