2. Circles –chords and tangents

1.	$6^2 = x(5 + x)$ $x^2 + 5x – 36 = 0$ $(x – 4)(x + 9) = 0$ x = 4 or -4 BC = 4cm	**M1** **M1** **A1**	Correct factorisation
		03	

1. a) i) $<DCF = \frac{180 – 92}{2} = 44° = < CAD$

ii) $< BAO = 50°$

Acute angle AOB = 80°

$\therefore$ obtuse angle = 360 – 80 = 280°

b) Area of the sector = $(^{80}/_{360}\, x\, ^{22}/_{7}\, x\ 7\ x\ 7) = 34.22cm^2$

Area of the Δ = ½ x 7 x 7 x sin80= $24.13cm^2$

Area of the shaded segment = 34.22 –

$\underline{24.13}$

$10.09cm^2$

2. $< COB = 2 x 50 = 100°$

$< OCA =< OAC = \frac{180 – 100}{2} = 40$

$\therefore < BAC = 180 – (50 + 70)$

$= 60$

3. $PB. PA (PT)^2$

$\frac{PB}{PT} = \frac{PT}{PA}$

$\frac{4}{12} = \frac{12}{4 + 2r}$

$\frac{4(4 +2r)}{4} = \frac{12^2}{4}$

$4 + 2r = 36$

$2r = 32$

$r = 16$ cm

4. (a) $\angle BOE = 2 \angle BCE = 2 x 20^0 = 40^0$

(b) $\angle BOE =40^0$

$\angle BEC = ½ (360^0 – 60^0) = 150^0$

Angels subtended at the centre is twice at the Circumference.

c) $\angle CEF = 90^0 – 80^0 = 10^0$

d) $\angle BCO = \angle CBO = 60^0$

Base angles isosceles triangle.

$\angle OXC = 180^0 – (60^0 + 20^0)$

$= 100^0$

e) $\angle BCE = 20^0$
$\angle CXE = 180^0 - 100^0 = 80^0$
$\angle CEX = 80^0$
$\angle OEF = 180^0 - (80^0 + 50^0 + 10^0)$
$= 40^0$

5. (a) $PQ = \sqrt{8^2 - 2^2}$
$= 60$
$= 7.746cm$
(b) $\angle PAS = 2\cos^{-1}$
$= 151^o$
$\therefore$Reflex $\angle PAS = 209^o$ OR $360^o - 151^o = 209^o$

(c) Length PYS $= \frac{209}{360} \times 2 \times 6 = 21.89cm$
Length QXR $= \frac{151}{360} \times 2 \times 4 = 10.54cm$

(d) Length of belt $= 7.74 \times 2 + 21.89 + 10.54$
$= 47.92cm$

6. a) i) In 1 hr; Tap A fills $^1/_3$
B - ¼
Capacity filled in 1 hr $= {}^1/_3 + ¼$
$= {}^7/_{12}$
$^7/_{12} = 1$ hr
$1 = 1 \times 1 \times {}^{12}/_7$
$= 1\,{}^5/_7$ hrs.
ii) $^1/_3 + ¼ - {}^1/_6 = {}^5/_{12} \Rightarrow$ in one hr
$^5/_{12} = 1hr$
$1 = 1 \times 1 \times {}^{12}/_5$
$= 2\,{}^2/_5$ hrs

7. $\angle ABD = 31^0$
$\angle CBD = 37^0$

8. $x(x+9) = 4x9$
$x^2 + 9x - 36 = 0$
$(x^2 - 3x) + (12x - 36 = 0)$
$x(x-3) + 12(x-3) = 0$
$(x+12)(x-3) = 0$
$x - 3 = 0$
$x = 3$ only

9. $PO.\ OQ = BO\ .OA$
$8 \times 6 = 4.5 \times y$

$$y = \frac{8 x 6}{4.5}$$
$$= 10.67$$

10. < DGB = < ABG = 40° (alt.seg <,s)

a) < DGE = < DBE = 25° (<s in same segment)

b) <EFG

< GEB = 40°, = <BDG and < BED = 45° = <BGD

$\therefore$ In Δ GED, <GDE = 180 – (25 + 40 + 45) = 70°

$\therefore$ <GFE = 180 – 70 = 110^o (Sup angles)

d) Angle CBD in ΔBGE, Angle GBE = 180 – (110) = 70^o

$\therefore$Angle CBD = 180 – (40 + 70 + 25) = 45^o

Or Angle CBD = Angle BGD = 45^o (Angles in Alt segment)

e) Angle BCD in Δ BCD, Angle BDC = 70 ^{o} Angles in a straight line

$\therefore$Angle BCD = 180 – (70 + 45) Angles of a triangle = 65^o

11. (a)$\sin\theta = \frac{4.5}{8} = 0.5025$

$\theta = \sin^{-1} 0.5625$

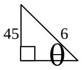
$= 34.23^o$

$\angle Apb = 68.46^o$

$\sin\alpha = \frac{4\text{-}5}{6} = 0.75$

$\alpha = Sin\text{-}10.75$

$= \angle 48.59^o$



$\angle Aqb = 97.18^o$

(b) Area Of Segment PAB = $\frac{68.46}{360} X \frac{22}{7} X 8 X 8 – ½ X 8 Sin 68.46$

$= 38.25 – 29.77$

$= 8.48cm^2$

Area Of Segment AQB = $\frac{97.18}{360} X \frac{22}{7} X 36 – ½ 36 Sin 97.18$

$= 30.65 – 17.86$ $= 12.68cm^2$

Area of quadrilateral APBQ = ½ 64 sin 68.46 + ½ x 36sin 92.18

$= 29.77 + 17.86$ $= 47.63$

Shaded area = 47.63 – (8.48 + 12.68) = 26.47cm^2

12. CBD = 90 - 42 = 48^o

Angle of triangle add to 180^o

DOB = 180^o – 42 = 138^o

Opposite angles of cyclic quadrilateral add to 180^o

DAB = $\frac{138^o}{2} = 69^o$

Angle at circumference is half the nagle substended at centre by same chord

CDA

ABD= 90 – 48 = 42o

ADB = 180 –(69+42)

180-111=69^o

$CDA = 90 + 69^o$ $= 159^o$

Show ΔADB is asoccesters

$\angle DAB = 69^o$

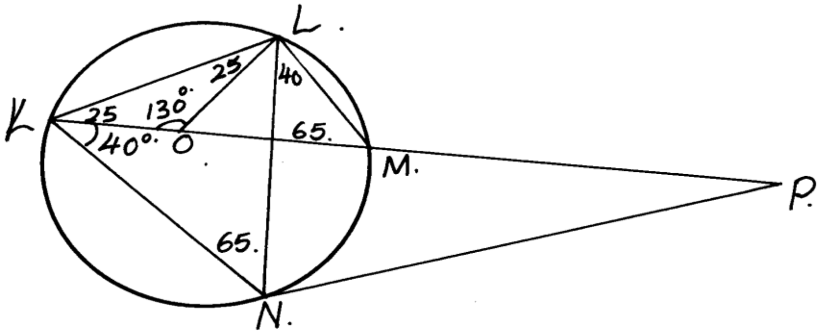
$\angle DAB = 69^o$

$\angle ADB = 69^o$

$\angle ABD = 42^o$

So two angles are equal hence it is asoccesters

13.



a) $MLN = 40^o$ angles subtended by same chord in the same segment are equal.

b) $OLN = 90 – 65 = 25^o$

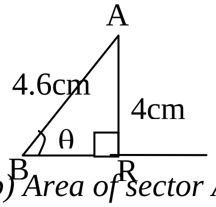
Angle sum of Δ is 180^o or angle subtended by > diameter is 90^o.

c) $LNP = 65^o$ exterior Δ is equal to opposite interior angle or angle btwn a chord and a tangent is equal to angle subtended by the same chord in the alternate segment.

d) $MPN = 180 – 170 = 10^o$ angle sum of a Δ is 180^o

e) $LMO = 65^o$ angles subtended by same chord.

14. (a)



$Sin = {}^4/_{4.6} = 0.869565$

$= sin\text{-}10.89565 = 60.408^o$

$ABR = 2 \times 60.408^o = 120.8163^oC$

$\approx 120.82^o$ (2d.p)

(b) Area of sector ABCR

$= \frac{120.8163^o}{360^o} x \pi x 4.62)cm^2$

$= 22.30994cm^2$

Area of sector OAPC

$= \frac{60^o}{360^o} x \pi x 8^2)cm^2$

$= 33.51032cm^2$

$= 33.51cm^2$(2d.p)

Area of $\Delta ABC = (½ x 4.6^2 sin 120.8163)cm^2 = 9.08625cm^2$

Area of $AOC = (½ x 82 sin 60) cm^2 = 27.7128cm^2$

Sum of area of $\Delta s = 36.799cm^2$ $36.80cm^2$

$\therefore$Area of shaded part = area of sectors – area of Δs

$= (22.31 + 33.51 – 36.80)cm^2 = 19.02cm^2$(2dp)

15. *(a) $\angle TDC = ABT$ (exterior opp. angle of a cyclic quadrilateral)*

$= 100^o$

(b) BAT = ATB (base s of isosceles ATB)

$= 180 – 100 = 40^o$

(c) ∠TCD = ∠X TD (angles in alternate segments)

$= 60^o$

Or $\angle BTC + 40^o = 100^o$(exterior angle of a Δ)

$\angle BTC = 100^o – 40^o = 60^o$

(d) DTC = $180^o - (58^o + 100^o)$ (angles in ΔTDC $= 12^o$

16. a) GBD = 90°

ABG = 180 – (90 + 36)

= 180 – 126 = 54°

GEB = ABG = 54°

b) BED = CBD = 36°

c) DGE = FEG = 20°

OEB = 90 – (36 +20)

= 90 – 56 = 34°

OBE = OEB = 34°

d) BGE = 36 + 20 = 56°

e) GFE = 180 – EDG

= 180 – 70 = 110°

17. $XZ^2 = 13.4^2 + 5^2 – 2 x 13.4 x 5 \cos 57.7^o$

$= 170.56 + 25 – 134 x 0.5344$

$= 204.56 – 71.6096$

$XZ^2 = 132.9504$

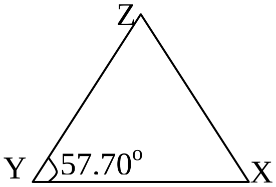
XZ = 11.5304cm

(ii) $2R = \frac{11.5304}{\sin 57.7^o}$

$$2R = \frac{11.5304}{0.8453}$$

2R = 13.60866

R = 6.08043cm



18. 52 = 62 + 62 – 2 x 6 x 6 cos A

72 cos A = 72 – 25 = 46

Cos A = $^{46}/_{72}$ = 0.6389

A = Cos-1 0.6389 = 50.29°

Area of the minor sector APQ

$= \frac{50.29}{360} x 3.142 x 6^2$

$= 15.801cm^2$

Area of the triangle APQ $= ½ \times 6 \times 6 \sin 50.29$ $= 13.847cm^2$

Area of the minor segment $= (15.801 – 13.847)cm^2$ $= 1.954cm^2$

Area of triangle PBQ

$\sqrt{6.5 (6.5 – 4) (6.5 – 4) (6.5 – 5)}$

$\sqrt{6.5 \times 2.5 \times 2.5 \times 1.5}$ $= 7.806cm^2$

Area of shaded region $= (7.806 – 1.954)cm^2$ $= 5.852cm^2$

19. a) $\measuredangle PQR = 180^o – 75^o$

$= 105^o$. NPQR is cyclic quadrilateral.

(b) $\measuredangle NRP = 90^o - 75^o$

$= 15^o$, Third angle of Δ NRP.

$\measuredangle PRS = 180^o - 65^o$, Angles on a

$= 115^o$, straight line.

$\therefore \measuredangle QSR = 180^o – (115^o – 35^o)$

$= 30^o$, 3rd angle of triangle PRS.

(c) Reflex $\measuredangle POR = 2 \measuredangle PQR$

$= 2 \times 105^o = 210^o$

(d) $\measuredangle MQR = \measuredangle MNR = 40^o$

Subtended by same chord MR

20.

(a) $\angle TDC = 100^o$ (Cyclic quadrilateral)

(b) $\angle TCB = 40^o$ (Cyclic quadrilateral)

(c) $\angle TCD = 58^o$ (Cyclic quadrilateral)

(d) $\angle BTC = 60^o$ (Sum angle of a Δ add upto 180^o)

(e) $\angle DTC = 22^o$ (angle sum of a straight line add upto 180^o)

21. $4 \times 10 = 5(5 + x)$

$40 = 25 + 5x$

$3 = x$

22. $T_{11} = a + 10d$

$T_2 = a + d$

$a + 10d = 4a + 4d$ ....(i)

$3a – 6d = 0$

$S7 = {}^7/_2\{2a + 6d\} = 175$ ...(ii)

$2a + 6d = 50$

$+$

$\underline{3a – 6d = 0}$

$5a = 50$

$a = 10$ $d = 5$

23. CBE $= 40^0$ (alt.segiment theoren)

$\angle BCE = 120^0$ (Suppl. To BCD $= 60^0$alt. seg.)

$\therefore(40 + 120 + E) = 180^0$ (Angle sum of Δ)

$\angle BEC = 20^0$

24. Taxable income p.a = 36,000+53142.86

=sh.412142.86

Monthly salary = $\frac{413142.86}{12}$ + 12,000

= 34428.57 + 1200 = Sh 35628.57

25. a) (i) $\angle PTQ = 180^o - 56^o = 124^o$

$124 + 38 = 162^o$

$180^o - 162^o = 18^o$

$90^o + 18^o = 108^o$

$180^o - 108^o = 72^o$

$180^o - (72^o + 56^o) = 52^o$

$\angle PRS = 52^o$ **√Value of the constant.**

(ii) $\angle RSQ = \angle RPQ = 18^o$

b) $A \propto B.\frac{1}{C^3}$ **√Substitution √Formulation**

$A = \frac{K.B}{C^3}$ **√Values of constants.**

√Substitution

$12 = \frac{3K}{2^3}$

$K = \frac{\overset{4}{\cancel{12}} \times 8}{\underset{1}{\cancel{3}}} = 32$

$\therefore A = \frac{32B}{C^3}$

$\frac{10 \times (1.5)^3}{32} = B$

$\therefore B = 1.055$

c) $y = K + Mx^2$ where K and M are constants

$7 = K + 100M$

$\underline{5.5 = K + 400M}^{-}$

$1.5 = 300M$

$100 \times 0.005 + K = 7$

$-0.5 + K = 7$

$K = 7.5$

$M = 0.005$

$y = 7.5 - 0.005 \times 18^2$

$y = 7.5 - 1.62$

$y = 5.88$

26. a) $PN^2 = 5^2 - 4^2$

$PN = 3cm$

$QN^2 = 6^2 - 4^2$

$QN = 4.47cm$

$\therefore PQ = 3 + 4.47 = 7.47$

b)i) < APB

Sin ½ $\theta\, {}^4/_5 = 0.8$

½ sin $\theta = 53.13$

< APB

ii) Sin ½ $\propto = 4/6 = 0.6667$

½ $\propto = 41.81$

$\propto 83.62$

A

$\therefore \angle AQB = 83.62°$

c) Area of the shaded region – Area of the segments

$= \frac{106.3}{360} \times \frac{22}{7} \times 5^2 - \frac{1}{2} \times 5 \times 5 \sin 106.3$

$= 23.19 - 11.998 = 11.192$

$\frac{83.6}{360} \times \frac{22}{7} \times 6 \times 6 - \frac{1}{2} \times 6 \times 6 \sin 83.6 = 8.38$

Total $11.192 + 8.38 = 19.52$

27. *Using cosine rule*

$7.8^2 = 6.6^2 + 5.9^2 - 2 \times 6.6 \times 5.9 \cos R$

$\text{Cos } C = \frac{6.6^2 + 5.9^2 - 7.8^2}{2 \times 6.6 \times 5.9}$

$= \frac{43.59 + 34.81 - 60.84}{77.88} \qquad = \frac{78.37 - 60.84}{77.88}$

$= \frac{17.53}{77.88} = 0.2251$

$\angle C = 77^o$

$\frac{7.8}{\text{Sin } 77} = 2r \Rightarrow r = \frac{7.8}{2 \times \sin 77}$

$= 4cm$

Area of circle $= 3.142 \times 4^2 \qquad = 50.27$

A$\triangle$rea of $PQR = \frac{1}{2}(6.6)(5.9) \sin 77$

$= 18.97$

$\therefore$*Area of shaded region* $= 50.27 - 18.97 = 31.30cm^2$

✓angle

✓angle

28. a) $\angle PAQ = 2\ PAB = 42^o \times 2 = 84^o$

$\angle PBQ = 2 \angle ABQ = 30^0 \times 2 = 60^o$ ✓

✓

(b) (i) *Area of sector APQ* $= \frac{84}{360} \times \frac{22}{7} \times 6 \times 6 = 26.4\ cm^2$ ✓

Area of sector PBQ $\frac{60}{360} \times \frac{22}{7} \times 8 \times = 33.5\ cm^2$ ✓

✓diff. areas

(ii) *Area of* $\triangle APQ = \frac{1}{2} \times 6 \times 65{:}- 84^o = 18 \times 0.9945$ **✓diff. areas**

$= 17.9\ cm^2$ **Exp. for total**

Area of $\triangle\ PBQ = \frac{1}{2} \times 8 \times 85{:} = 60^o = 32 \times 0.8660$ **✓answer.**

$= 27.7\ cm^2$

(iii) For each circle, shaded area = sector area – triangle Area.

= area of sector APQ – area of triangle APQ

$= 26.4 - 17.9 = 8.5\ cm^2$

2^{nd} *circle, shaded area*

= area of sector PBQ – area of $\triangle PBQ$

$= 33.5 - 27.7 = 5.8\ cm^2$

Total shaded area $= 8.5 + 5.8 \qquad = 14.3\ cm^2$

29. $\frac{90}{360} \times 3.142 \times 2 \times 6.5$

$\underline{10.2115 \text{ cm}}$

$= 10.21 \text{ cm}$