

W1-2-60-1-6

**JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY**

**UNIVERSITY EXAMINATIONS 2013/2014**

**SEMESTER II EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED GEOPHYSICS**

**SPH 3104: MATHEMATICAL PHYSICS**

**DATE: DECEMBER 2013 TIME: 3 HOURS**

**INSTRUCTIONS: ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS.**

The Laplacian operator in the polar spherical coordinate system is given by:



**QUESTION ONE**

1(a) The function f(x) of a complex variable ~~z~~ is given as f(z) =

Determine its singularities and their order. [3 marks]

(b)(i) Prove that the integral of a function of a complex variable can be made to depend on

line integrals for real functions. [3 marks]

(ii) Evaluate the complex integral

 along the parabola x=t, y=t2 where 1<t<2 [5 marks]

(c)(i) State and prove Cauchy’s theorem [6 marks]

(ii) Given that v= F(y-3x) is a general solution of the equation:



Determine the particular solution which satisfies the boundary condition

V(0,y) = 4 siny [4 marks]

(d) Plot graph of the following periodic function:

f(x) = 3 0<x<5

-3 -5<x<0 period = 10 [2 marks]

(e)(i) Given that the generating function of Hermite polynomials is;

 Show that Hn’ = 2nHn-1(x) [3 marks]

(ii) Hence determine H1(x) and H2(x) [4 marks]

**QUESTION TWO**

1. Consider Bessel’s differential equation

x2y”+xy’+(x2-n2)y = 0 n>0

(i) Write its general solution [2 marks]

(ii) Use the power series method to determine its solution. [10 marks]

(b)(i) A function of a complex function f(~~z~~) is given as f(~~z~~)= ~~z~~3 + ~~z~~ + 1

Separate it into the real and complex parts u(x,y) and v(x,y) respectively. [3 marks]

(ii) State the Cauchy – Riemann conditions. [2 marks]

(iii) Use the Caunchy – Riemmann conditions to show that the functions u(x,y) and v(x,y)

are harmonic. [3 marks]

**QUESTION THREE**

1. Find the solutions to Laplace’s equation;



In spherical coordinates if is independent of  [10 marks]

1. A drum consists of a stretched circular membrane of unit radius (fixed). If the membrane is struck so that its initial displacement is *F*() and is then released. Find the displacement at any time given that the wave equation for a general displacement z(,t) is

 [10 marks]

**QUESTION FOUR**

(a)(i) Find the fourier coefficients that correspond to the function:

 Period = 10 [10 marks]

(ii) Hence write the corresponding fourier series [4 marks]

(iii) How should *f(x)* to be defined at x= -5, x=0 and x=5

In order that the fourier series will converge to *f(x)* for -5<x<5? [3 marks]

(b) Determine the residue of the complex function;

*f(z)=*  at the pole ~~z~~ = *i* [3 marks]