121/2 SCHEME MOMALICHE CYCLE 3 2021

SECTION I

1. Jane mistypes $(x + y)^2$ as $x^2 + y^2$. Find the percentage error in the evaluation of $(x + y)^2$ when x= -2 and y= 12 (3mks)

$(x+y)^2 = (12-2)^2 = 100$

$x^2+y^2 = 12^2 + (-2)^2 = 148$ M1

error $= 148 - 100 = 48$

$\% = \frac{48}{100} \times 100$ M1

$= 48\%$ A1

2. An arc length a cm subtends angle of 0.39^c at the centre of a circle of radius 6cm. find the value of a (3mks)

$\theta = 57.3 \times 0.39$

$= 22.347$

$\frac{22.347}{360} \times 2 \times 3.142 \times 6$

2.34 cm

OR

$\frac{\text{arc length}}{\text{radius}} = \theta$

$0.39 = \frac{a}{6}$

$2.34 \text{ cm} = a$

3. Find the semi-interquartile range of the following set of numbers. 63, 65, 76, 65, 63, 51, 52, 95, 63, 71, 83. (3mks)

51, 52, 63, 63, 65, 71, 76, 83, 95

$Q_1 = \frac{52+63}{2} = 57.5$

$Q_3 = \frac{76+83}{2} = 79.5$ M1

$\frac{Q_3 - Q_1}{2} = \frac{79.5 - 57.5}{2}$ M1

$= \frac{22}{2} = 11$ A1

4. Solve the equation. (3mks)

$$\log(3x - 1) = \log(2x + 1) - \log 4$$

$3x - 1 = \frac{2x+1}{4}$ M1

$12x - 4 = 2x + 1$

$10x = 5$ M1

$x = \frac{1}{2}$ A1

5. Rationalize the denominator (3mk)

$$\frac{\sqrt{5} + \sqrt{3}}{\sqrt{7} - \sqrt{3}}$$

conjugate

$$\frac{\sqrt{5}+\sqrt{3}\,(\sqrt{7}+\sqrt{3})}{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})} \quad M_1$$

$$\frac{\sqrt{35}+\sqrt{15}+\sqrt{21}+3}{7-3} \quad M_1$$

$$\frac{\sqrt{35}+\sqrt{15}+\sqrt{21}+3}{4} \quad A_1$$

6. (i) Write down the first 4 terms in a seconding power of x in the expansion of

$(1-2x)^5$ (2mks)

$$1 - 5(2x) + 10(2x)^2 - 10(2x)^3 + --- \quad M_1$$

$$1 - 10x + 40x^2 - 80x^3 + --- \quad A_1$$

(ii) Use your expansion to estimate the value of $(0.96)^5$ (2mks)

$1-2x = 0.96$

$0.04 = 2x$

$0.02 = x$

$$1 - 10(0.02) + 40(0.0004) - 80(0.000008) \quad M_1$$

$$1 - 0.2 + 0.016 - 0.00064$$

$$0.81536 \quad A_1$$

7. Make q the subject of the formula (3mks)

$$T = \left(\frac{b - q}{q}\right)^{1/2}$$

$$T^2 = \frac{b-q}{q} \quad M_1$$

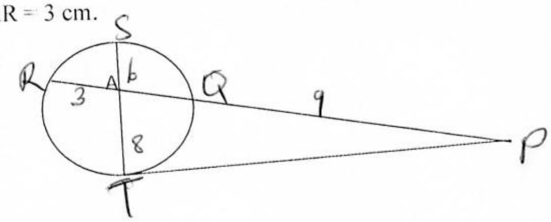
$$T^2 q = b - q$$

$$T^2 q + q = b \quad M_1$$

$$q\,(T^2+1) = b$$

$$q = \frac{b}{T^2+1} \quad A_1$$

8. In the figure below, PT is a tangent to the circle of T. PQ = 9cm, SA =6cm, AT = 8 cm and AR = 3 cm.



Calculate the length of

(i) AQ (2mks)

$SA \cdot AT = RA \cdot AQ$

$6 \times 8 = 3AQ$ M1

$16\text{cm} = AQ$ A1

(ii) PT (2mks)

$RP \cdot QP = PT^2$

$28 \cdot 9 = PT^2$ M1

$\sqrt{252} = PT$

$15.87\text{cm} = PT$ A1

9. Solve the simultaneous equations (~~3~~4mks)

$$x^2 + xy = 4$$

$$y - x = 2$$

$y = 2 + x$

$x^2 + x(2 + x) = 4$

$x^2 + 2x + x^2 = 4$

$2x^2 + 2x - 4 = 0$ M1

$x^2 + x - 2 = 0$

$x^2 - x + 2x - 2 = 0$

$x(x-1) + 2(x-1) = 0$

$(x+2)(x-1) = 0$

$x = -2$ or 1 A1

when $x = -2$, $y = 0$ B1

when $x = 1$, $y = 3$ B1

10. An arithmetic progressive whose first term is 2 and the n^{th} term 32 has the sum of n terms equal to 357. Find n. (3mks)

$a = 2$

$l = 32$

$\frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l) = 357$

$\frac{n}{2}(2+32) = 357$ M1

$34n = 714$ M1

$n = 21$ A1

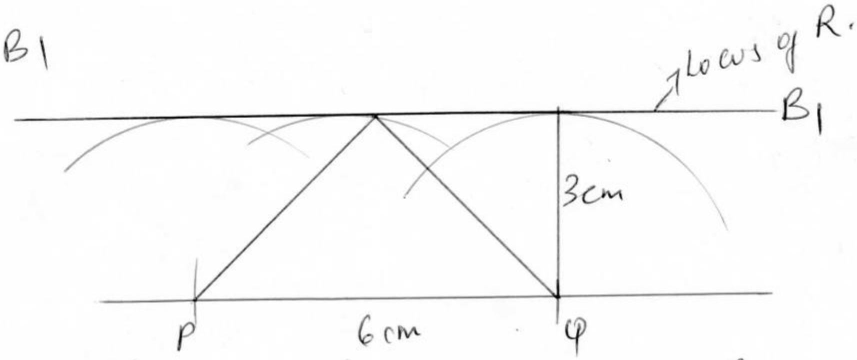
11. PQR is a triangle of area $9cm^2$. If PQ is the fixed base of the triangle and is 6cm long. On the upper side of PQ.

Draw ΔPQR and describe the locus of point R (3mks)

$\frac{1}{2}bh = 9$

$\frac{1}{2} \times 6h = 9$

$h = 3$ B1



R → Is a line parallel to PQ, 3 cm away. B1

12. Use matrix method to solve. (3mks)

$b = 4a + 6$

$3a - 2b = -2$

$4a - b = -6$

$3a - 2b = -2$

$\begin{pmatrix} 4 & -1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -6 \\ -2 \end{pmatrix}$ M1

det $= -8 + 3 = -5$

$\frac{1}{-5}\begin{pmatrix} -2 & 1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -2 & 1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} - \\ - \end{pmatrix}$ M

$\begin{pmatrix} a \\ b \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} 10 \\ 10 \end{pmatrix}$

$\left.\begin{matrix} a = -2 \\ b = -2 \end{matrix}\right\}$ A1 both

13. State the amplitude and the period of the wave $y = 3\sin\frac{3}{4}\theta$ (2mks)

Amplitude = 3 B1

Period = $360 \times \frac{4}{3} = 480°$ B1

14. Find the centre and radius of the circle whose equation is $4x^2 - 12x + 4y^2 - 8y - 3 = 0$ (3mks)

$x^2 - 3x + y^2 - 2y - \frac{3}{4} = 0$

$x^2 - 3x + \left(\frac{-3}{2}\right)^2 + y^2 - 2y + \left(\frac{-2}{2}\right)^2 = \left(\frac{3}{4}\right) + \frac{9}{4} + 1$ M1

$\left(x - \frac{3}{2}\right)^2 + (y-1)^2 = 4$

Centre $(1.5, 1)$ B1

radius = 2. B1

15. Twenty men can dig a trench 300m long in 15 days. Find the number of days it would take 30 men to dig a trench 360m long. (3mks)

men	Trench	Days
20	300	15
30	360	

$\frac{2\cancel{0}}{3\cancel{0}}$ M1 $\times \frac{36\cancel{0}}{30\cancel{0}}$ M1 $\times \cancel{15} = 12$ days A1

16. Use logarithms to evaluate $\frac{1.76\sqrt[3]{0.2876}}{379.5}$ (3mks)

No	Log
1.76	0.2455
$0.2876^{1/3}$	$\bar{1}.4588 \times \frac{1}{3}$
	$\frac{\bar{3}}{3} + \frac{2.4588}{3} = \bar{1}.8196$
	0.0651
379.5	2.5792 −
	$\bar{3}.4859$

3.061×10^{-3}

0.003061

SECTION II

17. The table below shows the marks scored by students in a mathematics test.

Marks	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No of students	3	5	6	21	12	6	4	2	1
C.f	3	8	14	35	47	53	57	59	60 B_1

(a) From the table above, determine the 25th percentile (2mks)

P_{25} $\frac{25}{100} \times 60 = 15$

$39.5 + \left(\frac{1}{21} \times 10\right) = 39.98$

(b) On the grid provided, draw an Ogive curve that represents the above information. (4mks)

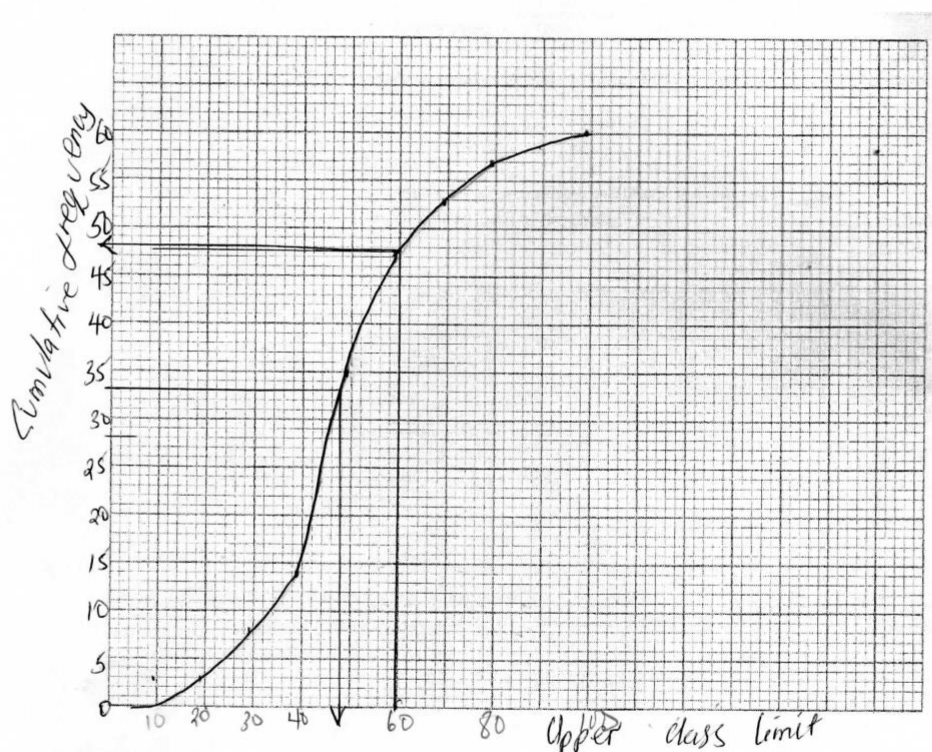


Table B_1 of C.f.

S_1

P_1

C_1

(c) Using the graph above

(i) Determine the pass mark if 45% of the students passed. (2mks)

$\frac{45}{100} \times 60 = 27$ passed

33 failed M_1

Pass mark $= 48 + 1$

$= 49$ A_1

(ii) If the pass mark was to be pegged at 60%. How many students passed? (2mks)

$\underset{M_1}{60 - 48} = \underset{A_1}{12}$ students

18. Three qualities P, Q and R are such that P varies directly as Q $\cancel{8}$ inversely as the square root of R.

Give that P = 2250 when Q = 450 and R = 64

(a) Write down an equation connecting P, Q, & R (4mks)

$P \propto \frac{Q}{\sqrt{R}}$

$P = \frac{kQ}{\sqrt{R}}$ ✓

$2250 = \frac{450k}{8}$ ✓

$k = 40$ ✓

$P = \frac{40Q}{\sqrt{R}}$ ✓

(b) If Q decreased by 16% and R increased by 44%.

Calculate the percentage change in (3mks)

$Q_1 = 0.84Q$

$R_1 = \sqrt{1.44R}$

$P_1 = \frac{k\,0.84Q}{1.2\sqrt{R}}$ ✓

$= \frac{0.84}{1.2}P$

$\left(\frac{0.7P - P}{P}\right) \times 100$ ✓

$\frac{P(-0.3)}{P} \times 100 = -30$

Decrease of 30% ✓

(c) In a soccer competition the number of goals (G) scored in a penalty shoot-out is partly constant and partly varies as he skill (S) of the player. Given that when S= 1 and G= 6 when S= 2 G=4.find the value G when S =3 (3mks)

$G \propto C + S$

$G = C + mS$

$6 = C + m$

$4 = C + 2m$

$2 = -m$

$m = -2$ ✓

$C = 8$ ✓

$G = 8 - 2S$

$G = 8 - 6$

$= 2$ ✓

19. The cost of a minibus was sh 950000. It depreciated in value by 5% per year for the first two years and by 15% per year for subsequent years.

(a) Calculate the value of the minibus after 5 years. (4mks)

first 2 yrs

$950\,000\left(1-\frac{5}{100}\right)^2$ M1

$950\,000(0.9025)$

sh 857375 A1

Subsequent 3 yrs

$857375\left(1-\frac{15}{100}\right)^3$ M1

$857375(0.614125)$

~~526535.9875~~

sh 526535 A1

OR

$950,000 \times 0.95^2 \times$ ...

526535.42

(b) After 5 years, the minibus was sold through a dealer at 25% more than its value to Mr. Nyeri. If the dealer's sale price was to be taken as its value after depreciation, calculate the average monthly rate of depreciation for the 5years. (6mks)

$\frac{125}{100} \times 526535 = \text{sh } 658168.75$

$658168.75 = 950\,000\left(1-\frac{r}{100}\right)^5$

$0.6928 = \left(1-\frac{r}{100}\right)^5$

$\sqrt[5]{0.6928} = 1-\frac{r}{100}$

$0.9292 = 1-\frac{r}{100}$

$\frac{r}{100} = 0.07077$

$r = 7.077$ p.a

Rate per month $= \frac{7.077}{12} = 0.58975$

=

20. A trader deals in two types of rice .Type P & Q .Type P costs ksh 1600 per bag and type Q sh 1400 per bag

(a) The trader mixes 30 bags of type P & 50 bags of type Q. If he sells the mixture at a profit of 20%,

Calculate the selling price of one bag of the mixture. (4mks)

Buying price of mixture

$$\frac{(30\times1600)+(50\times1400)}{30+50} \text{ M1}$$

$$\frac{118000}{80} = \text{sh } 1475 \text{ A1}$$

Selling price of mixture.

$$\frac{120}{100}\times 1475 \text{ M1}$$

1770 A1

(b) The trader now mixes type P with type Q in the ration x:y respectively. If the cost of the mixture is ksh 1534 per bag, find the ration. (4mks) 3

$$\frac{1600x+1400y}{x+y} = 1534 \text{ M1}$$

$$66x = 134y \text{ M1}$$

$$\frac{x}{y} = \frac{67}{33}$$

$x:y = 67:33$ A1

(c) The trader mixes one bag of the mixture in part (a) with one bag of the mixture in part (b) above.

Calculate the ration of type P rice to type Q rice in this mixture. (2mks) 3

Fraction of P in (a) $\frac{30}{80}$

in (b) $\frac{67}{100}$

$$\text{Total} = \frac{30}{80}+\frac{67}{100} = 1.045 \text{ M1}$$

Fraction of Q in (a) $\frac{50}{80}$

in (b) $= \frac{33}{100}$

$$\text{Total} = \frac{50}{80}+\frac{33}{100} = 0.955 \text{ M1}$$

1.045 : 0.955

209 : 191 A1

21. A point D' A' and Y' are images DAY with vertices D (4, 4) A (0,2) and Y(-2,4) respectively under a transformation given by matrix P=$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

Determine the co-ordinates of D', A' and Y' (3mks)

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 4 & 0 & -2 \\ 4 & 2 & 4 \end{pmatrix} = \begin{array}{ccc} D' & A' & Y' \\ \end{array}\begin{bmatrix} 12 & 2 & 0 \\ 8 & 2 & 2 \end{bmatrix}$$

B1 B1

D' (12, 8)
A' (2, 2)
Y' (0, 2) A1

(b) Triangle D'' A'' Y'' is the image of triangle D' A' Y' under another transformation whose matrix is Q=$\begin{pmatrix} -1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$

Determine the coordinates of D'' A'' Y'' (3mks)

$$\begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 12 & 2 & 0 \\ 8 & 2 & 2 \end{bmatrix} = \begin{array}{ccc} D'' & A'' & Y'' \end{array}\begin{bmatrix} -6 & -1 & 0 \\ -4 & -1 & -1 \end{bmatrix}$$

B1 B1

D'' (-6, -4)
A'' (-1, -1)
Y'' (0, -1) A1

(c) Find a single matrix of transformation that maps D" A" Y" onto triangle D A Y (2mks)

$(QP)^{-1}$

$$\begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = \begin{bmatrix} -1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}^{-1}$$

$$4\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -1 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -4 \end{bmatrix}$$

(d) A plane figure whose area is 20cm^2 undergoes transformation represented by QP. Find the area of image. (2mks)

$$QP = \begin{bmatrix} -1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

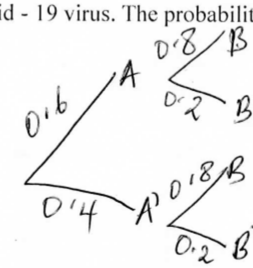
$$\det = \frac{1}{2} - \frac{1}{4} = \frac{1}{2} ✓$$

$$\frac{1}{2} = \frac{\text{Area of Image}}{20}$$

Area of Image = $\underline{\underline{10\text{cm}^2}}$ ✓

23. Patients A and B are to be tested for covid - 19 virus. The probability that A will be positive is 0.6 and that B will be negative is 0.2

Find the probability that



→ +ve

A' → -ve

(a) Both will be positive (2mks)

0.6×0.8

$\underline{\underline{0.48}}$

(b) Neither will be positive (2mks)

$0.4 \times 0.2 = 0.08$

(c) One will be positive (2mks)

$(0.6 \times 0.2) + (0.4 \times 0.8)$

$0.12 + 0.32 = \underline{\underline{0.44}}$

(d) At least one will be positive (2mks)

One or Both

$0.44 + 0.48 = 0.92$

e) At least B is negative . (2mks)

$(0.6 \times 0.2) + (0.4 \times 0.2)$

$0.12 + 0.08$

$\underline{\underline{0.2}}$

24. (a) Complete the table below

x	0	30	60	90	120	150	180	210	240	270	300	330	360
Sin x	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0
2 sin (x+30)	1	1.73	2	1.73	1	0	-1	-1.73	-2	-1.73	-1	0	1

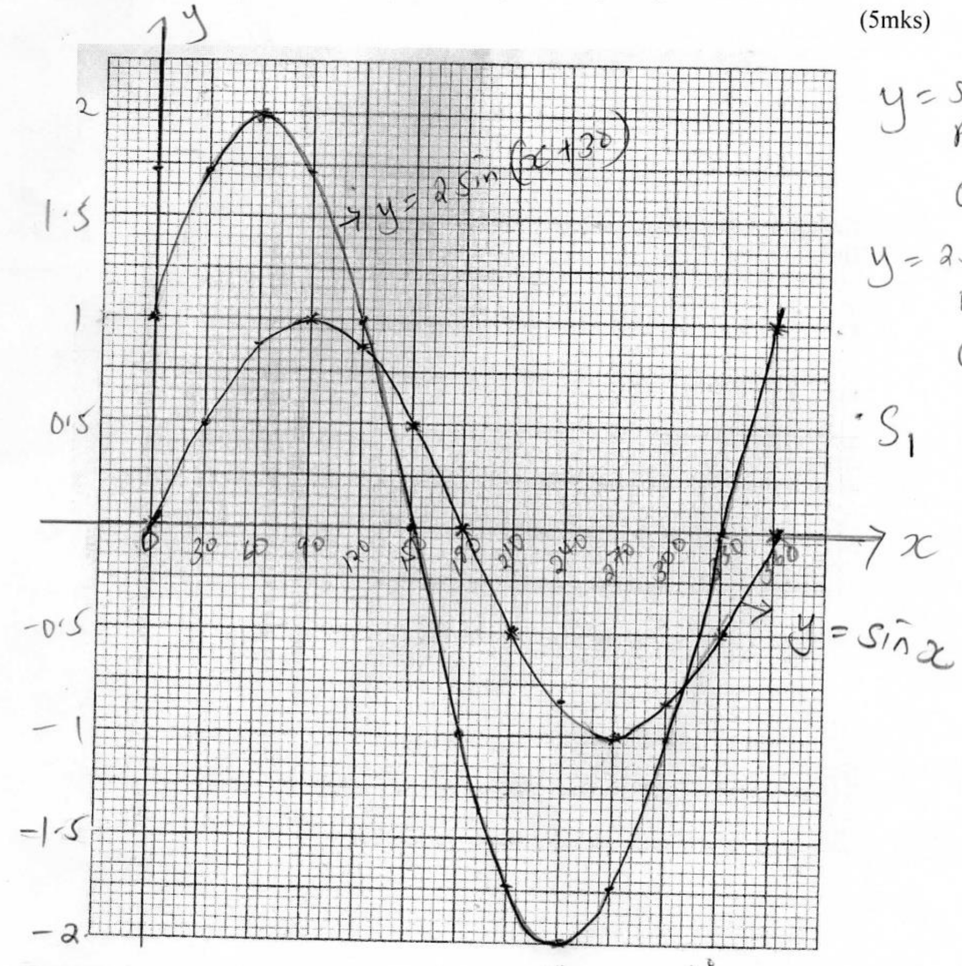
B_1 any

B_2 9

(2mks)

(b)On the same axes, draw the graphs of y = sin x and y = 2 sin (x+30) $0^0 \leq x \leq 360^0$

(5mks)



y = sin
P_1
C_1
y = 2 sin
P_1
C_1
S_1

(c) From the graph, find the roots of

$x = 126°$ or $306°$

2 sin (x + 30) – sin x = 0

(1mk)

(d) Describe fully the transformation that maps the graph of

Y = sin x onto that of Y = 2 sin (x + 30^0)

(2mks)

Stretch ~~scale~~ parallel to the y-axis scale factor 2, followed by translation $\begin{pmatrix} -30 \\ 0 \end{pmatrix}$.