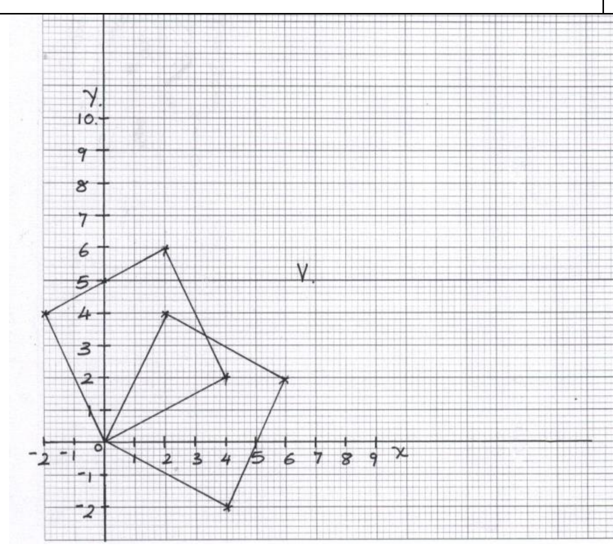
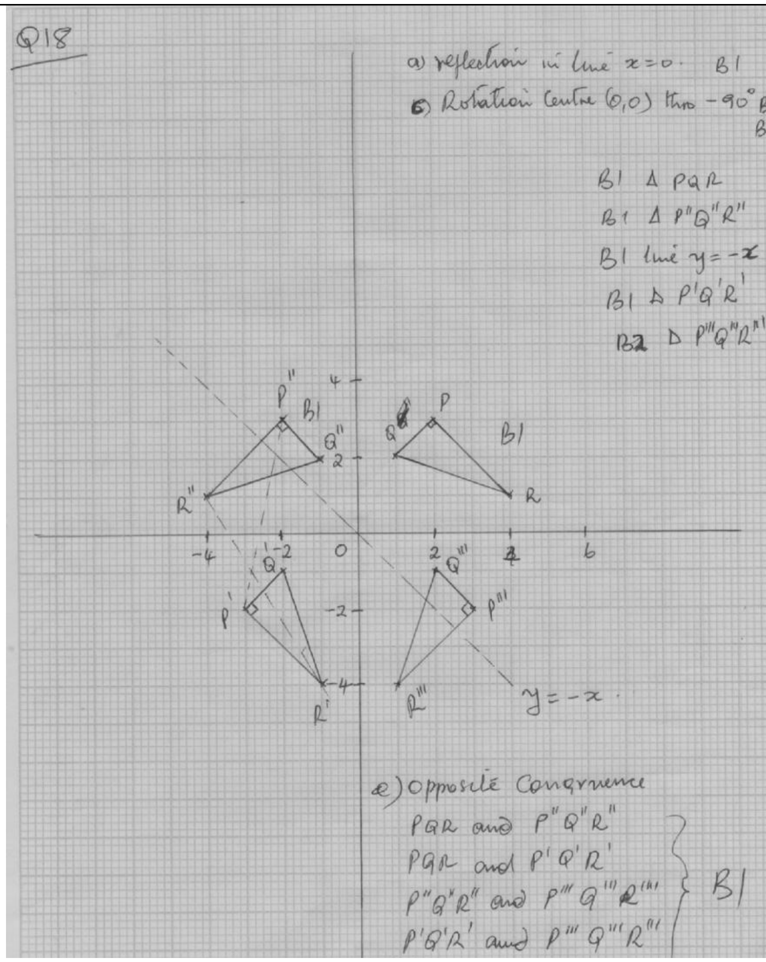
2. *Matrices and Transformations*

1	a) (i) (ii) $\begin{pmatrix}2 & -1\\1 & 2\end{pmatrix}\begin{pmatrix}0 & 2 & 2 & 0\\0 & 0 & 2 & 2\end{pmatrix}$ $=\begin{pmatrix}0 & 4 & 2 & -2\\0 & 2 & 6 & 4\end{pmatrix}$ $T \Rightarrow (0,0), (4,-2), (2,6), (-2,4)$ (iii) $\begin{pmatrix}2 & 1\\-1 & 2\end{pmatrix}\begin{pmatrix}0 & 2 & 2 & 0\\0 & 0 & 2 & 2\end{pmatrix} = \begin{pmatrix}0 & 4 & 6 & 2\\0 & -2 & 2 & 6\end{pmatrix}$ $U \Rightarrow (0,0), (4,-2), (6,2), (2,4)$ b) (i) $\begin{pmatrix}2 & -1\\1 & 2\end{pmatrix}\begin{pmatrix}2 & 1\\-1 & 2\end{pmatrix} = \begin{pmatrix}5 & 0\\0 & 5\end{pmatrix}$ $\begin{pmatrix}5 & 0\\0 & 5\end{pmatrix}\begin{pmatrix}0 & 2 & 2 & 0\\2 & 0 & 2 & 2\end{pmatrix} = \begin{pmatrix}0 & 10 & 10 & 0\\0 & 0 & 10 & 10\end{pmatrix}$ $V \Leftarrow (0,0), (10,0), (10,10), (0,10)$ (ii) Enlargement center (0,0) s.f = 5	B_1 M_1 A_1 B_1 B_1 B_1 B_1 B_1 B_1 B_1	Square S drawn ✓coordinates given Square T drawn ✓coordinates (implied) Square U drawn ✓coordinates (implied) Square V drawn
	 		
		10	

2.	 Total 10		
		10	

1. *a) B (4,-5), C (3,6 ½)*

Δ ABC drawn

Δ ABC drawn

a) ii) Shear maps

1

I (1, 1½)

Matrix = 1 0

1 ½

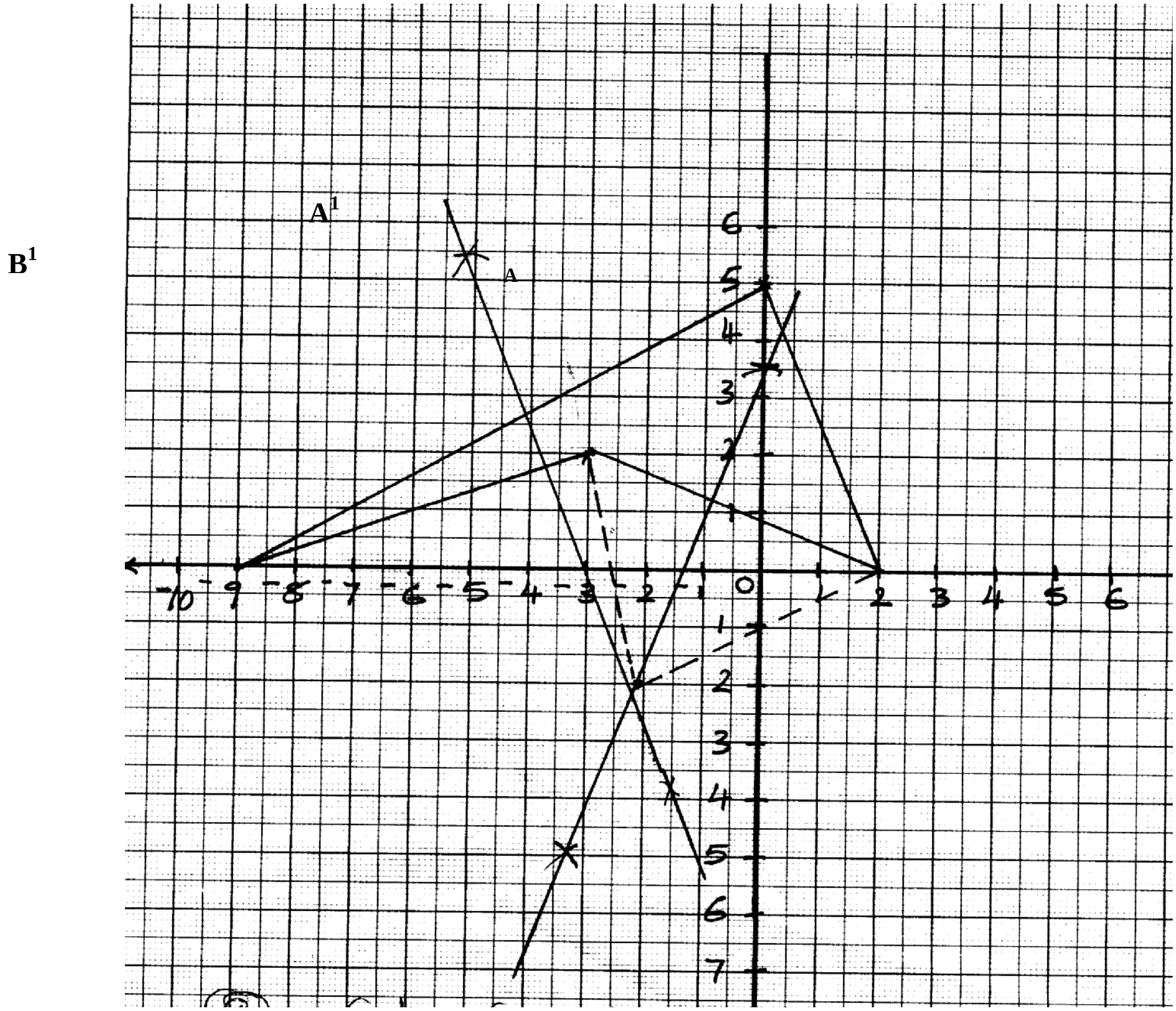
b) i) 1 1 1

$$\begin{pmatrix} -1 & 0 \\ \frac{3}{2} & -1 \end{pmatrix} \begin{pmatrix} A & B & C \\ -6 & -4 & 3 \\ -4 & -5 & 6\frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} A^{11} & B^{11} & C^{11} \\ 6 & 4 & -3 \\ -5 & -1 & -2 \end{pmatrix}$$

Δ A^{11} B^{11} C^{11} D^{11} *drawn* *ii) Half turn about (0,0)*

2.



(a) Centre (-2, -2) 90°
(b) A11 (-2 , -4) , B11 (0, 9)
(c) Half-turn about the centre (0, 2)

3.

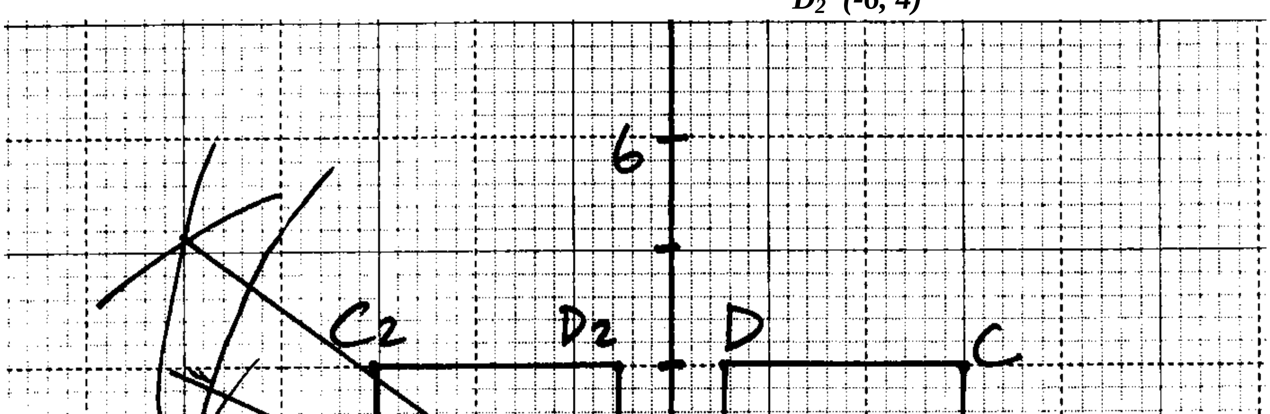
$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} A & B & C & D \\ 1 & 6 & 6 & 1 \\ 2 & 2 & 4 & 4 \end{pmatrix} \begin{pmatrix} A^1 & B^1 & C^1 & D^1 \\ -2 & -2 & -4 & -4 \\ -1 & -6 & -6 & -6 \end{pmatrix}$$

A_1 (-2, -1)
B_1 (-2, -6)
C_1 (4, -6
D_1 (-4, -1)

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} A_1 & B_1 & C_1 & D_1 \\ -2 & -2 & -4 & -4 \\ -1 & -6 & -6 & -6 \end{pmatrix} \quad \begin{pmatrix} A_2 & B_2 & C_2 & D_2 \\ -1 & -6 & -6 & -6 \\ 2 & 2 & 4 & 4 \end{pmatrix}$$

A_2 (-1, 2)
B_2 (-2, -6)
C_2 (-6, 4)
D_2 (-6, 4)

(b)



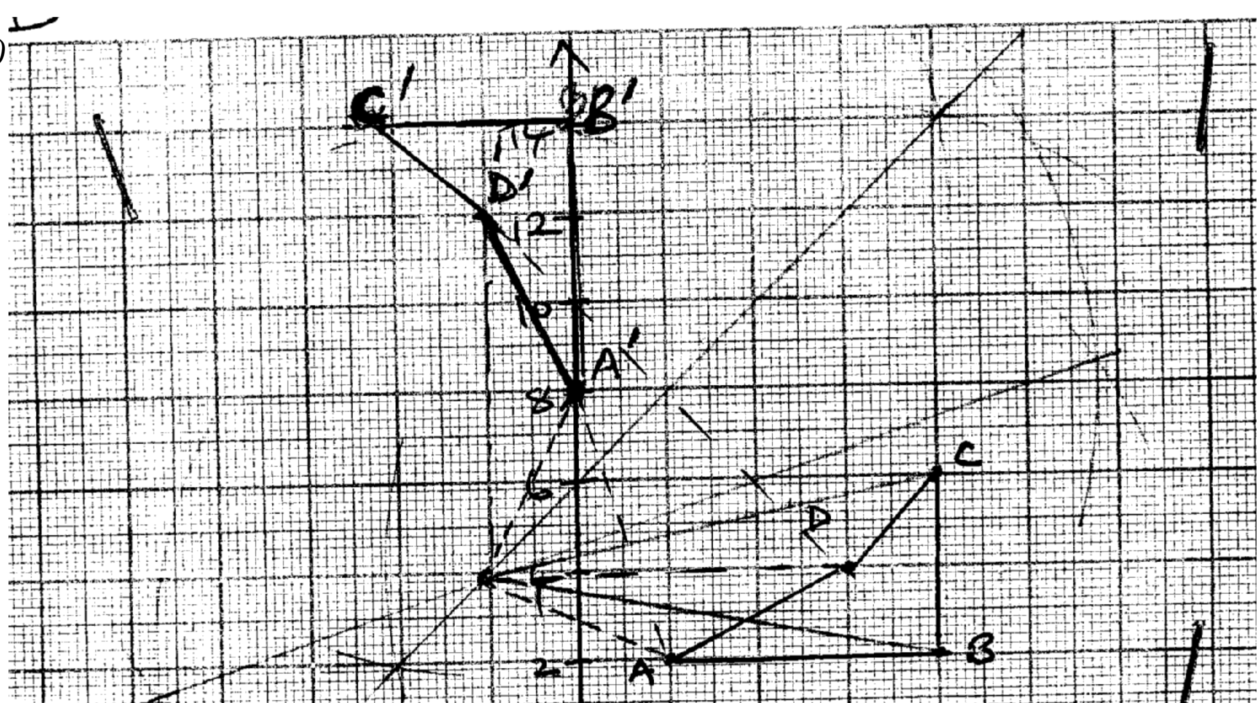
(c) (i) U - - positive three-quarter turn about the origin

(ii)UT – Reflection I the line x = 0

(d) IdetI = I2.5 x -2 – 1x 0 I= 5

∴Area = 5x(5x2) = 20sq. units

4. (a)



b) Centre (-2, 4)

Angle + 90°

5. $P(5,-3)$ $P^1\ (2, -5)$

$$\begin{pmatrix} 5 \\ -3 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$$

$$R^1 = \begin{pmatrix} -2 \\ -3 \end{pmatrix} + \begin{pmatrix} -3 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} -5 \\ -5 \end{pmatrix}$$

$$P^1R^1 = \begin{pmatrix} -5 \\ -5 \end{pmatrix} - \begin{pmatrix} -2 \\ -5 \end{pmatrix}$$

$$= \begin{pmatrix} -7 \\ 0 \end{pmatrix}$$

Mag. = 7units

6. $A^1 = (0+1, -1-2) = (1, -3)$

$B^1 = (4 + 1)\ , 3-2) = (4, 1)$

$C^1 = (\ 2 +1, 2-2) = (3-0)$

Matrix $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 5 & 3 \\ -3 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 15 & 9 \\ -9 & 3 & 0 \end{pmatrix}$$

A^{11} (3, -9) B^{11} (15, 3) C^{11} (9,0)

Determinant (0-9) =-9

Area = 9x24 = 216cm^2

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 3 & 15 \\ -9 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ -3 & 1 \end{pmatrix}$$

5(31 -9b =1   5(3c-9d=-3

<u>-15a+3b =5</u>   <u>15c +3d=1</u>

-48b =0   -48d = -16

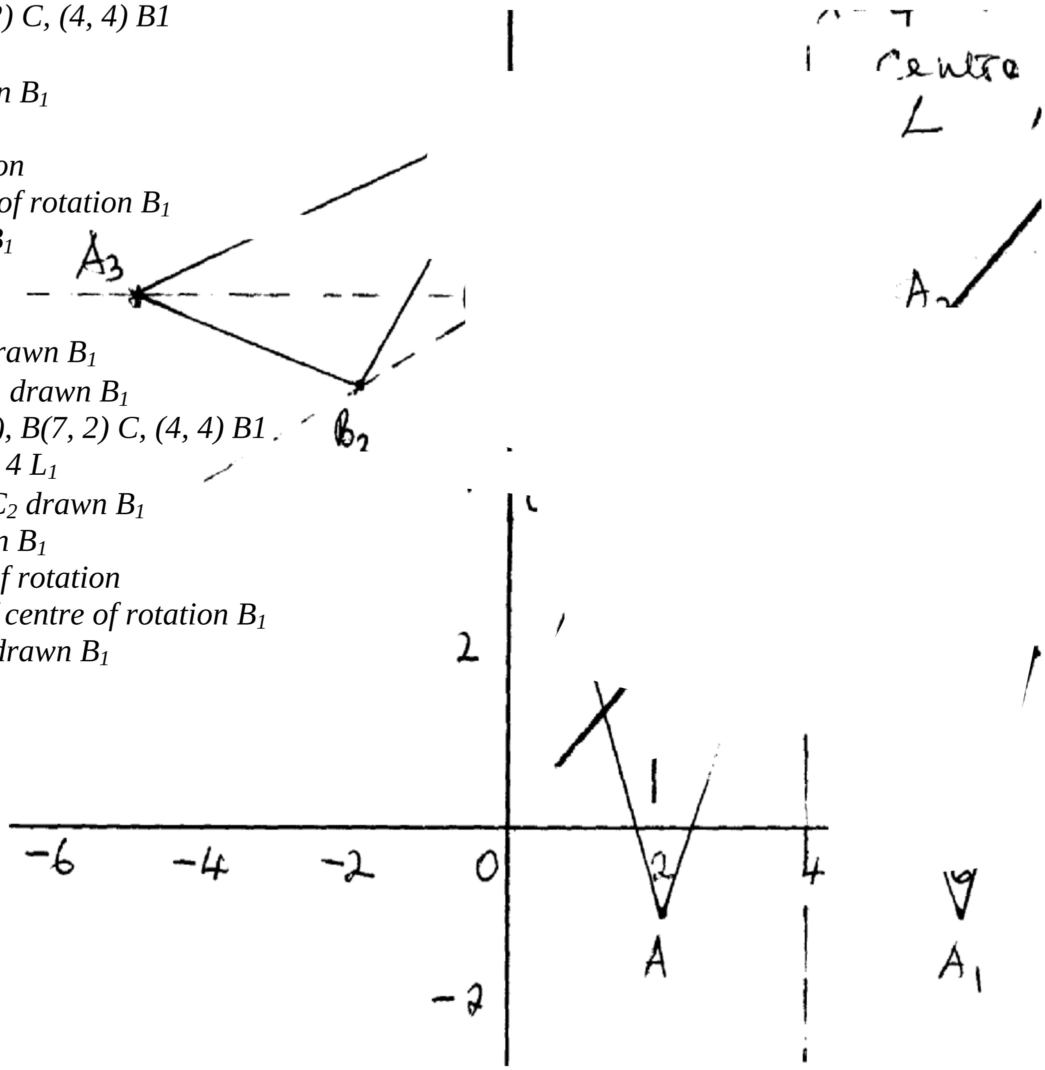
b = 0 $d = {}^1/_3$

$a = \frac{1}{3}$ $c=0$

matrix $\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$

7. *Scale used S_1*
ΔABC drawn B_1
$\Delta A_1B_1C_1$ drawn B_1
A, (6, -1), B(7, 2) C, (4, 4) B1
Line x = 4 L_1
$\Delta A_2\ B_2\ C_2$ drawn B_1
Two seen B_1
Centre of rotation
Angle of centre of rotation B_1
$A_3B_3C_3$ drawn B_1

Scale used S_1
ΔABC drawn B_1
$\Delta A_1B_1C_1$ drawn B_1
A, (6, -1), B(7, 2) C, (4, 4) B1
Line x = 4 L_1
$\Delta A_2\ B_2\ C_2$ drawn B_1
Two seen B_1
Centre of rotation
Angle of centre of rotation B_1
$A_3B_3C_3$ drawn B_1



8. (a) *P(6, -2)*
$X^1 = 6 - 3(-2) = 12$
$Y^1 = 2(6) = 12$
$(X^1, Y^1) = (12, 12)$

(b) *(i) $A^1(3, 4)$*
(ii) $B^1(3, 2)$
$C^1(1, 4)$
$D^1(4, 3)$

(c) (i) $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A^1 & B^1 & C^1 & D^1 \\ 3 & 3 & 1 & 4 \\ 4 & 2 & 4 & 5 \end{pmatrix}$

$\begin{pmatrix} A^{11} & B^{11} & C^{11} & D^{11} \\ -5 & -1 & -7 & -6 \\ 4 & 2 & 4 & 5 \end{pmatrix}$

=

A^{11} (-5, 4) , B^{11}(-1, 2), C^{11}(-7, 4) and D^{11}(-6, 5)

(ii) A stretch with y-axis invariant and a sketch factor (3)

2h = 6

h = 3

$\left.\begin{array}{l}-5a + 4b = 4 \\ \underline{- a + 2b = 2}\end{array}\right\}$ $\quad$ $-5c + 4d = -3$, $\underline{-c + 2d = 3}$

$\left.\begin{array}{l}-5a + 4b = 4 \\ \underline{-a + 4b = 4}\end{array}\right\}$ $\quad$ $-5c + 4d = -3$, $\underline{-c + 4d = -6}$

-4a = 0 $\quad$ - 4c = 3

a = 0 $\quad$ c = - ¾

b = 1 $\quad$ d = $^{15}/_{8}$

9. (a) X1(5, -1) y_1(7, -1) Z_1 (-2, 2)

xyz & $x_1y_1z_1$ well drawn

(b) 1-3 xyz x1y1z1

X_2(2, 10) y_2(2, 14)

$X_2y_2Z_2$ well drawn $\begin{pmatrix}0 & -2\\ 2, & 0\end{pmatrix}$ $\begin{pmatrix}5, & 7 & -2\\ -1, & -1, & 2\end{pmatrix}$ $\begin{pmatrix}5, & 7 & -2\\ -1, & -1, & 2\end{pmatrix}$

(c)

$\begin{pmatrix}0 & -2\\ 2, & 0\end{pmatrix}$ $\begin{pmatrix}1 & -1\\ 0, & 1\end{pmatrix}$ $\begin{pmatrix}0, & -2\\ 2, & -6\end{pmatrix}$

(d)) Area of $\Delta X_2y_2Z_2$

= 4x15 = 60cm^2

10. $\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ $\begin{matrix}2 & 4 & 4 & 2\\ 1 & 1 & 4 & 4\end{matrix}$ = $\begin{matrix}7 & 14 & 14 & 8\\ 8 & 7 & 16 & 16\end{matrix}$

2a + b = 8

$\underline{4a + b = 14}$

-2a = -6

6 + b -= 8

b = 2

∴6 + b = 8

b=2

2c + d = 7

$\underline{4c + d = 7}$

-2c = 0

c = 0

d = 7

∴ $\begin{pmatrix}3 & 2\\ 0 & 7\end{pmatrix}$

- it is an enlargement with scale factor 3 with centre (-1, -2)

(c) $\begin{pmatrix}8\\ 7\end{pmatrix}$ + $\begin{pmatrix}a\\ b\end{pmatrix}$ = $\begin{pmatrix}7\\ 9\end{pmatrix}$

a + 8 = 7 $\quad$ 7 + b = 9

a =-1 $\quad$ b = 2

∴T = $\begin{pmatrix}-1\\ 2\end{pmatrix}$

11. a) ABCD drawn B_1
Name – Parallelogram B_1

b) $A^1B^1C^1D^1$ drawn B_1
Attempt to joining any two points and bisecting. B_1
Description – Rotation + 90^0. B_1 or quarter turn about (0,0)

c) $A^{11}B^{11}C^{11}D^{11}$ drawn. B_1
Description – Enlargement centre (0, 0) Scale factor –Z. B_1

d) $A^{111}B^{111}C^{111}D^{111}$ – drawn. B_1
~~Attempt to reflect. B_1~~
Coordinates
A^{111} = 9-2, 4) C^{111} = (-8, 4) B_1 All correct
B^{111} = (-6, 0) D^{111} (-4, 8)

12. $\begin{pmatrix} -1 & 1 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 4 & 0 & -2 \\ 1 & -2 & 4 \end{pmatrix}$

$\begin{pmatrix} -3 & -2 & 6 \\ 5 & 6 & -16 \end{pmatrix}$
$A^|$(-3, 5) $B^|$(-2, 6) $C^|$(6, -16)

$\begin{pmatrix} 2 & -1 & -3 \\ 1 & 2 & 5 \end{pmatrix}$ = $\begin{pmatrix} -2 & 6 \\ 6 & -6 \end{pmatrix}$

$\begin{pmatrix} A^{||} & B^{||} & C^{||} \\ -11 & -10 & 18 \\ 7 & 10 & -6 \end{pmatrix}$
A^1(-11, 7) B^1(-10, 10) C'' (18, -6)

MN
= $\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 2 & -3 \end{pmatrix}$
= $\begin{pmatrix} -4 & 5 \\ 3 & -5 \end{pmatrix}$

p-1 = $\frac{1}{-12}\begin{pmatrix} 5 & -7 \\ -4 & 8 \end{pmatrix}$
$\begin{pmatrix} -^5/_{12} & ^7/_{12} \\ ^1/_3 & -^2/_3 \end{pmatrix}$

13. Det = 2 – 6
= - 4
A.S.F = 4
$\frac{25.6}{x} = 4$
$x = 6.4cm^2$
Area of $\Delta ABC = 6.4cm^2$

14. $T + \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$T = \begin{pmatrix} 4 - 2 \\ 0 + 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$\therefore \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$

$Q\ (1,6)$

16. $5x^2 + 6 = {}^{110}/_{10}$

$5x^2 + 6 = 11$

$x^2 = 1$

$x = \pm 1$