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**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTURIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS**

**1st YEAR 2nd SEMESTER 2016/2017 ACADEMIC YEAR**

**MAIN REGULAR**

**COURSE CODE: SMA 820**

**COURSE TITLE: OPERATOR THEORY I**

**EXAM VENUE: LR 1 STREAM: (Msc. Pure Mathematics)**

DATE: 21/04/17 EXAM SESSION: 11.30 – 2.30PM

TIME: 3.00 HOURS

**Instructions:**

1. **Answer any THREE questions only**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTION ONE [20 MARKS]**

1. Define: A normal operator, Hyponormal operator and Hermitian operator. (6 marks)
2. State and prove the spectral theorem for compact self-adjoint operator. (14 marks)

**QUESTION TWO [20 MARKS]**

1. Define the terms: Non-invariant subspace and reducing subspace. (6 marks)
2. Describe orthogonal direct sum and product of subspaces. (4 marks)
3. Show that an operator is self-adjointiff(10 marks)

**QUESTION THREE [20 MARKS]**

1. Differentiate between projection and idempotent giving two example s in each case in Hilbert spaces. (4 marks)
2. Let P be a nonzero projection. Prove that if P is positive it is an idempotent. (8 marks)
3. Prove that an operator is normal if and only if it is self-adjoint. (8 marks)

**QUESTION FOUR [20 MARKS]**

1. Define: left shift operator, adjoint of an operator and contractive operator. (6 marks)
2. Let *H* and *K* be Hilbert spaces and let be bounded operator. Prove that there exist which is bounded and linear. (14 marks)

**QUESTION FIVE [20 MARKS]**

1. Describe the process of diagonalization of self-adjointcomplex matrix operators. Give three example to show how diagonalization is carried out. (6 marks)
2. Let be compact and be an eigenvalue of *A*. Prove that Ker() is finite dimensional. (14 marks)