2. Trigonometric ratios 3

<table>
<thead>
<tr>
<th>X°</th>
<th>0°</th>
<th>30°</th>
<th>60°</th>
<th>90°</th>
<th>120°</th>
<th>150°</th>
<th>180°</th>
<th>210°</th>
<th>240°</th>
<th>270°</th>
<th>300°</th>
<th>330°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cos x</td>
<td>1.00</td>
<td>0.87</td>
<td>0.50</td>
<td>0</td>
<td>-0.5</td>
<td>-0.87</td>
<td>-1</td>
<td>-0.87</td>
<td>-0.5</td>
<td>0.5</td>
<td>0.7</td>
<td>1</td>
</tr>
<tr>
<td>2cos ½ x</td>
<td>2.00</td>
<td>1.93</td>
<td>1.73</td>
<td>1.41</td>
<td>1</td>
<td>0.52</td>
<td>0.00</td>
<td>-0.52</td>
<td>-1</td>
<td>-1.73</td>
<td>-1.93</td>
<td>-2.00</td>
</tr>
</tbody>
</table>

- (a) amplitude = 2 B1
- period = 720° B1
- (b) 2cos ½ x = cos x
  
  \[ X = 222° \pm 6° \]
1.  a) 

<table>
<thead>
<tr>
<th>X°</th>
<th>-225</th>
<th>-180</th>
<th>-135</th>
<th>-90</th>
<th>-45</th>
<th>0</th>
<th>45</th>
<th>90</th>
<th>135</th>
<th>180</th>
<th>225</th>
</tr>
</thead>
<tbody>
<tr>
<td>y = sin 2x</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>y = 2cos x</td>
<td>-2.0</td>
<td>0</td>
<td>1.4</td>
<td>1.4</td>
<td>0</td>
<td>-2.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

b) 

(y = sin 2x)

(d) (i) Highest point 1 unit
Lowest point  - 1.4

(c) -90° or 90°

2.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>150</th>
<th>180</th>
<th>210</th>
</tr>
</thead>
<tbody>
<tr>
<td>2sin(x+15°)</td>
<td>0.52</td>
<td>1.41</td>
<td>1.93</td>
<td>1.93</td>
<td>1.41</td>
<td>0.52</td>
<td>-0.52</td>
<td>-1.41</td>
</tr>
<tr>
<td>Cos(2x -30°)</td>
<td>0.87</td>
<td>0.87</td>
<td>0</td>
<td>-0.87</td>
<td>0.87</td>
<td>0</td>
<td>0.87</td>
<td>0.87</td>
</tr>
</tbody>
</table>

(i) Amplitudes: y = 2 sin ( x + 15)
= 2 units
y = cos (2x -30°)
= 1 unit

12°, 159°
3. Determine the
i) Altitude of the frustrum
Solution
\[ A^1C^1 = \sqrt{4^2 + 4^2} = \sqrt{32} \]
\[ AC = \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2} \]
\[ AM + XM = 10\sqrt{2} - 4\sqrt{2} = 6\sqrt{2} \]
\[ AM = \frac{6\sqrt{2}}{2} = 3\sqrt{2} \]
\[ \text{Height} = AM = \sqrt{5^2 - (3\sqrt{2})^2} = \sqrt{25 - 18} = \sqrt{7} = 2.646 \]
\[ \therefore \text{the altitude of the frustrum} = 2.646 \text{ cm} \]

ii) Angle between \( AC \) and the base
\[ AX = 3\sqrt{2} + 4\sqrt{2} = 7\sqrt{2} \]
\[ \tan \theta = \frac{CX}{AX} = \frac{\sqrt{7}}{7\sqrt{2}} = \frac{1}{\sqrt{2}} \]
\[
\theta = \tan^{-1} 0.2673 = 14.96^\circ
\]

iii) Volume of pyramid = \( \frac{1}{3} bh \)

\[AC = 10 \sqrt{2}\]
\[A_1C_1 = 4 \sqrt{2}\]
\[L.S.F = 10:4\]
\[\therefore h + 2.646 = \frac{10}{4A}\]
\[4(h + 2.646) = 10h\]
\[4h + 10.584 = 10h\]
\[6h = 10.584\]
\[h = 1.764\]
\[H = h + 2.646\]
\[= 1.764 + 2.646 = 4.410\]
\[V_f = \left(\frac{1}{3} \times 10 \times 10 \times 4.41\right) - \left(\frac{1}{3} \times 4 \times 4 \times 1.76\right)\]
\[= \frac{4410}{3} - \frac{28.224}{3}\]
\[= 137.592 cm^3\]

4. (a) table completed
   (b) (c) (i) 3 P1 - plotting
            S1 - scale
            C1 - smooth curve
   (ii) 180°
   (iii) Line y = 1 drawn
          \( x = 4.5^\circ \) or 72.8° - 107.2° - 175.4°

5. \( \left(\frac{A}{B}\right)^2 = p + 33q \)
   \[q - 3P\]
   \[A^2q - 3A^2P = BP + 3Bq\]
   \[Aq - 3Bq = BP + 3A^2P\]
   \[2(A^2 - 3B) = BP + 3A^2P\]
   \[Q = BP + 3A^2P\]
   \[\frac{A^2 - 3B}{A^2 - 3B} \]
8. a)  

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>150</th>
<th>180</th>
<th>210</th>
<th>240</th>
<th>270</th>
<th>300</th>
<th>330</th>
<th>360</th>
</tr>
</thead>
<tbody>
<tr>
<td>3sinx</td>
<td>1.5</td>
<td>2.6</td>
<td>1.5</td>
<td>-2.6</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2cosx</td>
<td>2</td>
<td>0</td>
<td>-1.0</td>
<td>-1.7</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) (i) Amplitude = 3
(ii) $x = 36^\circ$
$x = 216^\circ$
(iii) $33^\circ \leq x \leq 213^\circ$

9.  

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>90</th>
<th>180</th>
<th>270</th>
<th>360</th>
<th>450</th>
<th>540</th>
<th>630</th>
<th>720</th>
<th>810</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin $\frac{1}{2}x$</td>
<td>0</td>
<td>0.71</td>
<td>1</td>
<td>0.71</td>
<td>0</td>
<td>-0.71</td>
<td>-1</td>
<td>-0.71</td>
<td>0</td>
<td>0.71</td>
</tr>
<tr>
<td>$3\sin\left(\frac{1}{2}x + 60\right)$</td>
<td>2.6</td>
<td>2.9</td>
<td>1.5</td>
<td>-0.78</td>
<td>-2.6</td>
<td>2.9</td>
<td>-1.5</td>
<td>0.78</td>
<td>2.6</td>
<td>2.9</td>
</tr>
</tbody>
</table>
10. 

<table>
<thead>
<tr>
<th>x</th>
<th>0°</th>
<th>30°</th>
<th>60°</th>
<th>90°</th>
<th>120°</th>
<th>150°</th>
<th>180°</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 \sin x</td>
<td>0</td>
<td>1</td>
<td>1.73</td>
<td>2</td>
<td>1.73</td>
<td>1.00</td>
<td>0</td>
</tr>
<tr>
<td>1-\cos X</td>
<td>1</td>
<td>0.13</td>
<td>0.50</td>
<td>1</td>
<td>0.06</td>
<td>1.87</td>
<td>2</td>
</tr>
</tbody>
</table>

11. \[ \sin (x + 30) = 0.5 \]
\[ x + 30 = 30° \]
\[ x = 0 \]
\[ 0, 180, 360 \]
12. (c) $10 \sin x = -\frac{1}{50} + 5$
$Y = -\frac{1}{50} + 5$

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

$X_1 = 28^\circ \pm 1$
$X_2 = 70^\circ \pm 1$

12.

b) i) amplitude = 1
ii) Period = 360°
iii) 45°, 219°
13. \[ 2\theta + 10 = 210^\circ, 330^\circ, 570^\circ, 690^\circ \]
\[ 2\theta = 200, 320, 560, 680 \]
\[ = 100^\circ, 160^\circ, 280^\circ, 340^\circ \]
\[ = \frac{5\pi}{9}, \frac{8\pi}{9}, \frac{14\pi}{9}, \frac{17\pi}{9} \]

14. \[ 4\sin 2x + 4\cos x - 5 = 0 \]
\[ 4(1 - \cos^2 x) + 4\cos x - 5 = 0 \]
\[ 4\cos^2 x - 4 \cos x + 1 = 0 \]
\[ 4\cos x - 2\cos x - 2\cos x + 1 = 0 \]
\[ (2\cos x - 1)^2 = 0 \]
\[ X = 60^\circ, 300^\circ \]

15.

<table>
<thead>
<tr>
<th>( x )</th>
<th>15(^\circ)</th>
<th>60(^\circ)</th>
<th>150(^\circ)</th>
<th>165(^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4( \cos 2x )</td>
<td>3.46</td>
<td></td>
<td>3.46</td>
<td></td>
</tr>
<tr>
<td>2( \sin (2x + 30^\circ) )</td>
<td></td>
<td>1.00</td>
<td></td>
<td>-1.00</td>
</tr>
</tbody>
</table>

(b) graph

(c)(i) Amplitude = 4
(ii) period = 180\(^\circ\)
(d) $x = 30^\circ, 120^\circ$