MURANGA EAST JOINT MOCK EXAMINATION.

MATHEMATICS FORM FOUR

121/2

NAME......MARKING......   INDEX NO ......SCHEME.

SIGNATURE.................................   DATE................

INSTRUCTION TO CANDINDATES

a. Write your name and index number in the spaces provided.

b. This paper consist of two sections, section 1 and section 2.

C. Answer all Questions in section 1, and any Five questions in Section 2.

d. Show all steps in your calculations

e. Marks may be given for correct working even if the answer is wrong.

f. Electronic calculators may be used and KNEC mathematical tables   except where stated otherwise
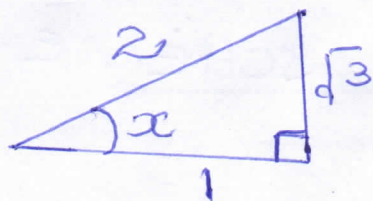
FOR EXAMINER'S USE ONLY

SECTION 1

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

SECTION 2

17	18	19	20	21	22	23	24	TOTAL

1. Given that $\sin^2 x = \frac{3}{4}$, Find sin x and tan x in surd form.   3mks

$$\sin x = \frac{\sqrt{3}}{2} \quad B_1 \qquad \cos x = \frac{1}{2} \qquad B_1$$

$$\tan x = \sqrt{3} \qquad B_1$$

2. The velocity of a particle after t seconds is given by $V = 4t^2 - 2t + 3$ m/s. Calculate the acceleration of the particle when t = 3 seconds 3mks.

$$a = \frac{dV}{dt} = 8t - 2 \qquad \qquad M_1$$

$$a = 8(3) - 2 \qquad \qquad M_1$$

$$= 22\, m/s^2 \qquad \qquad A_1$$

3.Write down the inverse of $\begin{bmatrix} 2 & -3 \\ 4 & 3 \end{bmatrix}$ hence solve the simultaneous equation : $2x - 3y = 7$ 4mks

$$4x + 3y = 5$$

$$DET = 6 - (-12) = 18$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 3 & 3 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ 5 \end{bmatrix} \quad M_1$$

$$\frac{1}{18}\begin{bmatrix} 3 & 3 \\ -4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \quad M_1$$

OR

$$\begin{bmatrix} \frac{3}{18} & \frac{3}{18} \\ \frac{-4}{18} & \frac{2}{18} \end{bmatrix} \quad B_1$$

$$\left.\begin{matrix} x = 2 \\ y = -1 \end{matrix}\right\} \quad A_1$$

OR Equivalent

4. Simplify the expression: $\dfrac{x2+3x+2}{x2-1}$ 3mks

$$\frac{(x+2)(x+1)}{(x-1)(x+1)} \quad \text{M1}$$

$$\text{M}$$

$$= \frac{x+2}{x-1} \quad \text{A}_1$$

5. The average of first and third term of a G.P is 120. Given that the first of the G.P is 24, Find the common ratio.   3mks

$$\frac{a+ar^2}{2} = 120 \text{ M}_1 \qquad r^2 = 9$$

$$a + ar^2 = 240 \qquad r = +3 \text{ or } -3 \Big\} \text{ A}_1$$

$$24 + 24r^2 = 240$$

$$24r^2 = 216 \text{ M}_1$$

6. The position vectors of points A and B are given as a = 3i – 3j and

b = 5i – j + 2k , respectively. Find the magnitude of A + B.   3 mks

$$\begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix} + \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ 2 \end{bmatrix} \text{ M}_1$$

$$|A+B| = \sqrt{8^2 + (-4)^2 + 2^2} \text{ M}_1$$

$$= \sqrt{84} = 9.165 \text{ A}_1$$

7. a) Write the simplified expansion of $(1 - 2x)^4$.   1mk

$$1^4 + 4 \cdot 1^3 \cdot (-2x) + 6 \cdot 1 \cdot (-2x)^2 + 4 \cdot 1 \cdot (-2x)^3 + (-2x)^4$$

$$1 - 8x + 24x^2 - 32x^3 + 16x^4 \quad B_1$$

b) Use the expansion to find the value of $(0.98)^4$, to 4 S.F   3mks

$$-2x = -0.02$$
$$x = 0.01$$

$$1 - 8(0.01) + 24(0.01)^2 - 32(0.01)^3 + 16(0.01)^4 \quad M_1$$

$$= 0.92236816 \quad A_1 \qquad = 0.9224 \quad B_1$$

8. If $x = 44.6$ and $y = 24.2$ both being to one D.P, Calculate the percentage error in $x - y$.   3mks

Max: $44.65 - 24.15 = 20.5$

Min: $44.55 - 24.25 = 20.3$ M_1

Error term $20.5 - 20.3 = 0.2$

Absolute Error $\dfrac{0.2}{2} = 0.1$

Original: $44.6 - 24.2 = 20.4$

$\dfrac{0.1}{20.4} \times 100\%$ $\dfrac{0.1}{20.4} \times 100\% \quad M_1 = 0.49\% \quad A_1$

9. Evaluate $\int_0^2 y\,dx$ given that $y = \dfrac{x^2 + 5x + 4}{x + 4}$ 4mks

$$\dfrac{(x+1)(x+4)}{(x+4)} \quad B_1 \qquad \left[\dfrac{1}{2}x^2 + x\right]_0^2 \quad M_1$$

$$\int_0^2 (x+1)\,dx = \qquad \left[\dfrac{1}{2} \cdot 4 + 2\right] - [0] \quad M_1$$

$$= 4 \quad A_1$$

10. R varies partly as square of V and partly as the cube of V.

When V = 2 , R = -20 , and when V = -3 , R = 135. Find the Law connecting R and V. 4mks

$$R = MV^2 + nV^3$$

$$4m + 8n = -20$$
$$m + 2n = -5$$
$$9m - 27n = 135$$
$$m - 3n = 15 \quad M_1$$

$$m + 2n = -5$$
$$-\underline{m - 3n = 15}$$
$$\qquad\qquad 5n = -20 \quad M_1$$

$$n = -4 \Big\} \quad A_1 \ (both)$$
$$m = 3$$

$$R = 3V^2 - 4V^3 \quad B_1$$

11. Make x the subject of the formula 3mks

$$T = \frac{a}{3m}\sqrt{\frac{r}{k-x}} \Rightarrow T^2 = \frac{a^2}{9m^2} \cdot \frac{r}{k-x} \quad M_1$$

$$T^2 = \frac{a^2 r}{9m^2 k - 9m^2 x}$$

$$\frac{1}{T^2} = \frac{9m^2 k - 9m^2 x}{a^2 r}$$
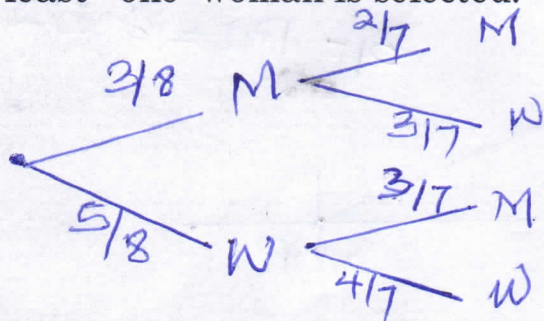
$$a^2 r = 9m^2 k T^2 - 9m^2 x T^2$$

$$9m^2 x T^2 = 9m^2 k T^2 - a^2 r$$

$$x = \frac{9m^2 k T^2 - a^2 r}{9m^2 T^2} \quad A_1$$

$$x = k - \frac{a^2 r}{9m^2 T^2}$$

12. In a nomination committee, two people were to be selected at random, from a group of 3 men and 5 women. Find the probability that at least one woman is selected. 2mks

$$1 - P(mm).$$
$$1 - \left(\frac{2}{7} \times \frac{3}{8}\right) \quad M_1$$
$$= \frac{25}{28} \quad A_1$$

13. Find the radius and the centre of the circle whose equation is

$4x^2 + 4y^2 + 20x - 24y = 39$ 3mks

$$x^2 + 5x + \underline{\quad} + y^2 - 6x + \underline{\quad} = \frac{39}{4} \quad M_1$$

$$\left(x + \frac{5}{2}\right)^2 + \left(y - 3\right)^2 = \frac{39}{4} + \frac{25}{4} + 9$$

$$\left(x + \frac{5}{2}\right)^2 + \left(y - 3\right)^2 = 25 \quad M_1$$

Centre $(-2.5, 3)$ Radius $= 5$ A_1

14. Determine the amplitude and period of the function,
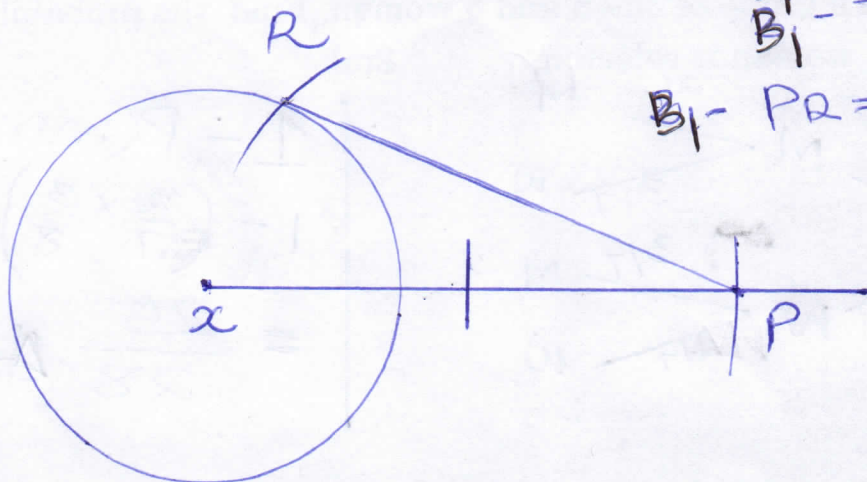
$Y = 3 \sin \left(\frac{1}{2}x + 30\right)^\circ$.   2mks

$$P = \frac{360}{\frac{1}{2}} = 720^\circ$$

$a = 3$ B_1 B_1

15. Construct a circle, centre x, and radius 2.5 cm. Construct a tangent from a point P, 7cm from x, to touch the circle at R. Measure the length PR.   3mks

B_1 - Bisector $\times P$
B_1 - Tangent PR
B_1 - PR $= 6.5 \pm 0.1$ cm

16. A rice blender mixes two types of rice, Type A which cost ksh 160 and Type B which cost ksh180. She sells the mixture at ksh 187.60 there by making a profit of 12%. Find the ratio at which she mixed the two types of rice. 3mks

$$\frac{100}{112} \times 187 \cdot 60 = 167 \cdot 5 \text{ ksh.} \rightarrow M1$$

$$\left. \frac{160A + 180B}{A + B} = \frac{167 \cdot 5}{1} \right\} \rightarrow \text{(M)}$$

$$160A + 180B = 167 \cdot 5A + 167 \cdot 5B$$
$$-7 \cdot 5A = -12 \cdot 5B$$

$$\frac{A}{B} = \frac{5}{3}$$

IM $A : B = 5 : 3$

OR,

A
160

B
180 M

}M1

167.5

12.5   7.5'

5   :   3 ⟶ A1

IM
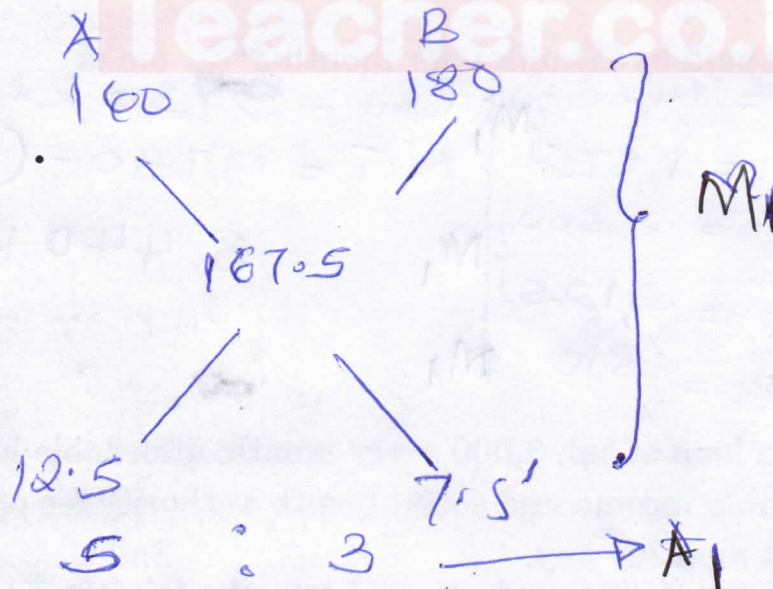
17. the table below shows income tax rates in a certain year

Monthly taxable income	Tax rates
0 - 14200	10%
14201 - 26700	15%
26791 - 39200	20%
39201 - 51700	25%
51701 and above	30%

Wanjiku is a civil servant and earns a basic salary of ksh 34,800, Medical allowance of ksh 2,400 , House allowance of ksh 10,000, commuter allowance of ksh 7,200 and non- taxable allowance of ksh12,000 . She has a personal relief of ksh 1,020   and also get relief equivalent to 10% of the premium paid. If she pays monthly insurance premiums of ksh 2,700 , calculate

a) Wanjiku's monthly taxable income.   2mks

$$34,800 + 2,400 + 10,000 + 7,200 \quad M1$$
$$= 54,400 \ ksh. \quad A1$$

b) Find the net tax paid by Wanjiku per month.   5mks

$$14200 \times \frac{10}{100} = 1420$$
$$12500 \times \frac{15}{100} = 1,875$$
$$12500 \times \frac{20}{100} = 2,500$$
$$12500 \times \frac{25}{100} = 3,125$$
$$2700 \times 0.3 = 810 \quad M1$$

Total - 9730 ksh. M1

$$N.T = 9730 - (1020 + 270)$$
$$= 8,440 \ ksh. \quad A1$$

c) If Wanjiku pays a loan of ksh 8,000 every month, affordable house levy of 1.5% of taxable income and social health authority fee of ksh 1,805. Find her net monthly pay.   3mks

$$Deductions = (8440 + 2700 + 8000 + 816 + 1805)$$
$$= 21761 \quad M1$$
$$Npay = (54,400 + 12000) - 21761 \quad M1$$
$$= 44,639 \quad A1$$

18. a) Complete the table below giving your values correct to 2 decimal places.   2mks

	0	15	30	45	60	75	90	105	120
y =3cosx	3.00	2·90	2.60	2·12	1.50	0.78	0.00	-0.78	−1·50
Y=4sin(2x-10)⁰	-0.69	1.37	3·06	3.94	3.76	2·57	0.69	−1·37	-3.06

B2

b) Taking 1 cm to represent 15^0 on the x – axis and 2cm to represent 1 unit on the y-axis, draw the graph of y=4sin(2x-10)⁰ and y=3cosx⁰ on the same set of axes on the grid provided.   5mks.

GRAPHICAL

c)Use your graph to find the values of x for which 3cosx – 4sin(2x-10) = 0   1mk

Points of Intersection.

$x = 25·5°$; $97·5° \pm 1·5°$ B1

d) State

i) The amplitude of graph y=3cosx   1mk

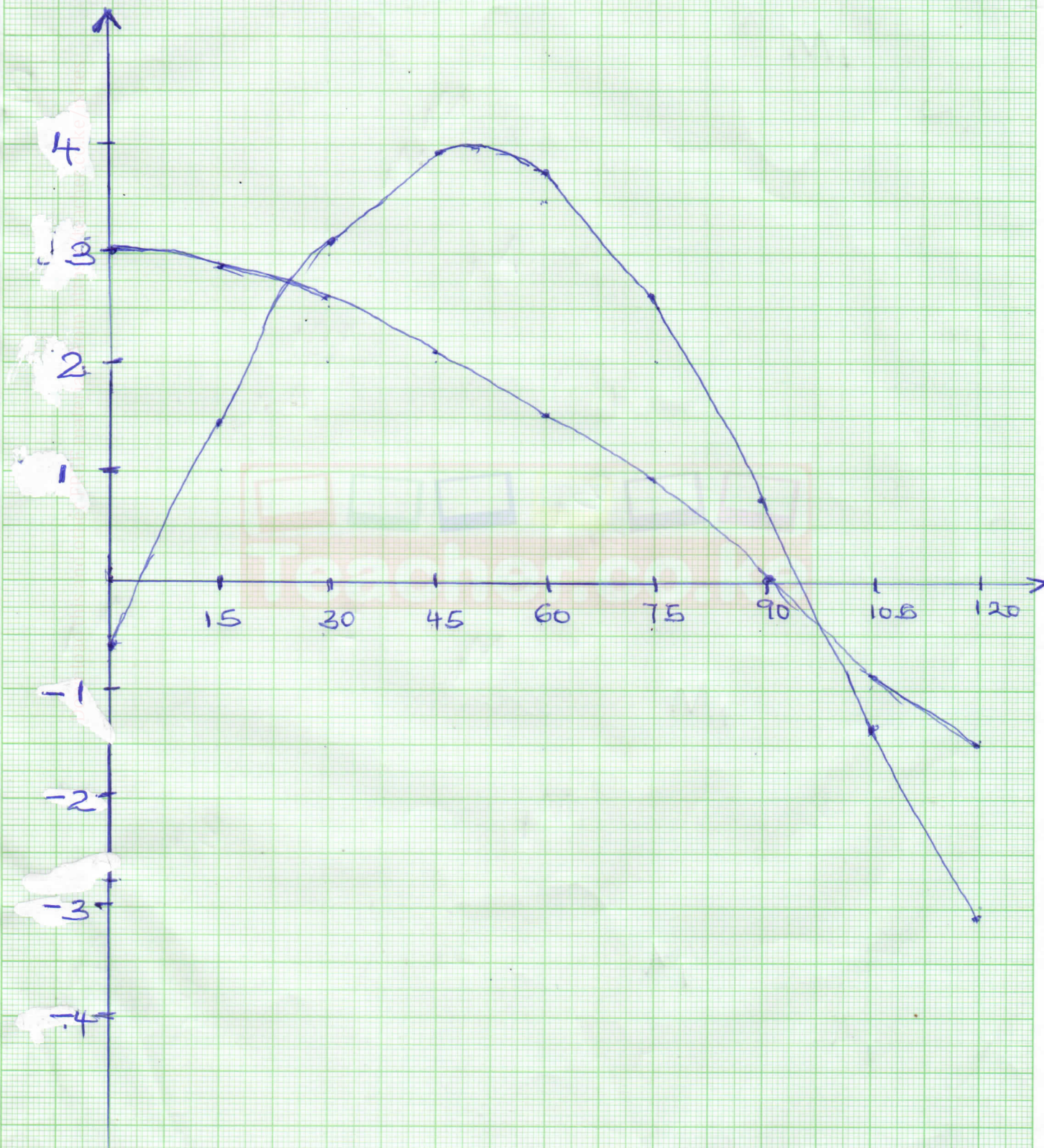3   B1

ii) The Period of the graph y=4sin(2x-10)

$180°$ B1

19. A Quantity P varies as the square of m, and partly as n.

When P = 3.8, m = 2, and n = -3. When P = -0.2, m = 3, and n = 2.

a). Find:

i) The equation that connect P, m and n.   4mks

$$P = K_1 m^2 + K_2 n$$
$$4K_1 - 3K_2 = 3.8 \quad M_1 \quad | \quad P = 0.2m^2 - n \quad \underline{B}_1$$
$$9K_1 + 2K_2 = -0.2 \quad M_1$$
$$\left. \begin{array}{l} K_1 = 0.2 \\ K_2 = -1 \end{array} \right\} \underline{A}_1$$

ii) The value of P when m = 10 and n = 4   1mk

$$P = 0.2 \times 100 - 4$$
$$P = \boxed{16} \quad \underline{B}_1$$

b) Express m in terms of P and n.   2mks

$$0.2m^2 = P + n$$
$$m^2 = \frac{P + n}{0.2} \quad \bigg| \quad M_1 \quad m = \pm\sqrt{\frac{P+n}{0.2}} \quad \underline{A}_1$$

c.) If P and n are each increased by 10%, Find the percentage increase in m correct to 2 decimal places.   3mks

$$m_x = \sqrt{\frac{1.1P + 1.1n}{0.2}} \quad M_1 \quad = 1.0488\sqrt{\frac{P+n}{0.2}} \quad M_1$$
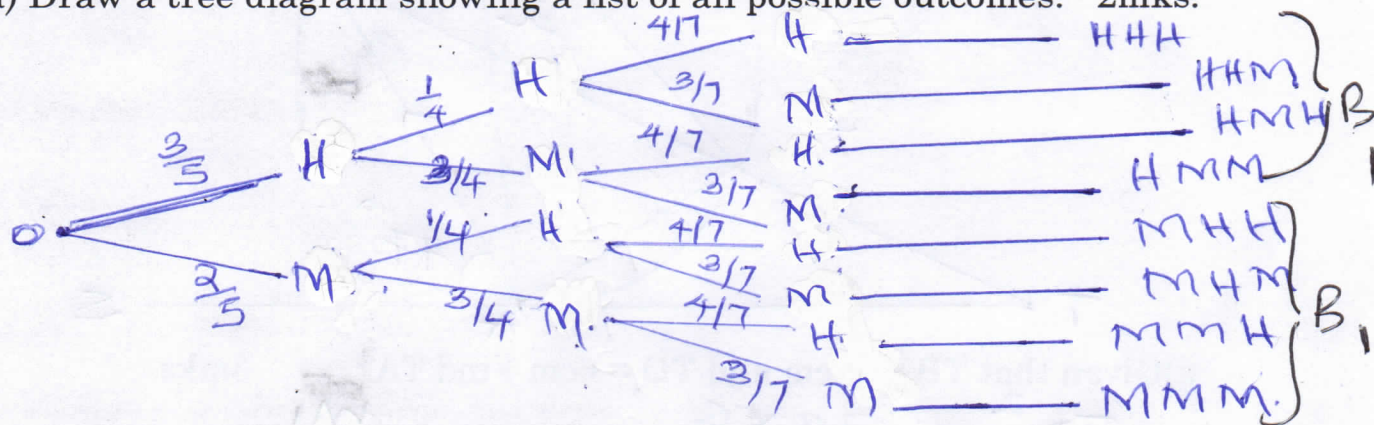$$= \sqrt{\frac{1.1(P+n)}{0.2}} \qquad \% \text{ increase} = 4.88\%$$
$$\underline{A}_1$$

20. Onyango, Chege and Mutiso are playing a game of darts. The probability of Onyango, Chege and mutiso hitting the bull's eye is $\frac{3}{5}, \frac{1}{4}$ and $\frac{4}{7}$ respectively.

a) Draw a tree diagram showing a list of all possible outcomes. 2mks.

b) Find the probability that, in one attempt

i) Only one will hit the bull's eye. 3.mks

$$P(HMM) + P(MHM) + P(MMH)$$

$$\left(\frac{3}{5} \times \frac{3}{4} \times \frac{3}{7}\right) + \left(\frac{2}{5} \times \frac{1}{4} \times \frac{3}{7}\right) + \left(\frac{2}{5} \times \frac{3}{4} \times \frac{4}{7}\right) M_1 m_1$$

$$\frac{27}{140} + \frac{6}{140} + \frac{24}{140} = \frac{57}{140} A_1$$

ii) Two hit the bull's eye. 3.mks

$$P(HHM) + P(HMH) + P(MHH)$$

$$\left(\frac{3}{5} \times \frac{1}{4} \times \frac{3}{7}\right) + \left(\frac{3}{5} \times \frac{3}{4} \times \frac{4}{7}\right) + \left(\frac{2}{5} \times \frac{1}{4} \times \frac{4}{7}\right) M_1 m_1$$

$$\frac{9}{140} + \frac{36}{140} + \frac{8}{140} = \frac{53}{140} A_1$$

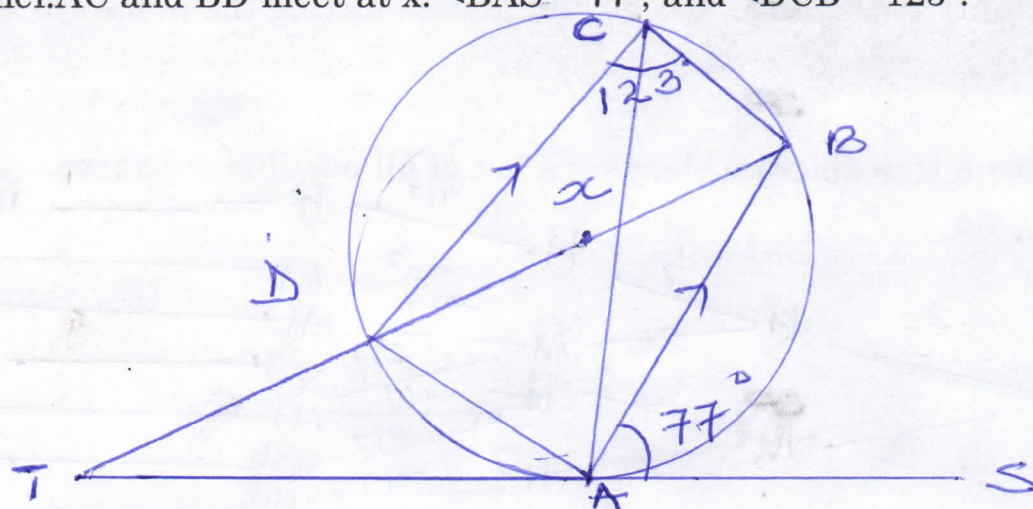iii) At least one hit the bulls eye. 2mks

$$1 - P(MMM)$$

$$1 - \left(\frac{2}{5} \times \frac{3}{4} \times \frac{3}{7}\right) M_1 = \frac{61}{70} A_1$$

21. In the diagram below TA is Tangent to the circle. AB and CD are parallel. AC and BD meet at x. <BAS = 77⁰, and <DCB = 123⁰.

a) Given that TB = 24cm and TD = 6cm Find TA.   3mks

$$TA^2 = 24 \times 6$$
$$TA = \sqrt{144}$$
$$= 12 cm.$$

M1
M1
A1

b) Giving Reasons find the values of the following angles

i) < DAB   2mks

$$180 - 123 = 57°$$ opposite Angles of a
                          cyclic quadrilateral.

B1

B1

ii) <DAT   1mk   Angles in a
$$180 - (77 + 57) = 46°$$ straight line.

B1

iii) <DCA   2mks

46° Angle in alternate segment
B1 of chord DA.

B1

iv) < DTA   2mks
$$180 - (103 + 46) = 31° - \text{Angles in a}$$
                          Triangle.
B1

B1.

22.The diagram below shows triangle OAB . OA = a and OB = b. C divides OA in the ratio 1:2 and D divides OB in the ratio2:3 . AD and BC meet at E.
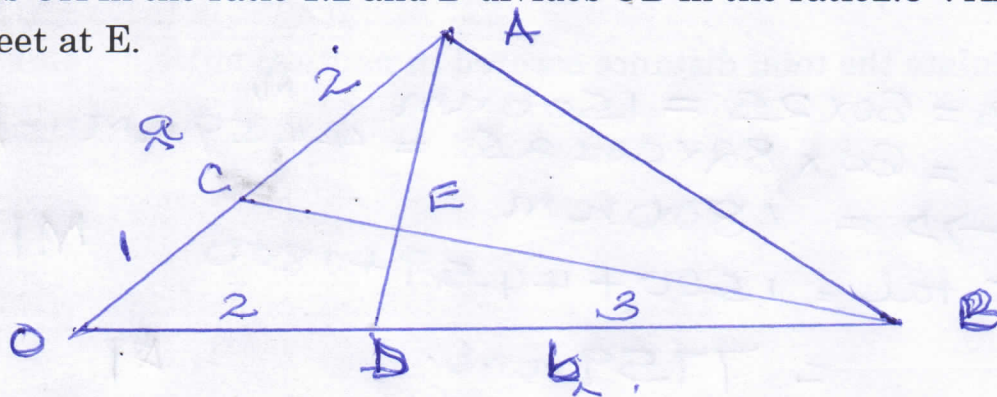
a)Find the following vectors in terms of , a and b .

i) AB $\quad = a + b$ **B₁** $\qquad$ 1mk

ii) CB $\quad -\frac{1}{3}a + b$ **B₁** $\qquad$ 1mk

iii)OC $\quad \frac{1}{3}a$ **B₁** $\qquad$ 1mk

iv) AD $\quad -a + \frac{2}{5}b$ **B₁** $\qquad$ 1mk

b) Given that CE = mCB and DE = nDA where m and n are scalars. Find:

i) Two expressions of OE $\qquad$ 2mks

$OE = OC + CE$ $\quad$ **B₁**

$OE = OD + DE$ $\quad$ **B₁** or equivalent

ii) Find the values of m and n. $\qquad$ 3mks

$OE = \left(\frac{1}{3} - \frac{1}{3}m\right)a + mb = na + \left(\frac{2}{5} - \frac{2}{5}n\right)b$ **M₁**

$\Rightarrow n = \frac{1}{3} - \frac{1}{3}M$ and $M = \frac{2}{5} - \frac{2}{5}n$ **M₁**

$n = \frac{3}{13}$ and $M = \frac{4}{13}$ **A₁** (Both)

iii) Find OE. $\qquad$ 1mk

$OE = \frac{3}{13}a + \frac{4}{13}b$ **B₁**

23. A ship sails from A(0°, 70°W) Due North to B(25°N, 70°W) then due east to C(25°N, 12°E) and finally a further 1800nm due east to D.

a) Calculate the total distance covered in nautical miles.   4mks

$$AB = 60 \times 25 = 1500nm \quad M_1$$
$$BC = 60 \times 82 \times \cos 25 = 4459nm \quad M_1$$
$$C \rightarrow D = 1800nm \quad M_1$$
$$Total = 1500 + 4459 + 1800$$
$$= 7759nm. \quad A_1$$

b) If the whole journey took a total time of 300 hours, Find its average velocity in knots correct to one decimal place.   2mks

$$V = \frac{7759}{300} = 25.9 \text{ Knots.}$$
$$M_1 \qquad A_1$$

c.) Find to the nearest degree the final position of the ship.   4mks.

$$\alpha \times 60 \times \cos 25 = 1800 \quad M_1$$
$$\alpha = \frac{1800}{60 \times \cos 25} = 33.10° \quad M_1 \ A_1$$
$$\text{longitude of } D = 33.10 + 12 = 45.10°E.$$
$$\text{Position of } D(25°N, 45°E) \quad B_1$$

24. The acceleration of a body moving in a straight line $(4-t)$ m/s² and its velocity is v m/s after t seconds.

a) If the initial velocity of the body is 3m/s,

i) Express its velocity V in terms of t .   3mks

$$V = 4t - \frac{1}{2}t^2 + C \quad M_1$$

when $t = 0$, $V = 3$ M_1

$$\Rightarrow C = 3$$

$$V = -\frac{1}{2}t^2 + 4t + 3 \quad A_1$$

ii) Find the velocity of the body after 2 seconds.   2mks

$$V = -\frac{1}{2}(2 \times 2) + 4(2) + 3 \quad M_1$$

$$= 9 \, m/s \quad A_1$$

b) Calculate :

i) The taken to attain maximum velocity.   2mks

$$V = -t^2 + 8t + 6 = 0$$

$$t = \frac{-8 \pm \sqrt{64 - (4 \times -1 \times 24)}}{-2} \quad M_1$$

$$t = \frac{-8 \pm 9.381}{-2}$$

$$t = 8.691 \, sec \quad A_1$$

ii) The distance covered by the body to attain the maximum velocity.
3mks

$$S = -\frac{1}{3}t^3 + 4t^2 + 6t \quad M_1 \quad \text{when } s = 0 \quad t = 0$$

$$= -\frac{1}{3}(8.691)^3 + 4(8.691)^2 + 6(8.691) \quad M_1$$

$$= -218.82 + 302.13 + 52.15$$

$$= 135.46 \, M \cdot A_1$$