FORM THREE TERM ONE
Mathematics Alt A

Instructions

- Write your name, class and admission number in spaces provided above.
- The paper contains **two** sections **A** and **B.**
- Answer **ALL** questions in section **A** and **anyfive** from section **B** in the spaces provided below each question.
- Marks may be given for correct working even if the answer is wrong.
- Non-programmable silent electronic calculator and KNEC mathematical tables may be used, except where stated or otherwise.

For Examiner's Use Only

SECTION A

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

SECTION B

17	18	19	20	21	22	23	24	TOTAL

GRAND TOTAL

SECTION A (50 MARKS)

*Answer **all** questions in this section in the spaces provided*

1. Use logarithms to evaluate to 4 significant figures (3 marks)

$$\frac{(0.5241)^2 \times 83.59}{\sqrt[3]{0.23 + 0.126}}$$

NO	
0.5241	T.7194 × 2
	T.4388 +
83.59	T̄.9222
	1.3610 → 1.3610
0.356	T.5514 × ⅓
	T.8505 → T̄.8505
3.240 × 10¹	T̄.5105 ← 1.5105

ANS = 32.40

2. Translation T is represented by the column vector $\binom{5}{4}$ and another translation U by column vector $\binom{-3}{2}$. A point P is mapped to a point Q by T and then Q is mapped to a point R by U. If R has coordinates $(7, -4)$ determine the coordinates of P. (3 marks)

$$Q + U = R$$
$$Q + \binom{-3}{2} = \binom{7}{-4}$$
$$Q = \binom{7}{-4} - \binom{-3}{2}$$
$$= \binom{10}{-6}$$

$$P + T = Q$$
$$P + \binom{5}{4} = \binom{10}{-6}$$
$$P = \binom{10}{-6} - \binom{5}{4} = \binom{5}{-10}$$

$$P(5, -10)$$

3. Solve for x in $\log_3(4 + 3x) + 3\log_3 3 - 2 = \log_3(x + 6)$ (3 marks)

$$\log_3(4 + 3x) + \log_3 27 - \log_3(x + 6) = 2$$

$$\log_3 \frac{(4 + 3x)27}{x + 6} = 2$$

$$\frac{108 + 81x}{x + 6} = 9$$

$$108 + 81x = 9x + 54$$

$$72x = -54$$

$$x = \frac{-3}{4}$$

4. A family spent $\frac{2}{5}$ of their income on food, $\frac{1}{3}$ of the remainder on water bill and saved the rest. If sh. 1200 more is spent on food than water bill. Find how much they saved? **(3 marks)**

Food $= \frac{2}{5}$

Water bill $= \frac{3}{5} \times \frac{1}{3} = \frac{1}{5}$

savings $= \frac{2}{5}$

$\frac{2}{5} - \frac{1}{5} = \frac{1}{5} \Rightarrow 1200$

if $\frac{1}{5} \Rightarrow 1200$

$\therefore \frac{2}{5} \Rightarrow 1200 \times \frac{2}{5} \times \frac{5}{1}$

$= \text{Ksh } 2,400$

5. Evaluate without using tables or calculator. $\sqrt[3]{\frac{0.729 \times 409.6}{0.1728}}$ **(2 marks)**

$\dfrac{0.729 \times 409.6}{0.1728} \quad \dfrac{\times 10000}{\times 10000}$

$\dfrac{\overset{27}{\cancel{729}} \times \overset{64}{\cancel{4096}}}{\underset{1}{\cancel{\underset{64}{1728}}}}$

$= \sqrt[3]{27 \times 64}$

$= \sqrt[3]{27} \times \sqrt[3]{64}$

$= 3 \times 4 = 12$

6. A DVD player whose cash price is sh. 15000 is bought on hire purchase by paying a deposit of sh 3000 and 12 monthly instalments of sh. 1250 each. Calculate the rate of interest per annum. **(3 marks)**

$P = 15,000 - 3,000$

$= \text{sh } 12,000$

$A = P(1 + \frac{r}{100})^n$

$15,000 = 12,000(1 + \frac{r}{100})^1$

$1.25 = 1 + \frac{r}{100}$

$r = 25\% \text{ p.a}$

7. A hollow cylindrical alloy of length 40mm has a mass of 352g. If its internal radius is 3cm and its 0.01m thick; calculate the density of the metal used to make the alloy in kg/m³. Take pi as 22/7. **(3 marks)**

$$\text{Cross-section Area} = \left(\frac{22}{7} \times 4^2 - \frac{22}{7} \times 3^2\right)$$

$$= 22\,cm^2$$

$$\text{Volume} = 22\,cm^2 \times 4\,cm$$

$$= 88\,cm^3$$

$$\text{Density} = \frac{352g}{88\,cm^3} = 4\,g/cm^3$$

$$1\,g/cm^3 = 1000\,kg/m^3$$

$$\therefore 4\,g/cm^3 = 4 \times 1000$$

$$= 4000\,kg/m^3$$

8. A salesman dealing in iphones earns a basic salary and commission as follows.

Sales	Commission
For sales up to ksh. 150,000	0%
For sales above ksh. 150,000	
- First ksh. 85,000	3%
- Next ksh. 85,000	4%
- Any amount above	5%

(a) In the month of December 2022, the salesman earned a basic salary of ksh. 25,000 and he sold 100 iphones at a discount of 10%. Calculate;

(i) His total sales in the month of December if the iphones were marked at Ksh. 5000. **(1 mark)**

$$5,000 \times 100 \times \frac{90}{100} = Ksh.\ 450,000$$

(ii) His total earnings that month. **(3 marks)**

$$85,000 \times \frac{3}{100} = 2,550$$

$$85,000 \times \frac{4}{100} = 3,400$$

$$130,000 \times \frac{5}{100} = 6,500$$

$$\overline{12,450}$$

$$Total = 25,000 + 12,450$$

$$= Ksh\ 37,450/=$$

9. If $\frac{\sqrt{3}}{2-\sqrt{5}} = a\sqrt{b} + c\sqrt{d}$, find the values of **a, b, c** and **d** where they are rational numbers. **(3 marks)**

$$\frac{\sqrt{3}}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}} = \frac{2\sqrt{3} + \sqrt{15}}{4-5} = -2\sqrt{3} - \sqrt{15}$$

$$a = 2$$

$$b = 3$$

$$c = 1$$

$$d = 15$$

10. Solve for x in the equation;

$25^x - 5^{2x-1} = 500$ **(3 marks)**

$5^{2x} - 5^{2x-1} = 500$

$5^{2x} - \dfrac{5^{2x}}{5} = 500$

$\dfrac{4}{5}(5^{2x}) = 500$

$5^{2x} = 625$

$5^{2x} = 5^4$

$2x = 4$

$\underline{\underline{x = 2}}$

11. Using completing the square method, solve **(3 marks)**

$10x^2 - 11x - 6 = 0$

$x^2 - \dfrac{11}{10}x = \dfrac{6}{10}$

$x^2 - \dfrac{11}{10}x + \left(\dfrac{11}{20}\right)^2 = \dfrac{6}{10} + \left(\dfrac{11}{20}\right)^2$

$\left(x - \dfrac{11}{20}\right)^2 = \dfrac{361}{400}$

$x - \dfrac{11}{20} = \sqrt{\dfrac{361}{400}}$

$x = \dfrac{11}{20} \pm \dfrac{19}{20}$

$x = \dfrac{30}{20} \quad \text{or} \quad -\dfrac{8}{20}$

$\underline{\underline{x = 1.5 \text{ or } -0.4}}$

12. Each side of a regular heptagon measures 11.2cm. What is the percentage error in calculating the heptagon's perimeter? **(3 marks)**

Actual Perimeter $= 11.2\text{cm} \times 7$

$= \underline{78.4\text{cm}}$

Maximum Perimeter $= 11.25\text{cm} \times 7$

$= \underline{78.75\text{cm}}$

Minimum Perimeter $= 11.15 \times 7$

$= \underline{78.05\text{cm}}$

% error $= \dfrac{(78.75 - 78.05)}{78.4} \times 100\%$

$= \underline{0.8929\%}$

13. A cone is made from a sector whose dimensions are given below. Calculate the volume of the cone. $\left(Take\ \pi = \frac{22}{7}\right)$. OA = OB = 7cm radius of the sector and $< AOB = 120^0$. Give your answer to 3 significant figures.

(4 marks)
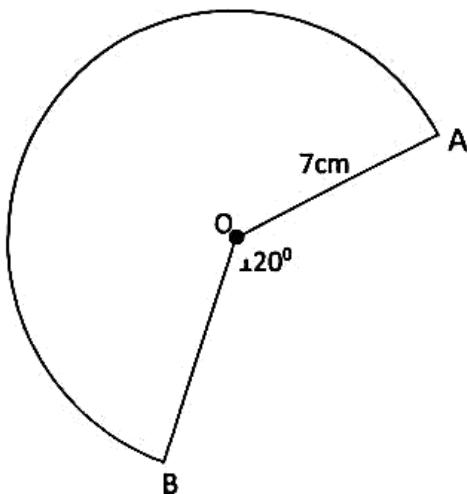
Arc length AB $= \frac{240}{360} \times \frac{22}{7} \times 14$

$= 29.33cm$

Circumference of the base of the cone $= 29.33cm$

$29.33 = \frac{22}{7} \times 2R$

$\frac{7}{44} \times \frac{88}{3} = \frac{44}{7}R \times \frac{7}{44}$

$R = \frac{14}{3}$ cm

$S = 7cm$

$H = \sqrt{S^2 - R^2}$

$= \sqrt{7^2 - \left(\frac{14}{3}\right)^2}$

$= \underline{5.217cm}$

$V = \frac{1}{3}\pi R^2 H$

$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{14}{3}\right)^2 \times 5.217$

$= 119.036$

$\simeq \underline{119\ cm^3}$ to 3 s.f

14. Given that, $6 \cos(x + 15)° - 6 \sin(2x - 30)° = 0$, find $\tan(2x - 10°)$ leaving your answer in surd form. **(3 marks)**

$\cos(x+15)° = \sin(2x-30)°$

$x+15 + 2x-30 = 90$

$3x - 15 = 90$

$3x = 105$

$\underline{x = 35°}$

$\tan(2x-10)° = \tan(70-10)°$

$= \tan 60° = \underline{\sqrt{3}}$

6

15. Solve the inequality $3x - 10 \leq 6x - 26 < 4x - 6$ hence express the solution as a combined inequality. **(2 marks)**

$$3x - 10 \leq 6x - 26 \qquad | \qquad 6x - 26 < 4x - 6$$

$$26 - 10 \leq 6x - 3x \qquad \qquad 6x - 4x < 26 - 6$$

$$16 \leq 3x \qquad \qquad \qquad 2x < 20$$

$$\frac{16}{3} \leq x \qquad \qquad \qquad x < 10$$

$$\frac{16}{3} \leq x < 10$$

16. Two trains T_1 and T_2 travelling in same direction, on parallel tracks, are just 12m apart. Train T_1 is 72m long and travelling at 108km/h. T_2 is x m long and travelling at 72km/h. Find the length of T_2 in metres if they pass each other completely after 16.2s. **(3 marks)**

Relative speed $= 108 - 72$

$$= 36 \text{ km/h} = 10 \text{ m/s}$$

Relative distance $= 72 + 12 + x$

$$= (84 + x) \text{ m}$$

Time $= 16.2 \text{ s}$

$$16.2 = \frac{84 + x}{10}$$

$$162 = 84 + x$$

$$x = 78 \text{ m}$$

SECTION B (50 MARKS)

*Answer any **FIVE** questions from this section in the space provided*

17. The frequency distribution table below shows the mass in kilograms (kg) of parcels delivered by a courier company to their office in Lubao.

Mass (kg)	50 – 99	100 – 199	200 – 249	250 - 349
Number of parcels	380	230	160	170
Frequency density	7·6	2·3	3·2	1·7

(a) Complete the table above to 1dp. **(2 marks)**

(b) On the grid provided below, draw a histogram to represent the information shown in the table above. **(3 marks)**

(c) Draw a vertical line in the histogram where the median lies. Show your calculations using the graph. **(3 marks)**

$$\frac{940}{2} = 470$$

$$470 - 380$$

$$= 90$$

$$\frac{90}{2.3} = 39.13$$

$$\simeq 39$$

$$99.5 + 39 = 138.5$$

(d) Use the histogram to determine the number of parcels with a mass of 300kg or more delivered by the courier company. **(2 marks)**

$$50 \times 1.7 = 85$$
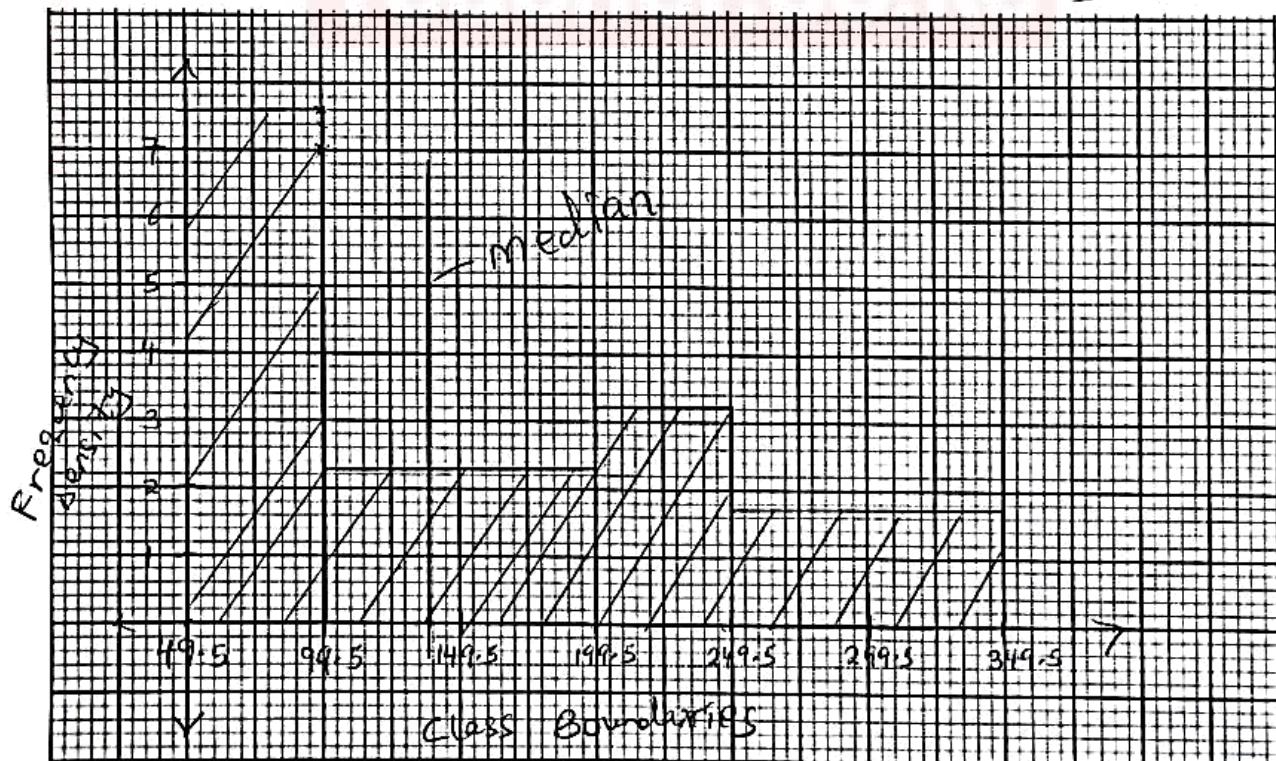
18. A solid which comprises of a cylindrical solid and a base which is a frustum of a pyramid. The cylindrical part which is open at the top has a portion of the cylinder cut off along a dotted line as shown. Radius of the cylinder is 9cm and height 28cm.
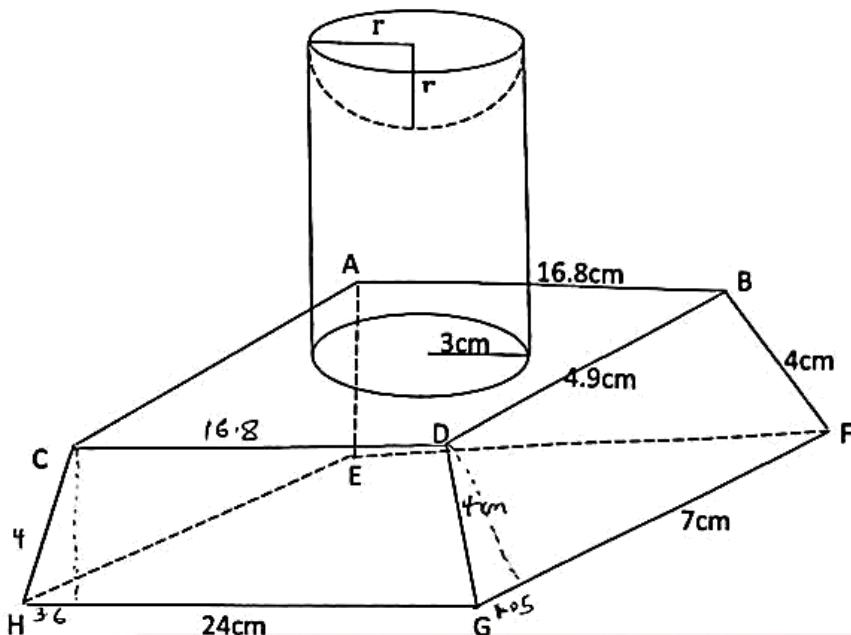
Given that the original pyramid had slant edge of 32.5cm and HG = 24cm, GF = 7cm, BC = 4.9cm and AB = 16.8cm; Find to 1d.p;

(a) the surface are of the solid. **(6 marks)**

Area of base = $24 \times 7 = 168 cm^2$

height of CDGH = $\sqrt{4^2 - 3 \cdot 6^2}$
$= 1 \cdot 744$

Area of CDGH + ABFE = $\left(\frac{1}{2}(24 + 16 \cdot 8)1 \cdot 744\right) \times 2$
$= 71 \cdot 1552 cm^2$

height of DBFG = $\sqrt{4^2 - 1 \cdot 05^2} = 3 \cdot 8597$

Area of DBFG + ACHE = $(\frac{1}{2}(7 + 4 \cdot 9)3 \cdot 8597) \times 2$
$= 45 \cdot 93 cm^2$

A of curved part of cylinder
$= \frac{22}{7} \times 18 \times 28 = 1584 cm^2$

A of curved part that was cut off = $\frac{1}{2}\pi r^2$
$= \frac{1}{2} \times \frac{22}{7} \times 9^2$
$= 509 \cdot 143$

TSA = $168 + 71 \cdot 155 + 45 \cdot 93 + 1584 + 509 \cdot 143 = 2378 \cdot 2cm^2$

(b) the volume of the solid. **(4 marks)**

vertical height of original pyramid
$= \sqrt{32 \cdot 5^2 - 12 \cdot 5^2}$
$= 30 cm$

height of smaller pyramid
$= 30 \times \frac{28 \cdot 5}{32}$
$= 26 \cdot 71875$

Volume of frustum = $\frac{1}{3} \times 24 \times 7 \times 30 - \frac{1}{3} \times 16 \cdot 8 \times 4 \cdot 9 \times 26 \cdot 71875$
$= 946 \cdot 8375$

Volume of cylindrical part
$= \frac{22}{7} \times 9 \times 9 \times 28 - \frac{1}{2} \times \frac{1}{3} \times \frac{22}{7} \times 9^3$
$= 7128 - 1527 \cdot 4287$
$= 5600 \cdot 57$

Total volume
$= 946 \cdot 8375 + 5600 \cdot 57$
$= 6547 \cdot 4 cm^3$

19. Point B is 110km on a bearing of S60⁰E from point A. Point C is 90km on a bearing of065 from B. Point D is 075 from A and 336⁰ from C.

(a) Using a scale of 1cm to represent 20km, draw a diagram to show the relative positions of A, B, C and D. **(5 marks)**
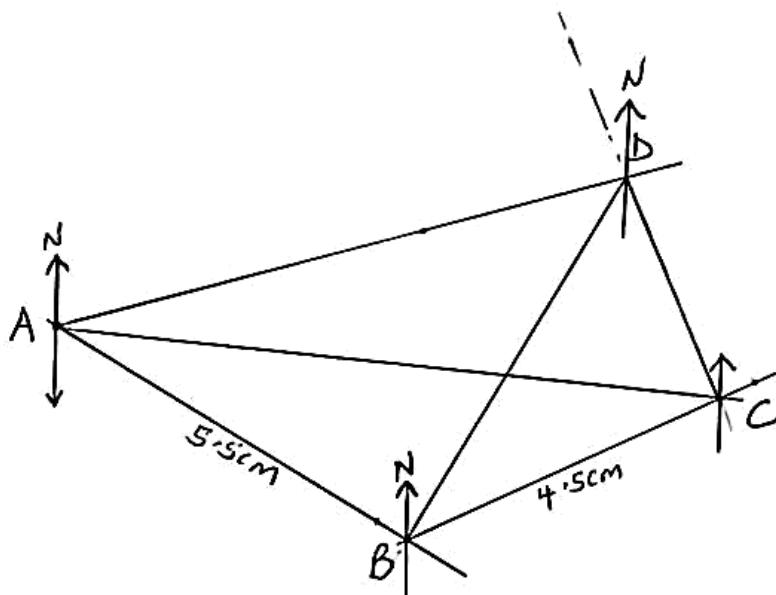
(b) Use your diagram to determine;

(i) the distance and the bearing of B from D. **(2 marks)**

$$5.5 \times 20 = 110 \text{KM} \pm 2\text{KM}$$

Bearing $210° \pm 1°$

(ii) the distance and compass bearing of A from C. **(3 marks)**

$$9 \times 20 = 180 \text{ km} \pm 2 \text{ km}$$

Bearing $N \, 84° \, W$

20. In the figure below, ABCD is a quadrilateral in which AB = 17 cm, AD = 16 cm and angle $ABC = 150°$. AB is parallel to DC and AB = BD.
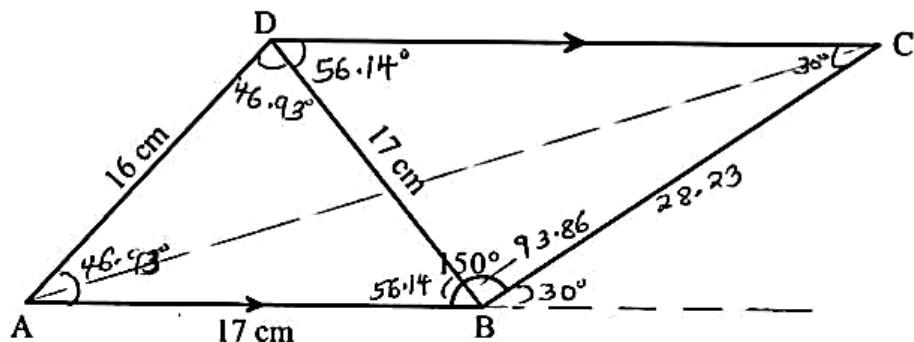
Calculate correct to 2 decimal places;

(a) The length of BC. **(3 marks)**

$b^2 = a^2 + d^2 - 2ad\cos B$

$256 = 289 + 289 - 578\cos B$

$578\cos B = 322$

$\cos B = 0.5571$

$B = 56.14°$

$\angle BDC = 56.14°$

$\dfrac{BC}{\sin 56.14} = \dfrac{17}{\sin 30}$

$BC = \dfrac{17\sin 56.14}{\sin 30} = \underline{28.23 cm}$

(b) The length of AC. **(2 marks)**

$AC^2 = 17^2 + 28.23^2 - 2(17 \times 28.23)\cos 150$

$= 289 + 796.9329 + 831.2285$

$AC^2 = 1917.1614$

$\underline{AC = 43.79 cm}$

(c) The size of angle ACD. **(3 marks)**

$\angle ADC = 103.07°$

$\dfrac{43.79}{\sin 103.07} = \dfrac{16}{\sin ACD}$

$\sin ACD = \dfrac{16\sin 103.07}{43.79}$

$\underline{\angle ACD = 20.85°}$

(d) Area of the quadrilateral. **(2 marks)**

$\frac{1}{2} \times 17 \times 17 \sin 56.14 + \frac{1}{2} \times 17 \times 28.23 \sin 93.86$

$= 119.993 + 239.411$

$= \underline{359.40 cm}$

21. Manu and Harry entered into a business partnership in which they contributed ksh. 120,000 and ksh. 150,000 every year respectively. After one year Cindy joined the business and contributed ksh. 90,000.

(a) Calculate the ratio of their investment after 3 years of business. (3 marks)

Manu = 120,000 X 3
      = 360,000
Harry = 150,000 X 3
      = 450,000
Cindy = 90,000 X 2
      = 180,000

Manu : Harry : Cindy

= 360,000 : 450,000 : 120,000

= Manu : Harry : Cindy = 4 : 5 : 2

(b) It was agreed that 30% of the profits after 3 years be used to cater for the cost of running the business, while the remaining would be shared proportionally. Calculate each person's share, if the profit made after three years has ksh. 187,000. (4 marks)

Amount shared = $\frac{70}{100}$ X 187,000

= Ksh 130,900

Manu's share = $\frac{4}{11}$ X 130,900 = Ksh 47,600

Harry's share = $\frac{5}{11}$ X 130,900 = Ksh 59,500

Cindy's share = $\frac{2}{11}$ X 130,900 = Ksh 23,800

(c) If each of them invested their shares back in the business, find their new individual investments at the beginning of the fourth year. (3 marks)

Manu = 360,000 + 47,600
     = 407,600
Harry = 450,000 + 59,500
      = 509,500
Cindy = 180,000 + 23,800
      = 203,800

Manu : Harry : Cindy

407,600 : 509,500 : 203,800

= $\frac{4076}{1019}$: $\frac{5095}{1019}$: $\frac{2038}{1019}$

= 4 : 5 : 2

22.   The table below shows the income tax rates in a certain year.

Total income in k£ per annum	Rate in shs per pound
1-3900	2
3901-7800	3
7801-11,700	4
11701-15600	5
15601-19500	7
Over 19500	7.5

Mrs. Masau earned a basic salary of ksh18600 per month and allowances amounting to ksh 7800 per month. She claimed a personal relief of ksh 1080 per month.

Calculate:

a) Total taxable income in k£ p.a   (2 marks)

$$18,600 + 7800$$

$$= Ksh\ 26,400 \times \frac{12}{20}$$

$$= K£\ 15,840\ P.a$$

b) i) the tax payable in ksh per month without relief   (4marks)

$$3900 \times 2 = 7,800$$
$$3900 \times 3 = 11,700$$
$$3900 \times 4 = 15,600$$
$$3900 \times 5 = 19,500$$
$$240 \times 7 = \underline{1,680}$$
$$\underline{56,280}$$

$$\frac{56280}{12} = \underline{Ksh\ 4,690\ P.M}$$

ii) the tax payable in ksh per month after relief   (2marks)

$$Ksh(4,690 - 1080)$$

$$= \underline{Ksh\ 3,610\ P.M}$$

c) Mrs. Musau's net monthly income   (2marks)

$$18600 + 7800 - 3610$$

$$= \underline{Ksh\ 22,790}$$

13

a. **(a)** Complete the below for the function $y=2x^2-3x-5$ for $-2 \leq x \leq 3$. **(2 marks)**

X	-2	-1	0	1	2	3
Y	9	0	-5	-6	-3	4

(b) On the grid provided, draw the graph of $y=2x^2-3x-5$ for $-2 \leq x \leq 3$. **(3 marks)**
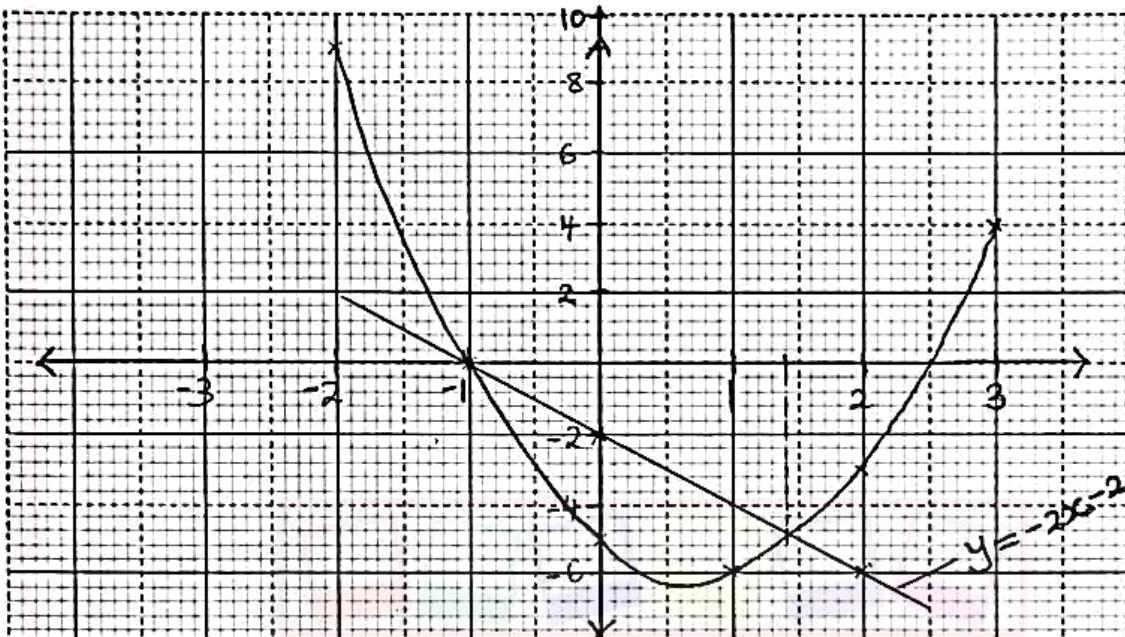
(c) Use your graph to find the roots of the equation $2x^2 -3x - 5 = 0$ **(1 mark)**

$$x = -1 \text{ or } 2.5$$

(d) (i) On the same axes, draw the graph of $y= -2x - 2$ **(1 mark)**

X	0	2
Y	-2	-6

(ii) From your graphs, find the values of x that satisfy the simultaneous equations $y= -2x - 2$ and $y=2x^2-3x-5$. **(1 mark)**

$$x = -1 \text{ or } 1.4$$

(iii) Hence write the quadratic equation in the form $ax^2 + bx + c = 0$ satisfied by the values of x where the two graphs intersect, where a, b and c are integers. **(2 marks)**

$$x+1 = 0 \text{ or } x-1.4=0$$

$$(x+1)(x-1.4)=0$$

$$x^2 -1.4x +x -1.4 = 0$$

$$x^2 -0.4x -1.4 = 0$$

$$\underline{5x^2 -2x -7 =0} \quad \text{or} \quad \underline{10x^2 -4x -14= 0}$$

14

24. (a) On the grid provided, draw a square (s) with vertices (6,3), (7, 0), (9, 4) and (10, 1). Draw also two straight lines AB and AC where the coordinates of A, B and C are(1, − 2). (3, 2)   and (0, 1) respectively.

    S^1 is the image of S under reflection in the line AB and S^{11} is the image of S^1 under reflection in the line AC. Draw S^1 and S^{11}. **(7 marks)**
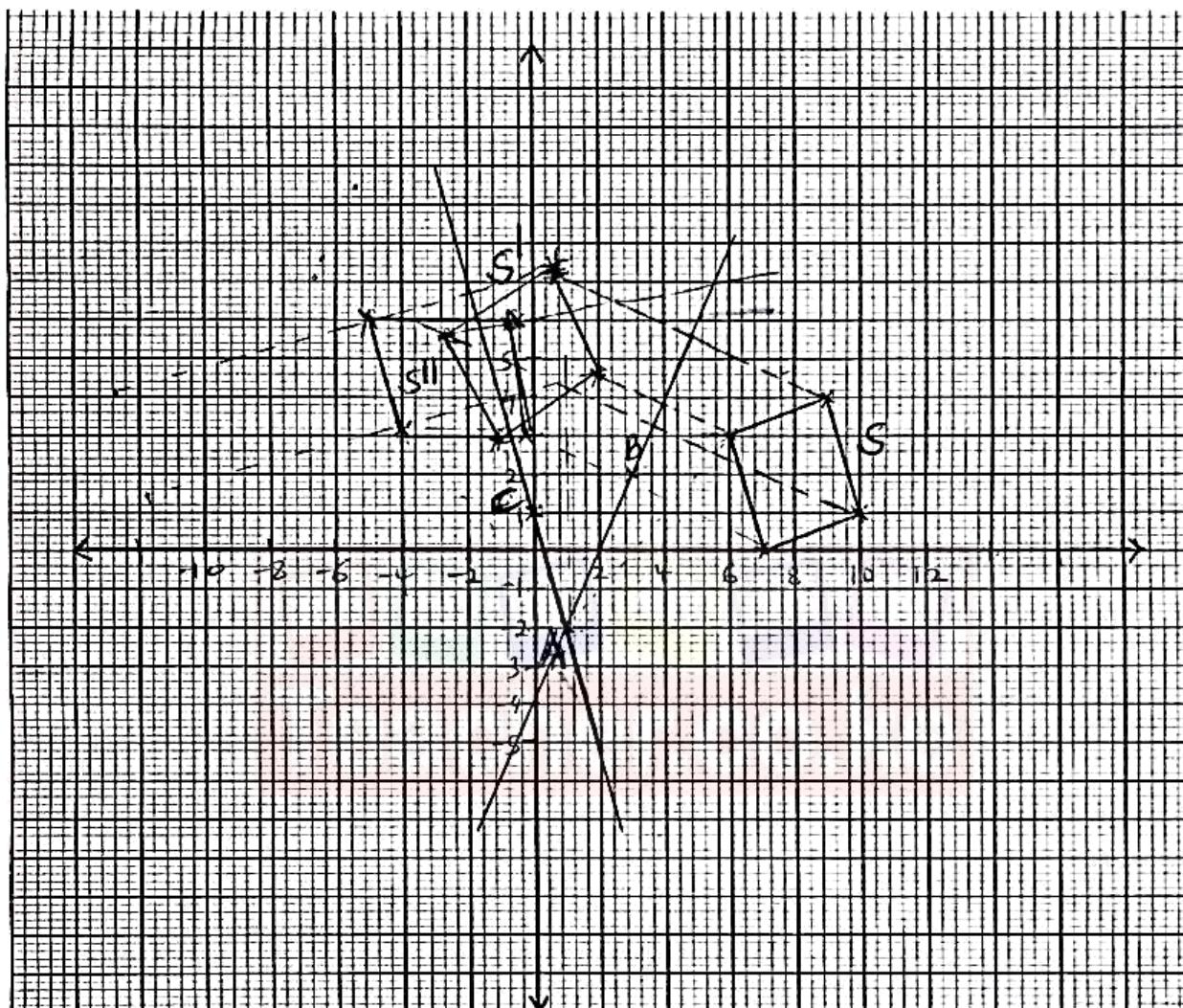
(b) Describe the transformation which maps S onto S^{11} if the transformation is;

    (i) Translation **(1 mark)**

$$\text{Translation} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} -10 \\ 0 \end{pmatrix}$$

    (ii) Rotation **(2 marks)**

+ Quarter turn centre (1, −2)

15