. Differentiation

1	$\int_1^2 (9t^2 - 6t + 2)dt$ $[3t^3 - 3t^2 + 2t + c]_1^2$ $(3\times 2^3 - 3\times 2^2 + 2\times 2) - (3 - 3 + 2)$ $(24 - 12 + 4) - (2)$ $16 - 2 = 14m$	M_1 M_1 A_1	
2.	(a) V = ds/dt = 8 – 2t (i) At t = 1 V = 8 – 2 = 6m/s (ii) At t = 3 v = 8-6 = 2m/s (b) At maximum ds.dt = 0 8 – 2t = 0 t = 4 secs therefore maximum displacement s = 8t – t2 S = 8 x 4 - 42 = 16m (c) Acceleration = dv/dt = 2m/s-2 (d) At starting point, displacement is zero = 8t – t2 = 0 t(8 – t) = 0 t = 0 or t = 8 body back after 8sec	B1 B1 B1 M1 A1 M1 A1 B1 M1 A1 	
		10	

1. *$S = t^3 - 3t^2 + 2t$*

(a) $V = \frac{ds}{dt} = 3t^2 - 6t + 2$

When t = 2

$V = 3(4) - 6(2) + 2$

$= 2m/s$

(b) At minimum velocity :

$\frac{dv}{dt} = 0$

$\frac{dv}{dt} = 6t - 6$

$6t - 6 = 0$

$t = 1$

Min-velocity = $3(1)^2 - 6(1) + 2$

$= -1 m/s$

(c) $3t^2 - 6t + 2 = 0$

$t = \frac{6 \pm \sqrt{(-6) - 4(3)(2)}}{6}$

$= \frac{6 \pm 5.2}{6}$

$t = 1.58 \text{ or } 0.4 sec$

(d) $acc = \frac{dv}{dt} = 6t - 6$

$a = 6(3) - 6 = 12 m/s^2$

2. a)

X	2	5	8	10
y	5	26	65	101

b) $A = h(2 + 10 + 26 + 50 + 82)$

$= 2 \times 170$

$= 34$ square units

c) $A = (x^2 + 1)\, dx$

$= ({}^{1000}/_{3} + 10) - 0$

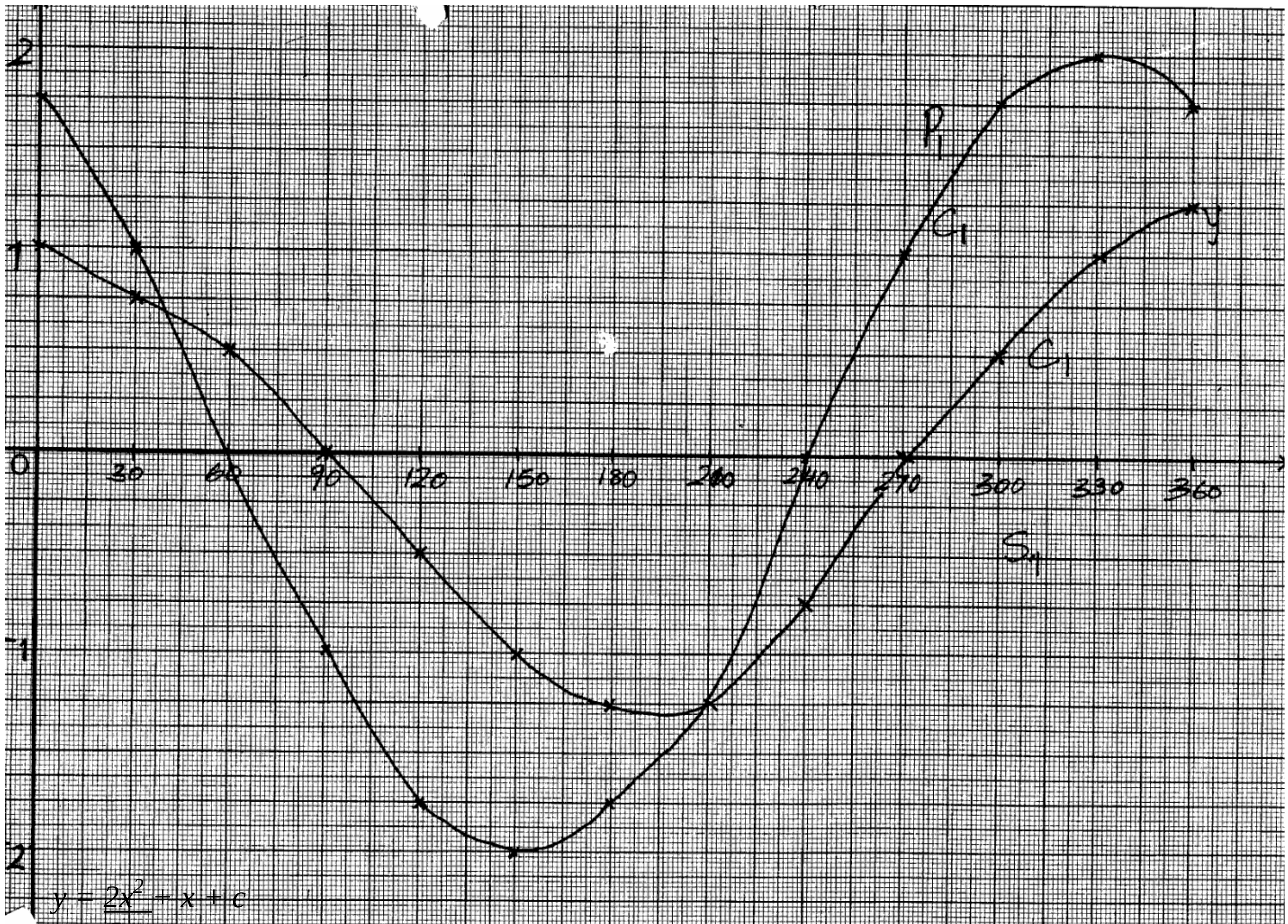
$= 333.33 + 10$

$= 343.33$

$= 343.33$ square units

d) Percentage error $= {}^{3.33}/_{343.33} \times 100\ \%$

$= 0.97\%$



3. $y = \frac{2x^2}{2} + x + c$

$a + x = -4 , y = 6$

$6 = (-4)^2 – 4 + c$

$c = -6$

$y = x^2 + x - 6$

4. a) $-2t^2 + t + 28 = 0$

$P = -56$

$S = 8, -7$

$-2t^2 + 8t – 7t + 28 = 0$

$-2t (t – 4) – 7(t – 4) = 0$

$t = 3.5$

$t = 4$

b) $AC = -4t + 1$

$- 4t + 1 = 0$

$T = ¼$

$V = -2(¼)2 + ¼ + 28$

$V = 28.125$

c) $Acc = - 4t + 1$

At rest $t = 3.5$, $t = 4$

$Acc = -4 \times 4 + 1$

$= -15 m/s^2$

At $t = 3.5$

$A = -13 m/s^2$

d)(i) $D = \frac{2t^3}{3} + \frac{t^2}{2} + 28t + 5$

Distance = $-2 \times 3^3/_3 + 3^2/_3 + 28 \times 3 + 5 = 75.5m$

ii) $D = \frac{2t^3}{3} + \frac{t^2}{2} + 28t + 5$

$D = -2 \times 3^3/_3 + 3^2/_3 + 28 \times 3 + 5$

$= -18 + 4.5 + 84 + 5$

$= 70.5 + 5 \quad = 75.5$

5. a i) $V = 15 + 4t - 3t^2$

$\frac{dv}{dt} = Acc = 4 - 6t$

ii) $V = 15 + 4t - 3t^2$

$V = \frac{dv}{dt} = 15 + 4t - 3t^2$

$\therefore S = \int (15 + 4t - 3t^2)\, dt$

$S = 15t + \frac{4t^2}{2} - \frac{3t^2}{3} + C$

$S = 15t + 2t^2 - t^3 + C$

b) i) Acc = 0 hence $\frac{dv}{dt} = 0$

$4 - 6t = 0$

$-6 = -4$

$t = {}^2/_3$ sec.

ii) $S = \left[15t + 2t^2 - t^3 + C \right]_0^{2/3}$

$= 15\left({}^2/_3\right) + 2\left({}^2/_3\right)^2 - \left({}^2/_3\right)^3$

$= \frac{10}{1} + \frac{8}{9} = \frac{8}{27}$

$= \frac{286}{27} \quad = 10.5925 \quad \simeq 10.59$

c) Acc. $4 - 6t$

$-4 = -6t$

$t = {}^2/_3$ Acc.= 0

$\therefore$ Time is 0 and ${}^2/_3$

Bth. 0 and ${}^2/_3$ sec.

6. (a) $x^2 = -x2 + 8$

$2x^2 = 8$

$x = 2 \quad a = -2, \quad b = 2$

(b) Area of $\int_{-}^{2} x^2 = \left[\frac{x^3}{3}\right]_{-2}^{2}$

$= \frac{8 - {}^-8}{3}$

$= \frac{16}{3}$

$Area = (x^2 + 8)dx$

$= \left(\frac{-x^3}{3}\right) + 8x$

$= \left(\frac{-80}{3} + 16\right)\left(-\frac{{}^-8}{3} - 16\right)$

$\frac{80}{3} = 26\frac{2}{3}$

(c) $Area = \frac{80}{3} + \frac{16}{3} = \frac{96}{3}$

$= 32$

7. $a = \frac{d^2s}{dt^2} = \frac{d^2}{dt^2}(t^3 - \frac{5t^2}{2} + 2t + 5)$

$= \frac{d}{dt} = 3t^2 - 5t + 2$

$= 6t - 5$

If $a = 0$

$6t - 5 = 0$

$t = {}^5/_6$

$v = \frac{ds}{dt} = 3t^2 - 5t^2 = 3 \times \frac{25}{36} - 5 \times \frac{5}{6} + 2$

$= -\frac{1}{12} m/s$

8. (a) $V = 6t + 4 = 3t^2 + 4t + c$

$5 = 3(0)^2 + 4(0) + c$

$5 = c$

$V = 3t^2 + 4t + 5$

(b) $V = 3(4)^2 + 4(4) + 5$

$= 69$ m/s

(c) (i) $\int 3t^2 + 4t + 5$

$= t^3 + 2t^2 + 5t + c$

When $t = 0$ $S = 0$

$S = t^3 + 2t^2 + 5t$

(ii) $S = \left[t^3 + 2t^2 + 5t\right]_1^4$

$= \left((4)^3 + 2(4)^2 + 5(4)\right) - \left((1)^3 + 2(1)^3 + 5(1)\right)$

$= 108$ m

9. a) $S = 3t + \frac{3t^2}{2} - 2t^3$

$\frac{ds}{dt} = v = 3 + 3t - 6t^2$

$\frac{dv}{dt} = a = 3 - 12t \qquad t = 0$

$a = 3m/s^2$

b)i) $O = -6t^2 = 3t + 3$ $\qquad -8t^2$ → $+6t - 3t$

$t = 1$

ii) $S = 3(1) + \frac{3(1)^2}{2} - 6(1)^3$

$= 3 + {}^3/_2 - 2$

$= {}^2/_2 + {}^3/_2 = {}^5/_2$

c) $V = 3 + 3(1) - 6(1)$

$= 3 + 3 - 6$

$= 0m/s$

10. $dy/dx = 12x2 - 4x - 3$ at $(2,23)$

$= 12(4) - 4(2) - 3$

$= 48 - 8 - 3$

$= 40 - 3$

$= 37$

$M = y - y$ or $y = mx + c$

$= \frac{23 - y}{2 - x}$

$23 - y = 37(2-x)$

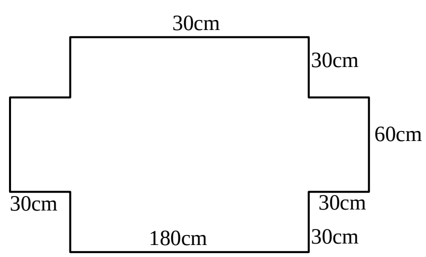
$23 - y = 74-x$

$23 = 37(2) + c$

$C = 23 - 74 = -51$

Hence equation is $y = 37x - 5$

11.



(i) $(180 \times 30 \times 2) = 10800$

$(60 \times 30 \times 2) = 3600$

$(180 \times 60 \times 1 = \underline{10800}$

Total area $= 25200cm^2$

(ii) Volume of the cuboid

$= (180 \times 60 \times 30)\ cm^3 = 324{,}000cm^3$

Mass $= (2.5 \times 180 \times 60 \times 30)$

$= \frac{810000g}{1000}$

$= 810kg$

Volume of water $= (324{,}000cm^3)$

Mass of water $= \frac{(324{,}000 \times 1)}{1000}$

$= 324kg$

Mass of cuboid $= 324 + 810$

Full of water $= 1{,}134kg$

12. Let length of square cut off be x

Length of box = 8 – 2x

Width of box = 5 – 2x

Height of box = x

$V = (8 - 2x)(5 - 2x)x$

$= 4x^3 - 26x^2 + 40x$

$\frac{dv}{dx} = 12x^2 - 52x + 40$

$12x^2 - 52x + 40 = 0$

$3x^2 - 13x + 10 = 0$

$3x^2 - 10x - 3x + 10 = 0$

$X(3x - 10) - 1(3x - 10) = 0$

$(x - 1)(3x - 10) = 0$

$x = 1 \qquad x = {}^{10}/_{3}$

${}^{d2}/_{dx2} = 24x - 52$

$x = 1$

${}^{d2}/_{dx2} = 24x - 52 = -28$

maximum

x = 1cm gives maximum vol

$(8-2)(5-2) X 1 = 6 x 3$

$= 18cm^3$

13. a) $\frac{dy}{dx} = 3x^2 - 2$

Gradient of the tangent is 1 so, gradient of the normal is -1

$\frac{y - {}^{-}2}{x - 1} = \frac{-1}{1}$

$\frac{y + 2}{x - 1} = \frac{-1}{1}$

$y = -x - 1$

(b) $dy = 3x^2 - 3 = 0$

$3x^2 - 1) = 0$

$(x-1) = 0$

$x = 1, y = 0$ & $x = -1, y = 4$

Coordinates of turning points

(1,0) and (-1, 4)

For (1,0) $x < 1$, $\frac{dy}{dx}$ is –ve

$x > 1$, $\frac{dy}{dx}$ is +ve

(1,0) is a minimum point for (-1, 4) $x < -1$, $\frac{dy}{dx}$ is +ve

(1, 0) is a minimum point for (-1, 4) $x < -1$, $\frac{dy}{dx}$ is +ve

$x > -1$, $\frac{dy}{dx}$ is –ve

⇒ (-1, 4) is a maximum point

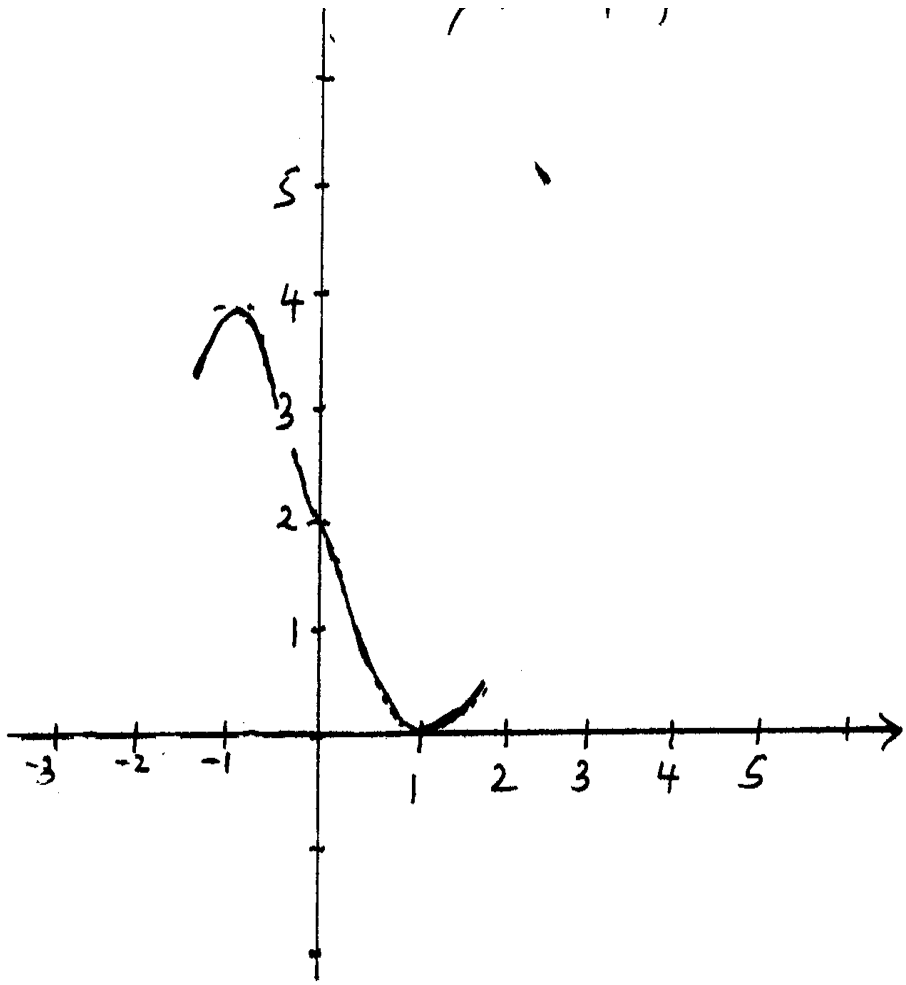
To sketch the curve we

(i) Its turning points and their nature

(ii) The points the graph cuts the x and y axis i.e the x and y-intercepts

(b) ⇒Indicating that the curve turns at (-1, 4) (1, 0) and cuts the y-axis at (0, 2) B_1

⇒C_1 for correct sketch



14. a) $-2t^2 + t + 28 = 0$

$t^2 - t - 28 = 0$

$2t^2 - 8t) + (7t - 28) = 0$

$+ (t-4) + 7(t-4) = 0$

$t + 7) (t-4) = 0$

$t = -3.5$ or 4

p.B at rest at t= 4seconds

(b) $a = 1-4t$

$1 - 4t = 0$

$0.25s = t$

$V = 28 + 25 - 2 (0.25)^2$

$= 28.25 - 0.125$

$V = 28.125m/s$

(c) (i) $S = 28t + \frac{t^2}{2} - \frac{2t^3}{3} + C$

when $t = 0, s = 0$

$\therefore S = 28t + t2 - 2t$

PB at rest after 4s

$\therefore S = 28 \times 4 \times 42 - \frac{2}{3} \times 4^3$

$= 112 + 8 - 42.667$

$= 120 - 42.6667 = 77.33m$

15. $S = t^3 - 3t^2 + 2t$

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(b) At minimum velocity :

$\frac{dv}{dt} = 0$

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$6t - 6 = 0$

$t = 1$

Min-velocity $= 3(1)^2 - 6(1) + 2$

$= -1m/s$

(c) $3t^2 - 6t + 2 = 0$

$$t = \frac{6 \pm \sqrt{(-6) - 4\,(3)\,(2)}}{6}$$

$$= \frac{6 \pm 5.2}{6}$$

$t = 1.58$ or $0.4sec$

(d) $acc = \frac{dv}{dt} = 6t - 6$

$a = 6(3) - 6 = 12m/s^2$