2. *Vectors*

1	$\begin{pmatrix}3\\2\\-2\end{pmatrix}-\begin{pmatrix}2\\1\\-3\end{pmatrix}=\begin{pmatrix}1\\1\\1\end{pmatrix}$	M_1 A_1	✓exp
2	(a) (i) 4p – 3q = $\begin{pmatrix}10\\5\end{pmatrix}\times 1$ P + 2q = $\begin{pmatrix}-14\\15\end{pmatrix}\times 4$ $4p-3q=\begin{pmatrix}10\\15\end{pmatrix}$ $4p+8q=\begin{pmatrix}-56\\60\end{pmatrix}$ -11q = $\begin{pmatrix}66\\-55\end{pmatrix}$ q = $\begin{pmatrix}-6\\5\end{pmatrix}$ $p+2\begin{pmatrix}-6\\5\end{pmatrix}=\begin{pmatrix}-14\\15\end{pmatrix}$ $p+\begin{pmatrix}-12\\10\end{pmatrix}=\begin{pmatrix}-14\\15\end{pmatrix}$ $p=\begin{pmatrix}-2\\5\end{pmatrix}$ $q=\begin{pmatrix}-6\\5\end{pmatrix}$ and $p=\begin{pmatrix}-2\\5\end{pmatrix}$ (ii) |p + 2q| = $\begin{pmatrix}-2 & -12\\5 & 10\end{pmatrix}$ with + between columns $\begin{pmatrix}-14\\15\end{pmatrix}=\sqrt{(-14)^2+(15)^2}=\sqrt{196+225}=\sqrt{421}=20.52$ (b) $\vec{AB}=\begin{pmatrix}5\\3\end{pmatrix}-\begin{pmatrix}-1\\1\end{pmatrix}=\begin{pmatrix}6\\2\end{pmatrix}$ $\vec{BC}=\begin{pmatrix}11\\5\end{pmatrix}-\begin{pmatrix}5\\3\end{pmatrix}=\begin{pmatrix}6\\2\end{pmatrix}$ $AB=kBC$ $AB=1BC$	M_1 M_1 A_1 A_1 M_1 A_1 B_1	

	B (3, 5) is common AB is a scalar multiple of BC. Hence A (1, -1), B (3,5) and C (5, 11) are collinear	B_1 B_1 A_1	 Scalar 1 Correct pt B
3	i) $\underset{-}{P} = 2\begin{pmatrix}1\\-2\\1\end{pmatrix} - \frac{1}{3}\begin{pmatrix}6\\-3\\9\end{pmatrix} + \begin{pmatrix}-3\\2\\3\end{pmatrix}$ $= \begin{pmatrix}2\\-4\\2\end{pmatrix} - \begin{pmatrix}2\\-1\\3\end{pmatrix} + \begin{pmatrix}-3\\2\\3\end{pmatrix}$ $= \begin{pmatrix}-3\\-1\\2\end{pmatrix}$ ii) $\lvert P\rvert = \sqrt{9+1+4}$ $= \sqrt{14} = 3.742$	M1 A1 B1	
		3	
4.	$\underset{\sim}{PQ} = \begin{pmatrix}-2\\-1\end{pmatrix} - \begin{pmatrix}-6\\-3\end{pmatrix} = \begin{pmatrix}4\\2\end{pmatrix}$ $\underset{\sim}{QR} = \begin{pmatrix}6\\3\end{pmatrix} - \begin{pmatrix}-2\\-1\end{pmatrix} = \begin{pmatrix}8\\4\end{pmatrix}$ $2\underset{\sim}{PQ} = \underset{\sim}{QR}$ multiples of each other Q is common point hence PQ and R are collinear	B1 B1 B1	
		03	

1.

$Sin\ \overline{60} = \sqrt{}^3/_2$

$Sin\ \overline{45} = {}^1/\sqrt{2}$

$$\frac{1}{\frac{\sqrt{3}}{2}\cdot\frac{}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} - \frac{1}{}}$$

$$= \frac{1}{\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{\sqrt{2}}}$$

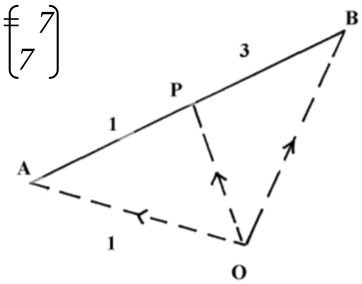
$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6} - 2\sqrt{2}}{4}$$

2. $\underset{\sim}{OP} = \underset{\sim}{OA} + \frac{1}{4}\underset{\sim}{AB}$
$= \underset{\sim}{OA} + \frac{1}{4}(\underset{\sim}{OB} - \underset{\sim}{OA})$
$= \underset{\sim}{OA} + \frac{1}{4}\underset{\sim}{OB} - \frac{1}{4}\underset{\sim}{OA}$
$= \frac{3}{4}\underset{\sim}{OA} + \frac{1}{4}\underset{\sim}{OB}$

$= \frac{3}{4}\underset{\sim}{OA} + \frac{1}{4}\underset{\sim}{OB}$

$= \frac{3}{4}\begin{pmatrix}12\\8\end{pmatrix} + \frac{1}{4}\begin{pmatrix}16\\4\end{pmatrix} = \begin{pmatrix}3\\6\end{pmatrix} + \begin{pmatrix}4\\1\end{pmatrix} = \begin{pmatrix}7\\7\end{pmatrix}$



3. $m\begin{pmatrix}4\\3\end{pmatrix} + n\begin{pmatrix}-3\\2\end{pmatrix} = \begin{pmatrix}5\\8\end{pmatrix}$
$4m - 3n = 5$ ……….. (i) x 2
$3m + 2n = 8$ ……….(ii) x 2
$8m - 6n = 10$
$\underline{9m + 6n = 24}$
$17m \quad = 34$
$m = 2$
$4 \times 2 - 3n = 5$
$-3n = -3$
$n = 1$
$\therefore m = 2, n = 1$

4. (a) (i) $BM = \frac{2}{5}a - b = \frac{1}{5}(2a - 5b)$
(ii) $AN = \frac{2}{3}b - a = \frac{1}{3}(2b - 3a)$

(b) $BX = \frac{t}{5}(2a - 5b)$
$AX = \frac{h}{3}(2b - 3a)$
$OX_1 = OB + BX = b + t\left(\frac{2}{5}a - 5b\right)$
$= (-t)b + \frac{2}{5} + a$
$OX = OA + AX = a + h(2b - 3a)$
$= (1-h)a + \frac{2}{3}hb$

(c) $OX_1 = OX_2$

$\frac{2}{5} + a + (\frac{1}{3} - t)b = (1-h)a + 2hb$

$\frac{2}{5}t = 1-h$ ....(i)

$(1 - t) = \frac{3}{4} h$ ....(ii) $\quad t = \frac{5 - 5h}{2}$

$1 - (\frac{5 - 5h}{2}) = \frac{2}{3}h = 11h = 9$

$h = \frac{9}{11}$

$t = \frac{5 - 5\left(\frac{9}{11}\right)}{2} = \frac{5}{11}$

(i) BX : XM = 1:10

(ii) AX: XN = 3:8

5. a) i) $MA = \frac{1}{2} a$

ii) $AB = a$

iii) $AC = a + c$

iv) $AX = {}^{2}/_{7} AC = {}^{2}/_{7} (-a + c)$

b) $MA = \frac{1}{2} a$

$AX = {}^{2}/_{7} c - {}^{2}/_{7} a$

$MX = \frac{1}{2} a + {}^{2}/_{7} - {}^{2}/_{7} a$

$= {}^{3}/_{14} a + {}^{2}/_{7} c$

Co-ordinates of P = $(\frac{1 + 3}{2}, \frac{6 + 0}{2}, \frac{8 + 4}{2})$

= (2, 3, 6)

$/OP/ = \sqrt{2^2 + 3^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49} \quad = 7$ units

c) Co-ordinates of O (0,0,0)

Co-ordinates of A (1, 6, 8)

Mid points of AO = $(\frac{1 + 0}{2}, \frac{6 + 0}{2}, \frac{8 + 0}{2})$

= (0.5, 3, 4)

6. a) $AB = DC \Rightarrow 1 - x = 2 \Rightarrow x = -1$

$6 - y = 4 \Rightarrow y = 2$

$\therefore D = \underline{(-1, 2)}$

b) (i) $\overrightarrow{RQ} = \underset{\sim}{Q} - R = \underset{\sim}{q} - {}^{3}/_{2}\underset{\sim}{q} - \frac{1}{2} p$

$= \left(-\frac{1}{2} \underset{\sim}{q}\right) - \underset{\sim}{p} \left(= \frac{1}{2} \underset{\sim}{p}\right) - \underset{\sim}{q}$ ✓

(ii) $\overrightarrow{PR} = {}^{3}/_{2}\, q - ½\, p – P$ ✓

$= {}^{3}/_{2}\left(q - p\right)$

${}^{3}/_{2}\, q = -½$ Also $-{}^{3}/_{2}\, p = ½\, kp$

$\Rightarrow k = -3$ $\Rightarrow k = -3$

Hence P, Q, R, Q Collinear.

(iii) $\overrightarrow{PQ} = q - p$, $QR = ½\,(q – P)$

$PQ : QR = 2 : 1$

7. (a) $PQ = PO + OQ = -p + q$

Or $= OP + PR = P + 2/3\, PQ$

$= P + 2/3\,(-p+q)$

$= {}^{1}/_{3}p + {}^{2}/_{3}q$

$QT = QO + OT = -q + ½\, OR$ since $OT = TR$

$= -q + ½\,({}^{1}/_{3}p – {}^{2}/_{3}q)$

$= {}^{1}/_{6}p – {}^{2}//_{3}q$ OR ${}^{1}/_{6}\,(p-4q)$

(b) $TS = TO + OS = -½\, OR + ¼\, OP$

$= -½\,({}^{1}/_{3}p + {}^{2}/_{3}q) + ¼\, p = {}^{-1}/_{6}p – {}^{1}/_{3}q + ¼\, p$

$= {}^{1}/_{12}p – {}^{1}/_{3}q$ or ${}^{1}/_{12}(p-4q)$

$QT: TS = {}^{1}/6(p-4q): {}^{1}/12(p-4q) = {}^{1}/6:{}^{1}/12 = 2:1$

$\therefore QT = 2TS$ $OT//TS$ but T is a common point hence Q, T, S are collinear

(c) Vector OT can be expressed in 2 ways

1st $OT = ½\, OR$ given

$= ½\,({}^{1}/_{3}\, P + {}^{2}/_{3}q) = {}^{1}/_{6}q + 1/_{3}q$………..(i)

2nd using OPT

$OT = OP + PT = P + {}^{5}/_{6}PM$

But $PM = PO + OM = -P + KOQ = -P + Kq$

$OT = P + {}^{5}/_{6}\,(-P + kq)$

$= P – {}^{5}/_{6}kq$

$= {}^{1}/_{6}p + n^{5}/_{5}kq$…………………(ii)

Aqn (i) and (ii) represent the same vector OT

${}^{1}/_{6}p + {}^{1}/_{3}q = {}^{1}/_{6}p + {}^{5}/_{6}kq$……………..(iii)

Comparing coefficients of q in eqn (iii) have ${}^{5}/_{6}k = {}^{1}/3$

$15k = 6$

8. $3a = 3\begin{pmatrix}-3\\2\end{pmatrix} = \begin{pmatrix}-9\\6\end{pmatrix}$

$½\, b = ½\begin{pmatrix}4\\-6\end{pmatrix} = \begin{pmatrix}2\\-3\end{pmatrix}$

${}^{1}/_{10}c = {}^{1}/_{10}\begin{pmatrix}5\\-10\end{pmatrix} = \begin{pmatrix}0.5\\-1\end{pmatrix}$

$P = \begin{pmatrix}-9\\6\end{pmatrix} - \begin{pmatrix}2\\-3\end{pmatrix} + \begin{pmatrix}0.5\\-1\end{pmatrix}$

$= \begin{pmatrix}-10.5\\8\end{pmatrix}$

$/P/ = \sqrt{(-10.5)^2 + 8^2}$

$= \sqrt{110.25} = 64$

$= \sqrt{174.25}$

$= 13.20037878$

$= 13.20$ (2 d.p)

9. (i) $BM = BO + OM$

$= {}^2/_5 a - b$

(ii) $AN = AO + ON$

$= \frac{2}{3} b - a$

(b) $OX = OB + BX$

$= b + k (2 a - b)$

$= \frac{2}{5} ka + b(1 - k)$

$OX = OA + AX$

$= a + h (\frac{2}{3}b - a)$

$= a (1-h) + 2 hb$

$= a(10h) 2hb$

(c) ${}^2/_5\ a = a (1-h)$ also $b(1-k) = 2hb$

$2k = 1-h \quad 1-k = 2h$

$k = \frac{5}{2} - \frac{5}{2}h$

$\therefore 1 - \frac{5}{2} + \frac{5}{2}h = \frac{2}{3}h$

$\frac{5}{2}h - \frac{2}{3}h = \frac{5}{2} - 1$

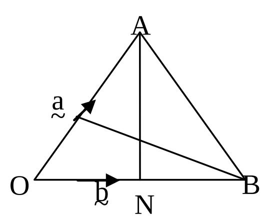
$1\frac{5}{6}h = \frac{3}{2}$

$h = \frac{3}{2} \times \frac{6}{11} = 9$

$k = \frac{5}{2} - \frac{5}{2} \quad \frac{9}{11}$

$= \frac{5}{2} - \frac{45}{22}$

$= \frac{5}{11}$



10. (i) $AN = AO + ON$

$= -a + \frac{4}{5} b$

(ii) $BM = BO + OM$

$= -b + {}^2/_5\ a$

(iii) $\underset{\sim}{AB} = \underset{\sim}{AO} + \underset{\sim}{OB}$

$= -\underset{\sim}{a} + \underset{\sim}{b}$

$\underset{\sim}{AX} = s\underset{\sim}{AN}$

$\underset{\sim}{BX} = t\underset{\sim}{BM}$

$\underset{\sim}{OX} = \underset{\sim}{OB} + \underset{\sim}{BX}$

$= \underset{\sim}{b} + t\underset{\sim}{BM}$

$= \underset{\sim}{b} + t(-\underset{\sim}{b} + {}^{2}/_{5}\underset{\sim}{a})$

$= \underset{\sim}{b} - t\underset{\sim}{b} + {}^{2}/_{5}t\underset{\sim}{a}$

$= \underset{\sim}{b}(1-t) + {}^{2}/_{5}t\underset{\sim}{a}$

$\underset{\sim}{OX} = \underset{\sim}{OA} + \underset{\sim}{AX}$

$= \underset{\sim}{a} + s\underset{\sim}{AN}$

$= \underset{\sim}{a} + s(-\underset{\sim}{a} + {}^{4}/_{5}\underset{\sim}{b})$

$= \underset{\sim}{a} - Sa + {}^{4}/_{5}\, s\underset{\sim}{b}$

$\underset{\sim}{a}(1-s) + {}^{4}/_{5}\, s\underset{\sim}{b}$

$b(1-t) + {}^{2}/_{5}t\underset{\sim}{a} = a(1-s)\, {}^{4}/_{5}sb$

$b(1-t) = {}^{4}/_{5}\, sb$

$1-t = {}^{4}/_{5}s$-------------(i)

$a(1-s) = {}^{2}/_{5}ta$

$1-s = {}^{2}/_{5}ta$

$s = 1-{}^{2}/_{5}t$-------------(ii)

$1-t = {}^{4}/_{5}(1-{}^{2}/_{5}t)$

$1-t = {}^{4}/_{5} - 8/_{25}t$

$-{}^{17}/_{25} = -1/_{5}$

$t = {}^{5}/_{17}$

$s = {}^{15}/_{17}$

11. $\frac{115800}{76.84} \times \frac{97.5}{100}$

$= 1469.35$ √

$= 1469.35 - 270$

$= 1199.35$ √

$= 1199$ dollars

12. $RM = \begin{pmatrix} -2 \\ 6 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 8 \\ -1 \end{pmatrix}$

$\sqrt{\ |\ }$

$|\underset{\sim}{RM}| = \sqrt{(-3)^2 + 82(-1)^2}$

$\sqrt{74} = 8.602$ units

13. (a) (i) $\underset{\sim}{OB} = \underset{\sim}{a} + \underset{\sim}{b}$

(ii) $BC = \underset{\sim}{BA} + AO + \underset{\sim}{OC}$

$= -\underset{\sim}{b} + -a + 2\underset{\sim}{b}$

$= \underset{\sim}{b} - \underset{\sim}{a}$

(b) $\underset{\sim}{CX} = \underset{\sim}{CO} + \underset{\sim}{OA} + \underset{\sim}{AB} + \underset{\sim}{BX}$

$= -2\underset{\sim}{b} + \underset{\sim}{a} + b + hBC$

$= a - b + h(b - a)$

$= \underset{\sim}{a} - \underset{\sim}{b} + hb - h\underset{\sim}{a}$

$= (1 - h)\underset{\sim}{a} + (h - 1)\underset{\sim}{b}$

(c) $\underset{\sim}{CX} = CO + OA + AX$

$= 2\underset{\sim}{b} + \underset{\sim}{a} + KAT$

but $AT = \underset{\sim}{AO} + \underset{\sim}{OT}$

$= -a + 3b$

$CX = 2b + a + K(3b - a)$

$= a - Ka + 3Kb + 2b$

$= (1- K)a + 3(K + 2)b$

(d) $I - h = 1 - k$ ....(i)

$h - 1 = 3k + 2$.......(ii)

from (i) $h = k$

sub in (ii) $h-1 = 3h + 2$

$h = {}^{-3}/_{2}$

$K = {}^{-3}/_{2}$

14. $a + b = (2 - 3)i + (1 + 4)j + (-2-1)k$

$= -i + 5j - 3k$

$|a + b| = \sqrt{(-1)^2 + (5)^2 + (-3)^2}$

$= \sqrt{35}$

$= 5.916$

15. i) $BD = BA + AD$

$= -b + {}^{3}/_{5}c$

$AE = AB + BE$

$= b + ½ BC = b + ½ (c - b)$

$= ½ b + ½ c$

ii) $BF = t({}^{3}/_{5}c - b)$

$AF = n(½ b + ½ c) = {}^{n}/_{2}(b + c)$

$AF = AB + BF$

$= b + t({}^{3}/_{5}c - b) = b + {}^{3}/_{5}tc + tb$

$= (1 - t)b + {}^{3}/5tc$

$(1 - t)b + {}^{3}/_{5}tc = {}^{n}/_{2}b + {}^{n}/_{2}c$

$1 - t = {}^{n}/_{2}$; $2 - 2t = n$ ………… (i)

$^{3}/_{5}t = {}^{n}/_{2}$; $6t - 5n = 0$ ………..... (ii)

Sub from équation (ii)
$6t - 5(2 - 2t) = 0$
$6t - 10 + 10t = 0$
$16t = 10$
$t = {}^{10}/_{16} = {}^{5}/_{8}$
$n = ¾$

iii) $BF = {}^{5}/_{8}\ BD$
F divides BD in the ratio 5 :3
$AF = ¾\ AE$
F divides AE in the ratio 3 :1

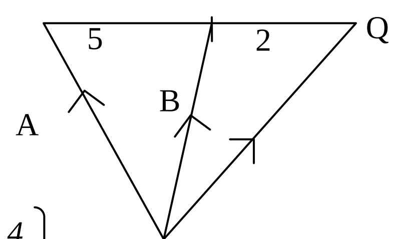
16. $BA = \begin{pmatrix} -8 \\ -2 \end{pmatrix}$

$½\ BC = ½ \begin{pmatrix} -3 \\ -4 \end{pmatrix} = \begin{pmatrix} 1\ ½ \\ -2 \end{pmatrix}$

$OP = \begin{pmatrix} -8 \\ -2 \end{pmatrix} + \begin{pmatrix} -1\ ½ \\ -2 \end{pmatrix} = \begin{pmatrix} -9\ ½ \\ -4 \end{pmatrix}$

Co-ordinates of P (-9 ½ , -4)

17. $OB = \frac{5}{7}OQ + \frac{2}{5}OA$



$OQ = \frac{7}{5}OB - \frac{2}{5}OA$

$OQ = \frac{7}{5}\begin{pmatrix} 2 \\ -1 \end{pmatrix} - \frac{2}{5}\begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} {}^{14}/_{5} \\ {}^{-7}/_{5} \end{pmatrix} - \begin{pmatrix} {}^{-6}/_{5} \\ {}^{8}/_{5} \end{pmatrix} = \begin{pmatrix} {}^{20}/_{5} \\ {}^{-15}/_{5} \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

Q= (4, -3)