Instructions to students

- This paper has two sections A and B
- Answer all questions in section A
- In section B answer **ONLY 2** questions

SECTION A

BODMAS

1. . Evaluate: $\dfrac{\frac{1}{2}\, of\ 18 \div -3+2\frac{1}{2}X\frac{3}{-5}}{\frac{1}{2}+3\frac{3}{4}\div\frac{3}{4}}$ (3 marks)

Num:

$\frac{1}{2} of\ 18 \div -3 + \frac{5}{2} \times \frac{-3}{5}$

$(9 \div -3) + \frac{5}{2} \times \frac{-3}{5}$

$-3 + \left(\frac{-3}{2}\right)$

$= -4\frac{1}{2}$ A1

Den:

$\frac{1}{2} + \frac{15}{4} \div \frac{3}{4}$

$\frac{1}{2} + \frac{15}{4} \times \frac{4}{3}$

$\frac{1}{2} + 5 = 5\frac{1}{2}$ A1

$\frac{-4\frac{1}{2}}{5\frac{1}{2}}$

$-\frac{9}{2} \div \frac{11}{2}$

$-\frac{9}{2} \times \frac{2}{11} = \frac{-9}{11}$ A1

2. The position vector of A and B are a = 4i + 4j – 6k and b = 10i + 4j + 12k. D is a point on AB such that AD:DB is 2:1. Find the co-ordinates of D (3 Marks)

A D B

2 1

$\begin{pmatrix}4\\4\\-6\end{pmatrix}$ $\begin{pmatrix}10\\4\\12\end{pmatrix}$

Using Ratio theorem

$D = \frac{2}{3}\begin{pmatrix}10\\4\\12\end{pmatrix} + \frac{1}{3}\begin{pmatrix}4\\4\\-6\end{pmatrix}$

$= \begin{pmatrix}20/3\\8/3\\24/3\end{pmatrix} + \begin{pmatrix}4/3\\4/3\\-6/3\end{pmatrix}$

$= \begin{pmatrix}20/3 + 4/3\\8/3 + 4/3\\24/3 - 6/3\end{pmatrix} = \begin{pmatrix}24/3\\12/3\\18/3\end{pmatrix}$ $D = \begin{pmatrix}8\\4\\6\end{pmatrix}$

$D = 8i + 4j + 6k$

3. A dealer has two types of grades of tea, A and B. Grade A costs Sh. 140 per kg. Grade B costs Sh. 160 per kg. If the dealer mixes A and B in the ratio 3:5 to make a brand of tea which he sells at Sh. 180 per kg, calculate the percentage profit that he makes (3 marks)

let the ratio be x:y

x:y : 3:5

$\frac{(140 \times 3) + (160 \times 5)}{3+5}$

price per kg of mixture

$\frac{420 + 800}{8} = \frac{1220}{8}$

Cost price $= 152.50$

profit $= 180 - 152.5$

$= 27.50$

% profit $= \frac{27.50}{152.50} \times 100$

$= 18.03\%$

4. A variable Z varies directly as the square of X and inversely as the square root of Y. Find the percentage change in Z if X increased by 20% and Y decreased by 19% (3 Marks)

$Z \propto \frac{x^2}{\sqrt{y}} \Rightarrow Z = \frac{kx^2}{\sqrt{y}}$

$Z_1 = \frac{(1.2)^2 k x^2}{\sqrt{\frac{81}{100}} \cdot \sqrt{y}}$

$Z_1 = \frac{1.44 kx^2}{0.9 \sqrt{y}}$

$Z_1 = 1.6Z$

% change $= \frac{1.6Z - Z}{Z} \times 100\%$

$\frac{\cancel{Z}(1.6 - 1)}{\cancel{Z}} \times 100\%$

$= 60\%$

5. Find the centre and radius of the circle whose equation is $2x^2 + 2y^2 - 8x + 12y - 2 = 0$ (3 Marks)

$\frac{2x^2}{2}+\frac{2y^2}{2}-\frac{8x}{2}+\frac{12y}{2}-\frac{2}{2}=\frac{0}{2}$

$x^2+y^2-4x+6y-1=0$ M1

$x^2-4x+y^2+6y=1$

$(x-2)^2+(y+3)^2=1+2^2+3^2$ M1

$(x-2)^2+(y+3)^2=14$

Centre $(2,-3)$

radius $=\sqrt{14}$ A1

$r = 3.742$ units

6. Pipe A can fill a tank in 2 hours, pipes B and C can empty the tank in 5 hours and 6 hours respectively. How long would it take

(a) To fill the tank if A and B are left open and C closed (2 Marks)

A ⇒ 1 hr — $\frac{1}{2}$ — inlet

B = 1 hr — $\frac{1}{5}$ outlet

C = 1 hr — $\frac{1}{6}$ outlet

A & B.

$\frac{1}{2}-\frac{1}{5}=\frac{3}{10}$ — 1 hr M1

1 hr — $\frac{3}{10}$

? ← $\frac{10}{10}$

$=\frac{10}{3}$ hr $=3\frac{1}{3}$ hrs. A1

(b) To fill the tank with all the pipes open (2 Marks)

$\frac{1}{2}-\frac{1}{5}-\frac{1}{6}=\frac{4}{30}$

$\frac{4}{30}\longrightarrow 1$ hr M1

$\frac{30}{30}\longrightarrow\frac{30}{4}=$

1 hr $\longrightarrow\frac{4}{30}$

$\frac{30}{30}$

$=\frac{30}{4}=\underline{7.5\text{ hrs}}$ A1

7. (a) Find the inverse of the matrix $\begin{pmatrix}4 & 3\\3 & 5\end{pmatrix}$ (1 Mark)

Det $= 20-9=11$.

Inverse $= -\frac{1}{11}\begin{pmatrix}5 & -3\\-3 & 4\end{pmatrix}$ B1

(b) Hence solve the simultaneous equation below using matrix method (3 Marks)

$4x + 3y = 6$

$5y + 3x - 5 = 0$

$4x+3y=6$

$3x+5y=5$

$\begin{pmatrix}4 & 3\\3 & 5\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}6\\5\end{pmatrix}$ M1

$-\frac{1}{11}\begin{pmatrix}5 & -3\\-3 & 4\end{pmatrix}\begin{pmatrix}4 & 3\\3 & 5\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=-\frac{1}{11}\begin{pmatrix}5 & -3\\-3 & 4\end{pmatrix}\begin{pmatrix}6\\5\end{pmatrix}$

$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=-\frac{1}{11}\begin{pmatrix}30-15\\-18+20\end{pmatrix}$ M1

$-\frac{1}{11}\begin{pmatrix}15\\2\end{pmatrix}$

$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-\frac{15}{11}\\-\frac{2}{11}\end{pmatrix}$

$x=-1\frac{4}{11}$ A1

$y=-\frac{2}{11}$

8. Evaluate by rationalizing the denominator and leaving your answer in surd form. (2 Marks)

$$\frac{\sqrt{8}}{1+Cos\,45}$$

$\frac{2\sqrt{2}}{1+\cos 45}$

$\frac{2\sqrt{2}}{1+\frac{1}{\sqrt{2}}}$

$\frac{2\sqrt{2}}{1+\frac{1}{\sqrt{2}}}\times\frac{\left(1-\frac{1}{\sqrt{2}}\right)}{\left(1-\frac{1}{\sqrt{2}}\right)}$

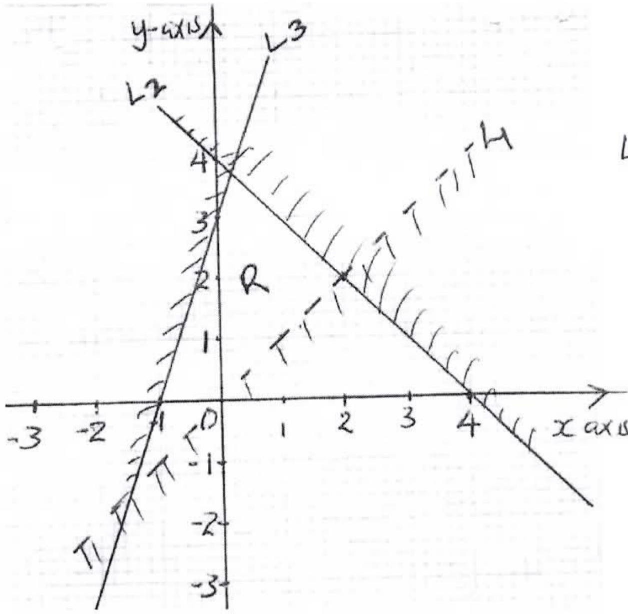
$\frac{2\sqrt{2}-2}{1-\frac{1}{2}}$

$\frac{2\sqrt{2}-2}{\frac{1}{2}}$

$2\left(2\sqrt{2}-2\right)$

$\underline{4\left(\sqrt{2}-1\right)}$

9. Form the three inequalities that satisfy the given region R (3 Marks)



$L_1 = y = x$.
$y \geq 1$ - - - B_1.

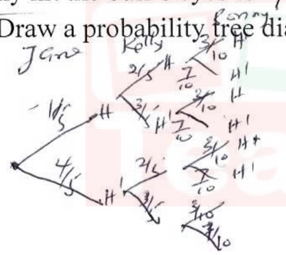
$L_2 = \frac{x}{4} + \frac{y}{4} = 1$.
$x + y = 4$.
$x + y \leq 4$ - - - B_1.

$L_3 = \frac{x}{-1} + \frac{y}{3} = 1$.
$-3x + y = 3$.
$y - 3x = 3$.
$y - 3x \leq 3$ - - - B_1.

SECTION II

10. Three darts players Jane, Kelly and Bony are playing in a completion the probability that Jane, Kelly and Brony hit the bull's eyes is $^1/_5$, $^2/_5$ and $^3/_{10}$ respectively.

(a) Draw a probability tree diagram to show all the possible outcomes for the players. (4mks)



(b) Calculate the probability that :

(i) Jane or Bony hit the bull's eye. (2mks)

$P(J \text{ or } B)$
$P(JK'B')$ or $P(J'K'B)$
$\left(\frac{1}{5} \times \frac{3}{5} \times \frac{7}{10}\right) + \left(\frac{4}{5} \times \frac{3}{5} \times \frac{3}{10}\right)$

$\frac{21}{250} + \frac{36}{250} = \frac{57}{250}$

(ii) All the three fail to hit the bull's eye. (2mks)

$P(\text{all fail}) = \left(\frac{4}{5} \times \frac{3}{5} \times \frac{7}{10}\right) = \frac{42}{125}$

(iii) Only two fails to hit the bull's eye. (2mks)

$P(\text{only two fail})$
$P(JK'B') + P(J'KB') + P(J'K'B)$
$\left(\frac{1}{5} \times \frac{3}{5} \times \frac{7}{10}\right) + \left(\frac{4}{5} \times \frac{2}{5} \times \frac{7}{10}\right) + \left(\frac{4}{5} \times \frac{3}{10} \times \frac{3}{10}\right)$
$\frac{21}{250} + \frac{56}{250} + \frac{36}{250} = \frac{113}{250}$

11. The fourth, seventh and sixteenth term of an arithmetic progression are in geometric progression. The sum of the first six terms of the arithmetic progression is 12.

Determine the

(a) First term and the common difference of the arithmetic progression. (6mks)

4th term $= a+3d$

7th $= a+6d$

16th $= a+15d$

$\frac{a+6d}{a+3d} = \frac{a+15d}{a+6d}$ m1

$a^2+12ad+36d^2 = (a+3d)(a+15d)$

$a^2+12ad+36d^2 = a^2+15ad+3ad+45d^2$

$12ad+36d^2 = 18ad+45d^2$ m1

$12ad-18ad = 45d^2-36d^2$ m1

$-\frac{6ad}{d} = \frac{9d^2}{d}$ m1

$-6a = 9d$

$a = -1.5d$

$S_6 = 3(2a+5d) = 12$ m1

$2a+5d = 4$

$2(-1.5d)+5d = 4$

$-3d+5d = 4$

$2d = 4$

$d = 2$

$a = -3$

$a = -3$

$d = 2$ A1

(b) Common ratio of the geometric progression. (2mks)

$r = \frac{a+6d}{a+3d}$ m1

$= \frac{-3+12}{-3+6} = \frac{9}{3} = 3$ A1

(c) Sum of the first six terms of the geometric progression. (2mks)

$S_n = a\left(\frac{r^n-1}{r-1}\right)$

$a = a+3d$

$= -3+6$

$= 3$

$\frac{3(3^6-1)}{3-1}$

$= \frac{3(3^5-1)}{2}$

$= 1.5 \times 242 = 363$

12. The relationship between the variables a and y is believed to be y = a/x + bx. Where a and b are constants. The table below shows corresponding values of x and y

xy	5	14	29	50	77	B1
X	1	2	3	4	5
Y	5.00	7.00	9.67	12.50	15.40
x^2	1	4	9	16	25	B1

(a) Write the relationship in the form of y = mx + c (1mk)

$y = \frac{a}{x} + bx$ $\quad xy = a + bx^2$ $\quad xy = bx^2 + a$

$y = mx + c$ B1

(b) By drawing a suitable straight line graph estimate the values of a and b (7mks)

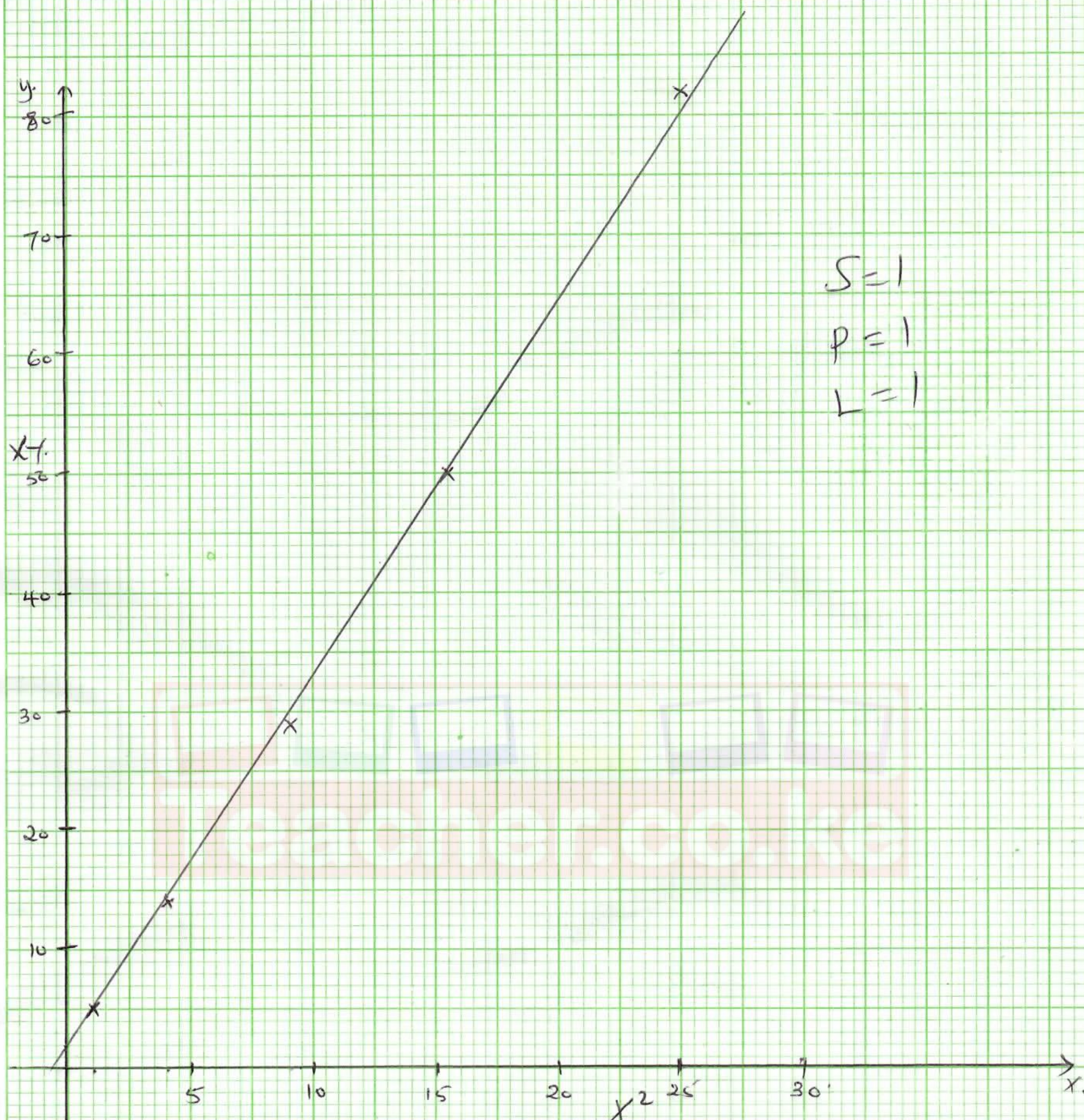
(c) Find the value of y when x = 1000 (2mks)

$y = \frac{a}{x} + bx$

$y = \frac{2}{x} + 3x$ B1

$y = \frac{2}{1000} + (3 \times 1000)$

$y = 3000.002$ B1



$a = 2$ B_1

$b = \frac{50-5}{16-1}$

$= \frac{45}{15} = 3$

$b = 3$ B_1