

2. Graphical methods

1. $x^2 + 4x + y^2 = 5$
 $x^2 + 4x + (\frac{1}{2}x + 4)^2 + y^2 = 5 + (\frac{1}{2}x + 4)^2$

$$(x + 2)^2 + (y + 0)^2 = 5 + 4$$

$$(x + 2)^2 + (y + 0)^2 = 9$$

Centre (-2,0)

Radius $\sqrt{9}$

$$r = 3 \text{ units}$$

2. $x^2 + 6x + (3)^2 + y^2 - 10y + (-5) = 2 + 9 + 25$

$$(x + 3)^2 + (y - 5)^2 = 36$$

$$(x - (-3))^2 + (y - (+5))^2 = 6^2$$

\therefore centre (-3, 5)

Radius 6 units

Completing of sq. for expression in x and y.

$\sqrt{\text{Expression.}}$

$\sqrt{\text{Centre}}$

$\sqrt{\text{Radius}}$

3. $\angle CBE = 40^\circ$ (alt. segment theorem)
 $\angle BCE = 120^\circ$ (Suppl. To $\angle BCD = 60^\circ$ alt. seg.)
 $\therefore (40 + 120 + E) = 180^\circ$ (Angle sum of Δ)
 $\angle BEC = 20^\circ$

4. $X^2 + Y^2 - 10Y + 25 = 25 - 16$
 $(X - 0)^2 + (Y - 5)^2 = 9$
 $(X - 0)^2 + (Y - 5)^2 = 3^2$
 Centre (0, 5)
 Radius = 3

5.

x	-5	-4	-3	-2	-1	0	1
x^3	-125	-64	-27	-8	-1	0	1
$6x^2$	150	96	54	24	6	0	6
8x	-40	-32	-24	-16	-8	0	8
y	-15	0	3	0	-3	0	15

$$x^3 + 6x^2 + 8x > 1$$

Between

(i) $x = -3.85 \pm 0.1$ and $x = -2.15 \pm 0.1$

(ii) $x > 0.5 \pm 0.1$

6. $y = x^3 - 3x + 2$
 $x = 0, y = 2$
 $(0, 2) \Rightarrow y$ -intercept.

$$\frac{dy}{dx} = 3x^2 - 3 = 0$$

$$x^2 = 1$$

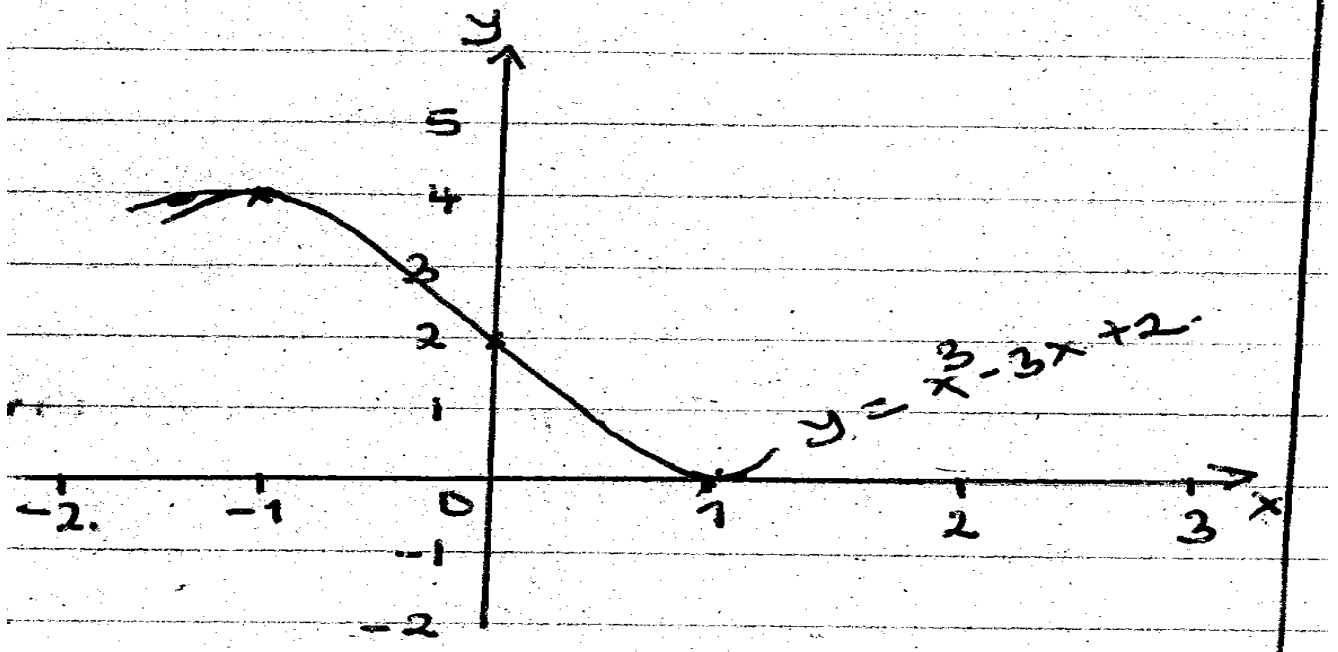
$$x = \mp 1$$

$$x = 1 \quad y = 0$$

Point (1, 0) min point
 $x = -1, y = 4$

Point (-1, 4) max point.

Point $(-1, 4)$ max point



7. $4x^2 - 12x + 4y^2 + 12y = 7$
 $x^2 - 3x + y^2 + 3y = 7/4$
 $x^2 - 3x + (3/2)^2 + y^2 + 3y + (3/2)^2 = 7/4 + 9/4 + 9/4 = 25/4$
 $(x - 3/2)^2 + (y + 3/2)^2 = 25/4$
 \therefore Centre $(1.5, -1.5)$ Radius 2.5 units

8. $\log R = n \log p + \log K$

Log P	0.48	0.54	0.60	0.65	0.70
Log R	1.56	1.69	1.81	1.91	2.00

$$\text{Gradient} = \frac{2 - 0.6}{0.7}$$

$$= \frac{1.4}{0.7} = 2$$

$$\text{Log R intercepts} = 0.6 = \log k$$

$$K = 4$$

The law connecting R and P is $R = 4P^2$

$$900 = 4P^2$$

$$P^2 = \frac{900}{4}$$

$$225 = P^2$$

9. $(x+2)^2 (y-3)^2 = 3^2$
 $X^2 + 4x + 4 + y^2 - 6y + 9 = 3^2$
 $X^2 + y^2 + 4x - 6y + 4 = 0$

10.

V	0	2	4	6	8	10
$\frac{1}{T}$	2.04	3.33	4.17	5	6.25	7.30

$$T = a$$

$$b + V$$

$$\frac{I}{T} = \frac{b + V}{a}$$

$$\frac{I}{T} = \frac{I}{a}V + \frac{b}{a}$$

$$y = mx + C$$

$$\frac{b}{a} \text{ (i) } = \frac{\text{Grad}}{\Delta x} \Rightarrow \frac{\Delta y}{\Delta x} = \frac{7.3 - 5}{10 - 6} = 2.3 = 0.575$$

$$a = 1.739$$

$$\frac{b}{a} = y - \text{Intercept} \Rightarrow 2.04$$

$$\frac{b}{1.739} = 2.04 \quad b = 2.04 \times 1.739$$
$$= 3.547556$$

$$b \simeq 3.548$$

$$\text{(ii) } T = 0.38$$

$$\frac{I}{T} = 2.63 \text{ shown on graph}$$

$$V = 1$$

$$\text{(iii) } \frac{I}{T} = 4.45$$

$$T = (4.45)$$
$$= 0.2247$$

$$\simeq 0.22$$

$$11. \quad y = 2x^3 + x^2 + 3x - 1$$

$$\frac{dy}{dx} = 6x^2 + 2x + 3$$

gradeindent at (1, -5)

$$= 6 + 2 + 3 = 11$$

$$\frac{y - (-5)}{x - 1} = 11$$

$$x - 1$$

$$y + 5 = 11x - 11$$

$$y = 11x - 16$$

$$12. \quad 3^5 = 3^{-4} \times 3^{-x}$$

$$3^5 = 3^{-4-x}$$

$$-4 - x = 5$$

$$-x = 9$$

$$x = -9$$

$$13. \quad x^2 + 2x + 1 + y^2 - 4y + 4 = 4 + 1 + 1$$

$$(x+1)^2 + (y-2)^2 = 9$$

Centre (-1, 2)

Radius 3 units

$$14. \quad c)$$

X	-4	-3	-2	-1	0	1	2
-6	-6	-6	-6	-6	-6	-6	-6
X	-4	-3	-2	-1	0	1	2
$4x^2$	64	36	16	4	0	4	16
X^3	-64	-27	-8	-1	0	1	8
$Y = -6 + x + 4x^2 + x^3$	-10	0	0	-4	-6	0	20

$$y = x^3 + 4x^2 + x - 6$$

$$0 = x^3 + 4x^2 + x - 6$$

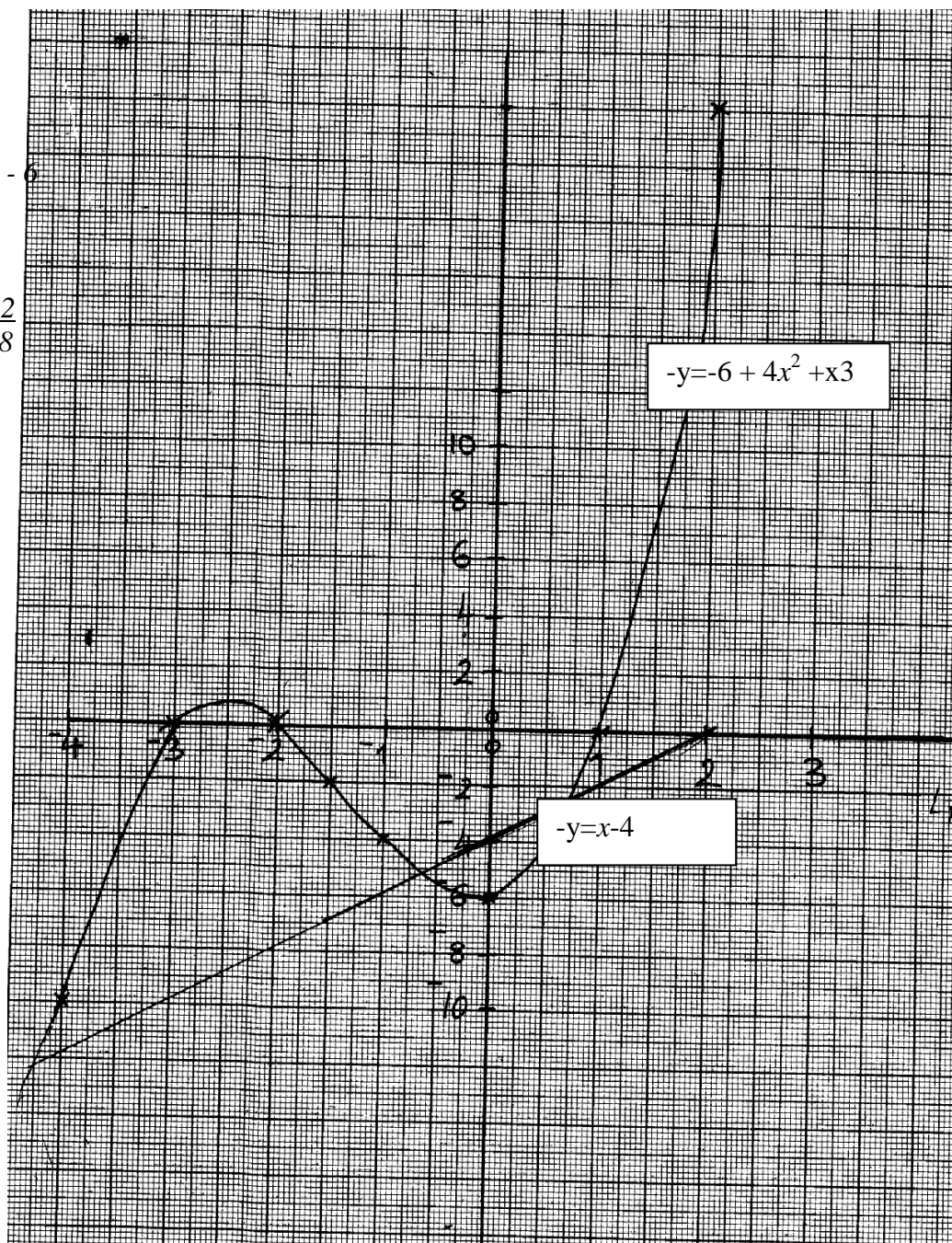
$$y = -2$$

(iii) $y = x^3 + 4x^2 + x - 6$

$$0 = x^3 + 4x^2 + 0 - 2$$

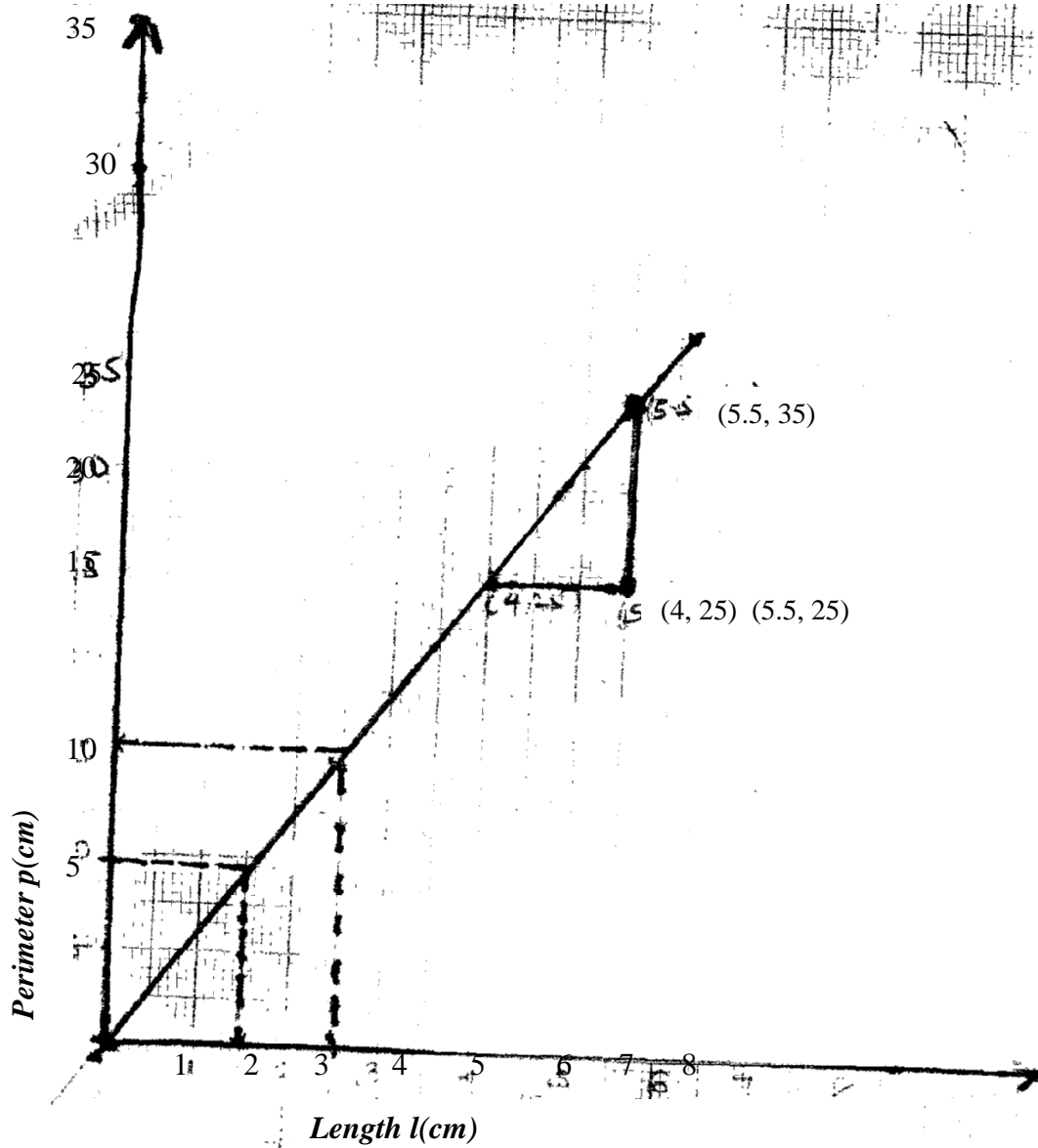
$$y = x - 4$$

x	1	0	-2
y	-3	-4	-8



- c (i) solution 0.8
-1.5
And -3.2
(c) 1, -2, -3

15.



(i) $P = 15.75\text{cm}$

(ii) $l = 1.5\text{cm}$

(iii) $m = \frac{35 - 25}{5.5 - 4.0} = \frac{10}{1.5} = c$

(c) choose $P(5, 31.4)$

$$\frac{p - 31.4}{l - 5} = \frac{10}{1.5}$$

$$\frac{p - 31.4}{l - 5} = \frac{100}{15}$$

$$15p - 471 = 100l - 500$$

$$15p = 100l - 29$$

$$15 \cdot 15$$

$$2k = \frac{100}{15}$$

$$k = \frac{100}{2 \times 15} = 3.33$$

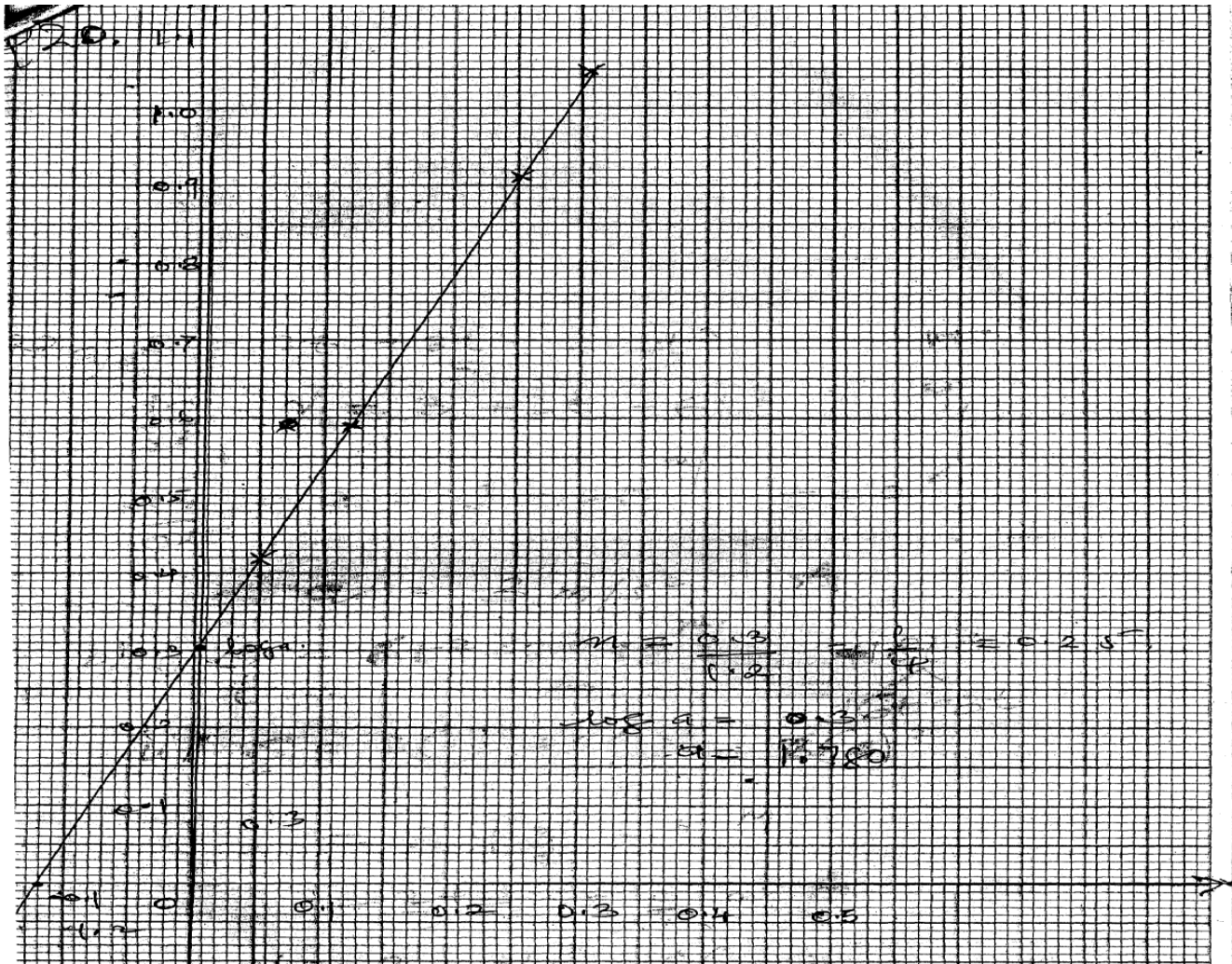
$$c = 1.93$$

$$P + 0.6 = ar^h$$

$$\text{Log}(P + 0.6) = \log a + n \log R$$

$$= n \log R + \log 9$$

$P + 0.6$	1.33	2.65	3.85	8.04	11.22
$\log(P + 0.6)$	-0.13	0.42	0.59	0.91	1.05
$\log R$	-0.05	0.05	0.12	0.25	0.30



$$\log 0.3 = \frac{1}{4} = 0.25$$

$$\log a = 0.3$$

17.

$$x^2 + y^2 - 6x = 3 - 4y$$

$$x^2 - 6x + \left(\frac{6}{2}\right)^2 + y^2 + 4y + \left(\frac{4}{2}\right)^2 = 3 + \left(\frac{6}{2}\right)^2 + \left(\frac{4}{2}\right)^2$$

$$(x - 3)^2 + (y + 2)^2 = 3 + 9 = 12$$

$$(x - 3)^2 + (y + 2)^2 = 16$$

$$C(3, -2)$$

$$\text{Gradient } \frac{\Delta y}{\Delta x} = \frac{7 - (-2)}{6 - 3} = 3$$

$$\Delta x = 6 - 3$$

18.

x	-3	-2	-1	0	1	2	3	4
$-x^3$	27	8	1	0	-1	-8	-27	-64
$2x^2$	18	8	2	0	2	8	18	32
$-4x$	12	8	4	0	-4	-8	-12	-16
2	2	2	2	2	2	2	2	2
y	59	26	9	2	-1	-6	-19	-46

b) Check on the graph paper.

c) $x = 0.5 \pm 0.1$

d) $-x^3 + 2x^2 - 5x + 3 = 0$

Line to allow: $y = x - 1$

$$\frac{x}{y} \left| \begin{array}{c|c|c} 0 & 1 \\ -1 & 0 \end{array} \right.$$

$x = 0.65$

19. $\frac{Dy}{dx} = 12x^2 - 12$

$12x^2 - 12 = 0$

$12(x^2 - 1) = 0$

$x = 1$

$x = -1$

At $x = 1$

At $x = -1$

0	1	2	-2	-1	0
GRD = 12	0	36	36	0	-12

-
0
+
+
0
-

(1, 7)

(-1, 9)

Minimum

maximum

20. (a) table

(b) plotting

scale

smooth curve

(c) (i) $-0.5 < x < 1$ and $x > 1$

(iii) $x = 2.5 \pm 0.1$

21. $2x^2 + 2y^2 - 6x + 10y + 9 = 0$

$x^2 + y^2 - 3x + 5y + 9/2 = 0$

$x^2 + y^2 - 3x + 5y = -9/2$

$x^2 - 3x + \underline{9} + y^2 + 5y + \underline{25} = 8.5 - 4.5$

$(x - \underline{3})^2 + (y + \underline{5})^2 = 4$

Radius = 2 units

Centre = (1.5, -2.5)