SECTION I (50 marks)

Answer all questions in this section in the spaces provided

1. Evaluate $4.4\dot{1}\dot{4}\dot{1} - 2.2\dot{1}\dot{4}\dot{1}$ (2 marks)

$r = 4.4141$

$100r = 441.4141$

$99r = 437$

$r = \frac{437}{99}$ — M_1

$x = 2.2121$

$100x = 221.2121$

$99x = 219$

$\frac{437}{99} - \frac{219}{99}$ M_1

$\frac{218}{99}$ — M_1 M_1

$2\frac{20}{99}$ A1

2. Given that $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$ without using tables or calculator find $\log 0.036$ correct to 4 significant figures. (3 marks)

$\log 0.036 = \log\left(\frac{36}{1000}\right)$

$= \log\left(\frac{2^2 \times 3^2}{10^3}\right)$ M_1 M_1

$= 2\log 2 + 2\log 3 - 3\log 10$ A1

$= 2(0.3010) + 2(0.4771) - 3$

$= -1.4438$

3. Simplify $\frac{ax - 2a - 5x + 10}{8 - 2x^2}$ (3 marks)

$\frac{a(x-2) - 5(x-2)}{2(4 - x^2)} = \frac{(a-5)(x-2)}{2(2-x)(2+x)}$ M_1 M_1

$\frac{(a-5)(x-2)}{2(2-x)(2+x)}$ A1

$\frac{-1(a-5)}{4+2x}$

$\frac{5-a}{4+2x}$

4. Solve the equation $\sin(150 \div x)^\circ = \cos(12x)^\circ$ for which x is acute. (3 marks)

$$\frac{150}{x} + 12x = 90^\circ$$

$$12x^2 - 90x + 150 = 0$$

$$x = \frac{90 \pm \sqrt{8100 - 4(12 \times 150)}}{24}$$

$$x = \frac{90 \pm 30}{24}$$

$$= 5^\circ \text{ or } 2.5^\circ$$

M1 M1 A1

5. Under an enlargement, the image of the points A(3,1) and B(1,2) are A′(3,7) and B′(7,5). Find the centre and scale factor of enlargement. (4 marks)

Let centre (x, y)

$$\frac{3-x}{3-x} = \frac{7-x}{1-x}$$

$$(3-x)(1-x) = (3-x)(7-x)$$

$$3 - 3x - x + x^2 = 21 - 3x - 7x + x^2$$

$$-18 = -6x$$

$$x = 3$$

$$\frac{5-y}{2-3} = \frac{7-y}{1-y}$$

$$5 - 5y - y + y^2 = 14 - 2y - 7y + y^2$$

$$y = 3$$

Centre (3,3)

$$\text{Scale factor} = \frac{7-3}{1-3}$$

$$= -2.$$

M1 M1 M1 A1

6. A shopkeeper bought a bag of sugar. He intends to repack the sugar in 40 g, 250 g and 750 g. Determine the least mass in grams of sugar that was in the bag. (3 marks)

L.C.M

10	40	250	750
4	4	25	75
25	1	25	75
3	1	1	3
	1	1	1

L.C.M $= 10 \times 4 \times 25 \times 3$

$= 3000\text{g}.$

M1 M1 A1

7. Solve for x and y in $3^{3y} \div 3^{2x} = 6561$ and $2^{2x} \times 2^{3y} = 1$. (4 marks)

$3^{3y} \div 3^{2x} = 3^8$

$2^{2x} \times 2^{3y} = 2^0$

$3y - 2x = 8$ — M1

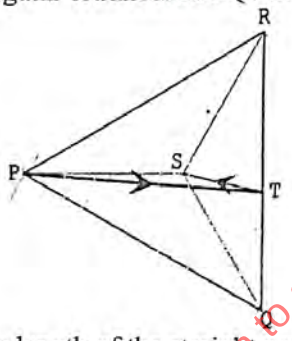
$3y + 2x = 0$

$-4x = 8$ M1

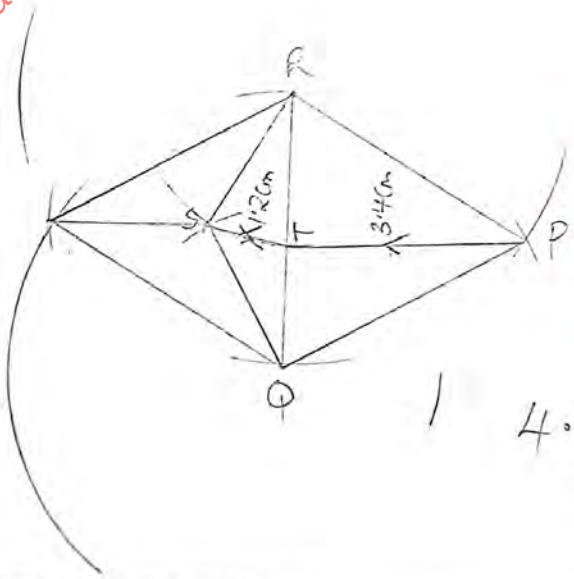
$x = -2$ A1

$y = 1\frac{1}{3}$ B1

8. The figure below shows a regular tetrahedron PQRS.



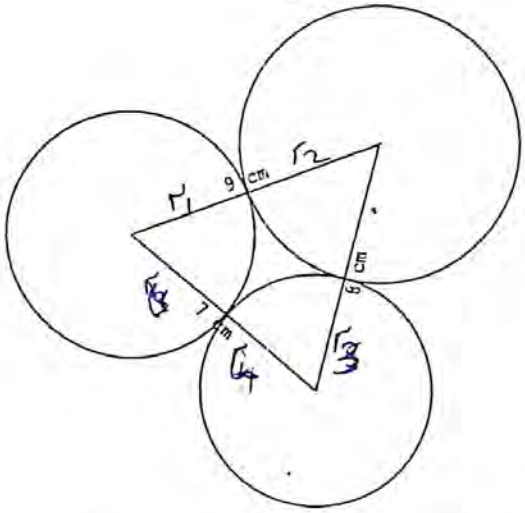
Draw its net and measure the length of the straight path of PS through the midpoint T over the edge QR. (3 marks)



B1 B1 B1

(4 marks)

9. The figure below shows three circles touching each other externally.



If the centres of the circle form a triangle with sides of length 9 cm, 8 cm and 7 cm, calculate the radii of each the circles. (3 marks)

$r_1 + r_2 = 9$
$r_1 + r_3 = 7$
$r_2 + r_3 = 8$

$r_2 - r_3 = 2$
$r_1 - r_2 = -1$

$-r_3 + r_1 = 1$ ---(i) M1
$r_3 + r_1 = 7$ ---(ii) +
$2r_1 = 8$ M1

$r_1 = 4$ Cm
$r_2 = 5$ Cm
$r_3 = 3$ Cm A1

10. A Kenyan company received M Us dollars. The money was converted into Kenya Shillings in a bank which buys and sells foreign currencies as shown below.

	Buying (Kshs)	*Selling (Kshs)*
1 Sterling Pound	*125.78*	*126.64*
1 US Dollar	*75.66*	*75.86*

a) If the company received Kshs.15, 132, 000, calculate the amount, M Us Dollars. (2 marks)

$75.66M = 15\,132\,000$

$M = \frac{15,132,000}{75.66}$ M1

200,000 Us Dollars A1

b) The company exchanged the above Kenyan shillings into Sterling pounds to buy a car in Britain. Calculate the cost of the car to the nearest Sterling Pound. (2 marks)

$\frac{15\,132\,000}{126.64} = 119\,488.3133$ M1

$= £\ 119,488$ A1

11. A bus travelling at an average speed of x km/h left station at 8.15 am. A car, travelling at an average speed of 80km/h left the same station at 9.00 am and caught up with the bus at 10.45 am. Find the value of x. (3 marks)

Distance by bus in 45 min
$= x \times \frac{45}{60} = 0.75x$ km B1

Time to catch up is $1\frac{3}{4}$ hrs.

$\frac{0.75x}{80-x} = \frac{7}{4}$ M1

$3x = 560 - 7x$

$x = 56$ km/hr A1

12. The position vector of points A and B are $-10i-6j+9k$ and $-5i-k$ respectively. Calculate $|\underset{\sim}{AB}|$ leaving your answer in surd form. (3 marks)

$\underset{\sim}{OA} = \begin{pmatrix} -10 \\ -6 \\ 9 \end{pmatrix}$

$\underset{\sim}{OB} = \begin{pmatrix} -5 \\ 0 \\ 1 \end{pmatrix}$

$\underset{\sim}{AB} = \begin{pmatrix} -5 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} -10 \\ -6 \\ 9 \end{pmatrix}$ M1

$\underset{\sim}{AB} = \begin{pmatrix} 5 \\ 6 \\ -8 \end{pmatrix}$

$|\underset{\sim}{AB}| = \sqrt{5^2+6^2+(-8)^2} = \sqrt{125}$ M1

$= 5\sqrt{5}$ units A1

13. Solve the inequality $5-2x < \frac{1}{2}x < \frac{x+2}{3}$ and represent the solution on a number line (3 marks)

$5 - 2x < \frac{1}{2}x$

$5 < \frac{5}{2}x$

$10 < 5x$

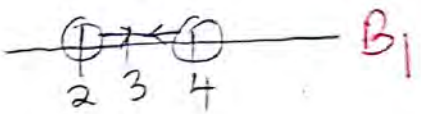
$2 < x$ B1

$\frac{1}{2}x < \frac{x+2}{3}$

$3x < 2x + 4$

$x < 4$ B1

$2 < x < 4$



B1

14. Use tables of reciprocals and square roots to evaluate $\frac{3}{0.521}+\sqrt{0.4036}$ (3 marks)

$3\left(\frac{1}{5.21\times10^{-1}}\right)+\left(40.36\times10^{-2}\right)^{1/2}$

$3(0.1919\times10)+6.353\times10^{-1}$ M1

$5.757+0.6353$ M1

6.3923 A1

15. Two farmers Salat and Ahmed share a grazing field. Salat puts 120 cows for 18days and Ahmed puts 150 cows for 16 days. If the rented land was charged Kshs.8 550, how much should Ahmed pay if they share proportionally? (3 marks)

$= 8550\times\frac{150}{270}\times\frac{16}{34}$

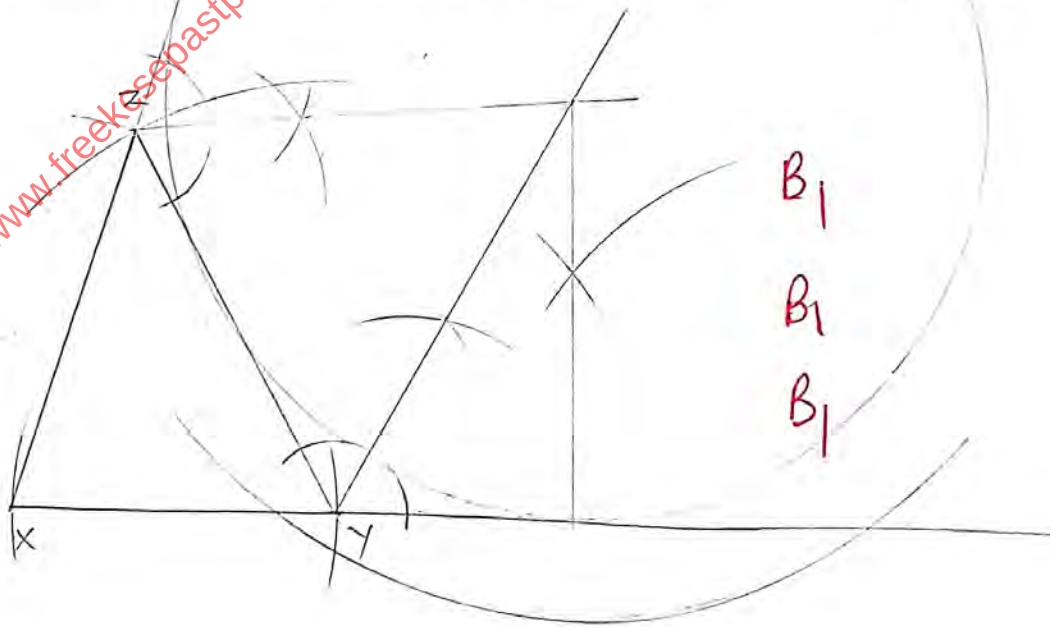
$= \text{Ksh. } 2235\frac{5}{17}$

Amount per day $= \frac{8550}{18} = 475$ M1

Ahmed $= 475\times\frac{150}{270}\times16$ M1

$= \text{ksh. } 4222\frac{2}{9}$ A1

16. Construct triangle XYZ in which XY = 4.5 cm, YZ = 6 cm and XZ = 5.5 cm. Construct a circle that touch ZY, XY and XZ produced and also opposite to ∠ YXZ. (3 marks)



B1 B1 B1

SECTION II (50 marks)

Answer only five questions from this section in the spaces provided

17. A salesman dealing in mattresses earns a basic salary and commission as follows:

	Commission
For sales up to Ksh 150 000	0%
For sales above Ksh 150 000	
First Ksh 85 000	3%
Next Ksh 85 000	4%
Any amount above Ksh 320 000	5%

(a) In the month of January 2023, the salesman earned a basic salary of Ksh 45 000 and he sold 110 mattresses at Ksh 5 000 each. Calculate:

(i) His total sales in the month of January 2023. (1 mark)

110×5000

Ksh. 550,000 B1

(ii) His total earnings that month. (3 marks)

$45{,}000 + \frac{3}{100} \times 85{,}000 + \frac{4}{100} \times 85{,}000 + \frac{5}{100} \times 230{,}000$ M1

$45000 + 2550 + 3400 + 11500$ M1

Ksh. 62450 A1

(b) In the month that followed, his basic salary was decreased by 10%. If he received a total earning of Ksh 47 450 in that month, calculate:

(i) Total sales that month. (4 marks)

B.Salary $= \frac{90}{100} \times 45000 =$ Ksh. 40500 M1

Total Commission $= 47450 - 40500$ M1

$=$ Ksh. 6950

First 85,000 = Sh. 2550 Commission.

Next 85,000 = Sh. 3400 Commission.

Amount above 320,000 = Ksh. 1000 Commission.

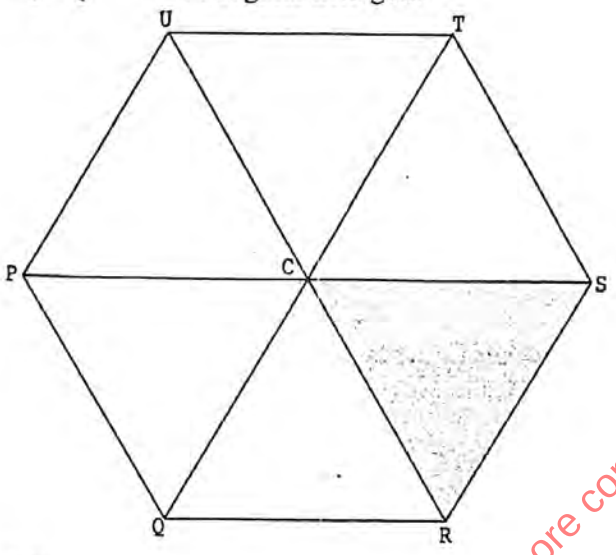
$\frac{5}{100}$ of $(x - 320{,}000) = 1000$ M1

Total sales $(x) = 340{,}000$ A1

(ii) The number of mattresses sold in that month. (2 marks)

$\frac{340{,}000}{5000} = 68$ mattresses M1 A1

18. In the figure below, PQRSTU is a regular hexagon.



(a) Describe fully:

(i) a reflection that maps ΔSCR onto ΔSCT. (1 mark)

Rotation through an angle of +60° about C. √B1

(ii) an enlargement that maps ΔSCR on ΔPCU. (2 marks)

An enlargement √B1 centre C, scale factor −1. √B1

(iii) a rotation that maps ΔSCR to ΔUCT. (3 marks)

Rotation √B1 about C √B1 through an angle of +120°. √B1

(b) The ΔPQC is reflected on the line RU. The image of ΔPQC under the reflection is then rotated through an angle -120^0 about point C. Determine the images of P and Q:

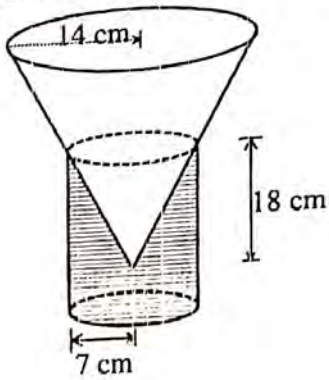
(i) Under the reflection. (2 marks)

T √B1 and S √B1 respectively

(ii) After the two successive transformations. (2 marks)

R √B1 and Q √B1 respectively.

19. A cylindrical tin of radius 7 cm contains water to a height of 19 cm. When a conical solid of radius 14 cm is fitted into the cylindrical tin to a depth of 18 cm as shown in the figure below, the water completely fills the space between the cylindrical tin and part of the cone inside the tin.



(a) Taking π to be $\frac{22}{7}$, calculate correct to one decimal place:

(i) The volume of part of the cone that is not in contact with water. (3 marks)

$\frac{14}{7} = \frac{H}{18}$ ✓M1

$H = 36cm$

$V = \frac{1}{3} \times \frac{22}{7}(14^2 \times 36 - 7^2 \times 18)$ ✓M1

$= 6468 cm^3$ ✓A1

(ii) The surface area of part of the cone that is not in contact with water. (4 marks)

$\pi(RL - rl) + \pi R^2$

$L = \sqrt{14^2 + 36^2} = \sqrt{1492} = 38.63cm$ } ✓B1

$l = \sqrt{7^2 + 18^2} = \sqrt{373} = 19.31cm$

Surface Area $= \frac{22}{7}(\sqrt{1492} \times 14 - 7\sqrt{373}) + \frac{22}{7} \times 14^2$ ✓M1 ✓M1

$= 1,274.67 + 616$

$= 1890.7 cm^2$ ✓A1

(b) Calculate the height of the cylindrical tin. (3 marks)

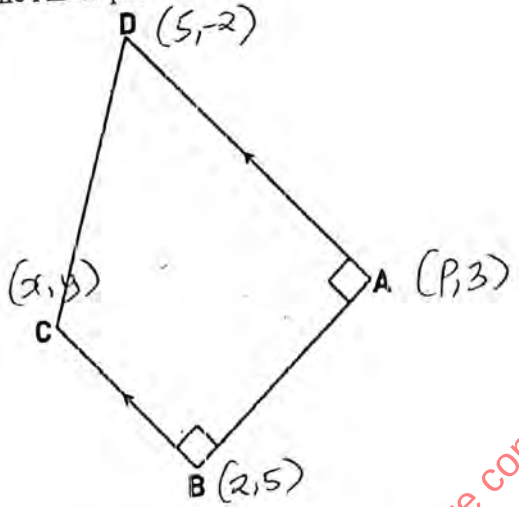
Volume of water $= \frac{22}{7} \times 7^2 \times 19 = 2926 cm^3$

Volume of the Cone in Contact with water $= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 18 = 924 cm^3$ } ✓B1

Volume of the Cylinder $= 2926 + 924 = 3850$ ✓M1

Height $= \frac{3850}{\frac{22}{7} \times 7^2} = 25cm$ A1

20. The figure below shows a trapezium on a Cartesian plane with the co-ordinates $A(p,3)$, $B(2,5)$, $C(x,y)$ and $D(5,-2)$. Line AD is parallel to BC.



Given that the line AB makes an angle of 45^0 with the positive $x-axis$

(a) Determine the equation of line AB in the form $y=mx+c$ (2 marks)

Gradient of AB = tan 45° = 1 √

$\frac{y-5}{x-2} = 1$ √M1

$y = x+3$ √A1

(b) State the value of p in the coordinates of point A (2 marks)

$\frac{3-5}{p-2} = 1$ √M1

$p = 0$ √A1

(c) Find the equation of BC in the form $ax+by=c$ (2 marks)

$\frac{y-5}{x-2} = -1$ √M1

$x+y = 7$ √A1

(d) Given that the gradient of $CD=-5$. Find its equation hence coordinates of point C. (4 marks)

Eqn of CD √M1

$\frac{y+2}{x-5} = -5 \Rightarrow 5x+y = 23$

$5x+y = 23$ ---(i)

$x+y = 7$ ---(ii)

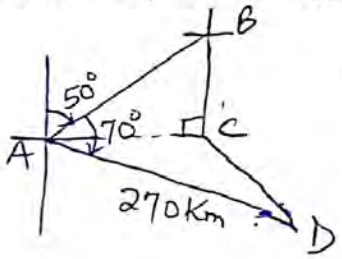
$4x = 16$ √M1

$x = 4, y = 3$ C(4,3) A1

21. Two aeroplanes T_1 and T_2 leave airport A for airports B and D respectively. Aeroplane T_1 flies on a bearing 050° at an average speed of 300 km/hr for 40 minutes. Aeroplane T_2 flies on a bearing 120° at an average speed of 360 km/hr for 45 minutes. An airport C is directly south of B and east of A.

(a) Calculate the distance between:

(i) Airports A and B. (1 mark)



$300 \times \frac{40}{60}$

$= 200\text{ km}$ $\checkmark B_1$

(ii) Airports A and D. (1 mark)

$360 \times \frac{45}{60}$

$= 270\text{ km}$ $\checkmark B_1$

(b) Calculate the distance correct to four significant figures between:

(i) Airports A and C. (2 marks)

$AC = 200\cos 40°$ $\checkmark M_1$

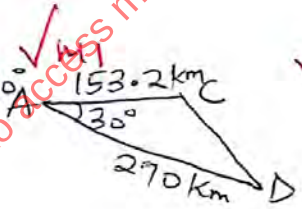
$= 153.2\text{ km}$ $\checkmark A_1$

(ii) Airports C and D. (3 marks)

$CD^2 = 270^2 + 153.2^2 - 2 \times 270 \times 153.2\cos 30°$ $\checkmark M_1$

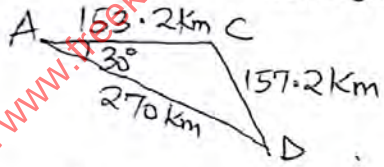
$= 24725.6904$

$CD = 157.2\text{ km}$ $\checkmark A_1$



$\checkmark B_1$ 30°

(c) Calculate the true bearing to the nearest degree of airport D from C. (3 marks)



$\frac{157.2}{\sin 30°} = \frac{270}{\sin \angle ACD}$ $\checkmark M_1$

$\angle ACD = \sin^{-1}\left(\frac{270 \sin 30°}{157.2}\right)$

$= \sin^{-1} 0.8585$

$= 59.15°$

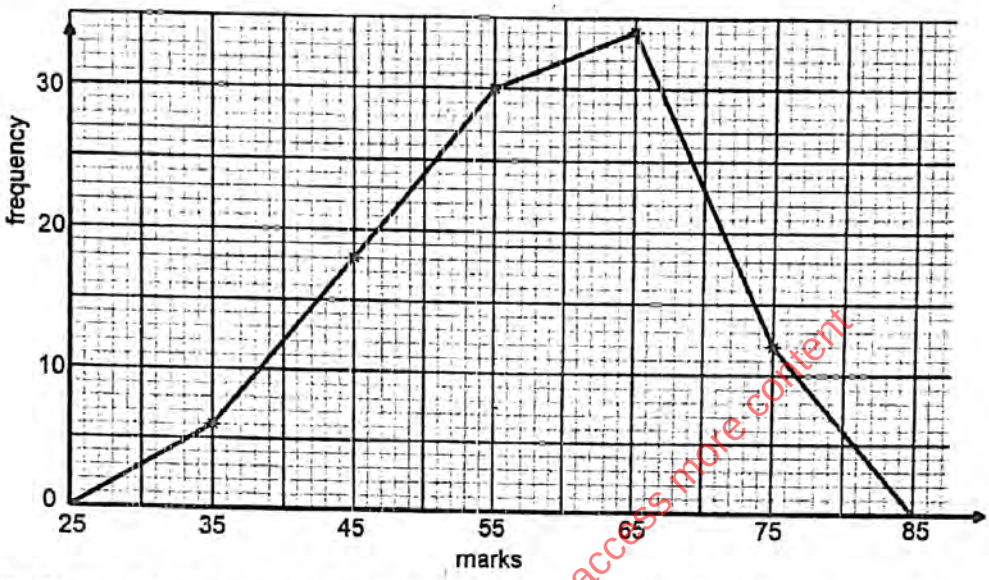
$\angle ACD = 180 - 59.15$ $\checkmark M_1$

$= 120.85°$

True bearing $= 270° - 120.85°$

$= 149.15°$ $\checkmark A_1$

22. The figure below shows a frequency polygon representing the scores of Form 4 students in a Mathematics Test.



a) Generate the Frequency Distribution of the data under the columns given below in the table below.

(5 marks)

Marks	Frequency (f)	Mid points (x)	fx	$c.f.$
30-40	6	35	210	6
40-50	18	45	810	24
50-60	30	55	1650	54
60-70	34	65	2210	88
70-80	12	75	900	100
			$\Sigma fx = 5780$	

√B1 √B1 √B1 √B1 √B1

b) State the modal class. (1 mark)

60-70 √B1

c) Estimate

(i) the mean mark (2 marks)

$\frac{\Sigma fx}{\Sigma f} = \frac{5780}{100}$ $\surd_{M1}$

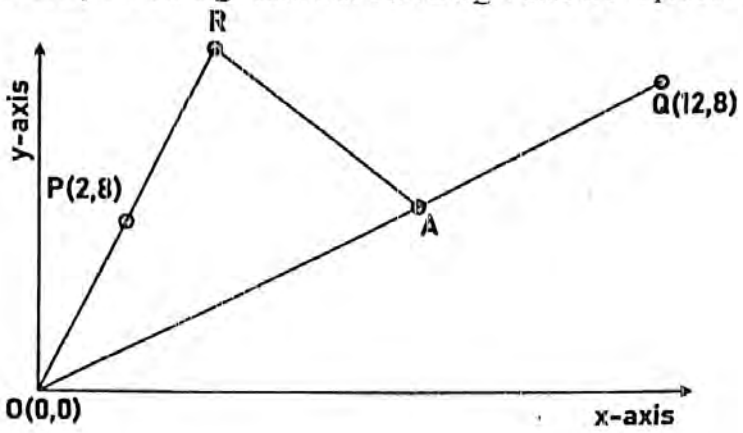
$= 57.80$ $\surd_{A1}$

(ii) the median mark (2 marks)

$50 + \frac{(50-24)}{30} 10$ $\surd_{M1}$

58.67 $\surd_{A1}$

23. In the diagram below, the coordinates of points O, P and Q are (0, 0), (2, 8) and (12, 8) respectively. A is a point on $\overrightarrow{OQ}$ such that $2\overrightarrow{OA} = \overrightarrow{OQ}$. Line **OP** is produced to R is such as $\overrightarrow{OR} = 3\overrightarrow{OP}$.



a) Find vector $\overrightarrow{RA}$. (3 marks)

$\underset{\sim}{OA} = \frac{1}{2}\underset{\sim}{OQ}$ ✓M1

$= \frac{1}{2}\begin{pmatrix}12\\8\end{pmatrix} = \begin{pmatrix}6\\4\end{pmatrix}$

$\underset{\sim}{OR} = 3\underset{\sim}{OP} = 3\begin{pmatrix}2\\8\end{pmatrix} = \begin{pmatrix}6\\24\end{pmatrix}$

$\underset{\sim}{RA} = \begin{pmatrix}6\\4\end{pmatrix} - \begin{pmatrix}6\\24\end{pmatrix}$ ✓M1

$= \begin{pmatrix}0\\-20\end{pmatrix}$ ✓A1

b) Given that point L is on $\overrightarrow{PQ}$ such that $\overrightarrow{PL} : \overrightarrow{LQ} = 2:3$, find vector $\overrightarrow{RL}$. (4 marks)

$\underset{\sim}{OL} = \frac{3}{5}\begin{pmatrix}2\\8\end{pmatrix} + \frac{2}{5}\begin{pmatrix}12\\8\end{pmatrix} = \begin{pmatrix}6\\8\end{pmatrix}$ ✓M1 ✓A1

$\underset{\sim}{RL} = \begin{pmatrix}6\\8\end{pmatrix} - \begin{pmatrix}6\\24\end{pmatrix} = \begin{pmatrix}0\\-16\end{pmatrix}$ ✓M1 A1

c) Show that R, L and A are collinear. (3 marks)

$\underset{\sim}{RL} = K\underset{\sim}{RA}$

$\begin{pmatrix}0\\-16\end{pmatrix} = \begin{pmatrix}0\\-20K\end{pmatrix}$ ✓B1

$-20K = 16$

$K = \frac{4}{5}$

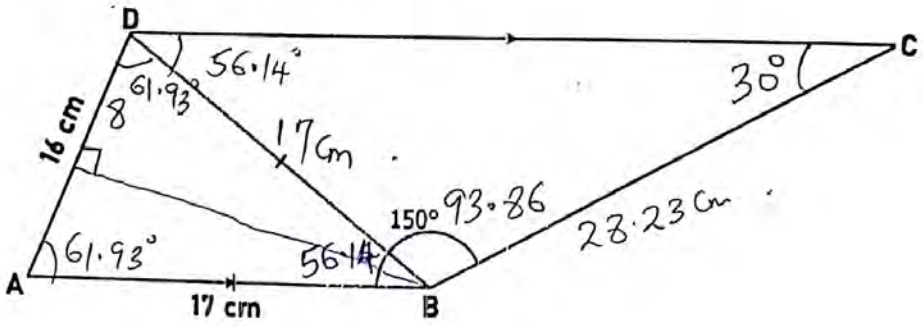
$\underset{\sim}{RL} = \frac{4}{5}\underset{\sim}{RA}$ ✓B1

$RL \parallel RA$.

The two vectors share a common point R. ✓B1

Hence R, L and A are Collinear.

24. In the figure below, ABCD is a trapezium in which AB = 17 cm, AD = 16 cm and angle ABC = 150^0. AB is parallel to DC and AB = BD.



(a) Calculate the area of triangle ABD (2 marks)

$\sqrt{25(25-16)(25-17)(25-17)}$ √M1

$\sqrt{14400}$

$= 120\,cm^2$ √A1

(b) Calculate correct to two decimal places:

(i) The length BC (2 marks)

$\angle DAB = \angle ADB = \cos^{-1}\frac{8}{17} = 61.93°$

$\frac{17}{\sin 30°} = \frac{BC}{\sin 56.14}$ √M1

$BC = \frac{17 \sin 56.14}{\sin 30°}$ √M1

$= 28.23\,cm$ √A1

(ii) The length AC (3 marks)

$AC^2 = 17^2 + 28.23^2 - 2 \times 17 \times 28.23 \cos 150°$ √M1

$AC = \sqrt{1917.1614}$ √M1

$= 43.79\,cm$ √A1

(iii) The size of angle ACD (3 marks)

$\frac{43.79}{\sin 118.07} = \frac{16}{\sin \angle ACD}$ √M1

$\angle ACD = \sin^{-1}\left(\frac{16 \sin 118.07}{43.79}\right)$

$= \sin^{-1} 0.3224$ √M1

$\angle ACD = 18.81°$ √A1