X

ACK MOCK JOINT EXAMINATION SCHEME

SECTION A (50 MARKS)

*(Answer **all** questions in this section in the spaces provided)*

1. Use logarithm table to evaluate. (4mks)

$$\sqrt[4]{\frac{(27\times0.0293)^2}{(825-94)\div0.2861}}$$

731

No	Std form	log table
27	2.7×10^1	1.4314
0.0293	2.93×10^{-2}	$\bar{2}.4669$ +
		$\bar{1}.8983$
		× 2
		$\bar{1}.79166$
731	7.31×10^2	2.6139
0.2861	2.861×10^{-1}	$\bar{1}.4566$ + −
		3.3073
		$\bar{4}.2893\times\frac{1}{4}$
0.1181	$10^{-1}\times1.181$	$\bar{1}.07231$

M1 ✓ logs

M1 ✓+ ✗ −

M1 use of powers

A1

2. Three sisters, Ann, Beatrice and Caroline together invested Ksh. 48,000 as capital and started a small business. If the share of profit is Ksh. 2,300, Ksh. 1,700 and Ksh. 800 respectively, shared proportionally. Find the capital invested by each of them. (3mks)

A : B : C

~~2300~~ : ~~1700~~ : ~~800~~

23 : 17 : 8

$\frac{23}{48}$: $\frac{17}{48}$: $\frac{8}{48}$

Ann $= \frac{23}{48}\times48,000 = $ Sh. $\underline{23,000}$ B1

Beatrice $= \frac{17}{48}\times48,000 = $ Sh. $\underline{17,000}$ B1

Caroline $= \frac{8}{48}\times48,000 = $ Sh. $\underline{8,000}$ B1

3. Make t the subject of formula in $x = (\frac{p+t}{t})^{\frac{1}{3}}$ (3mks)

$$(x)^3 = \left[\left(\frac{p+t}{t}\right)^{\frac{1}{3}}\right]^3$$

$$\frac{x^3}{1} = \frac{p+t}{t}$$

$$x^3t = p+t$$

$$x^3t - t = p$$

$$\frac{t(x^3-1)}{(x^3-1)} = \frac{p}{x^3-1}$$

$$\therefore t = \frac{p}{(x^3-1)}$$

M1 ✓ removing root

M1 ✓ collecting like terms

A1 answer

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4. Without using a calculator or mathematical tables, express $\frac{\sqrt{3}}{1-\cos 30^0}$ in surd form and simplify. (3mks)

$\cos 30 = \frac{\sqrt{3}}{2}$ ✓B1

$$\frac{(\sqrt{3})2}{\left(1-\frac{\sqrt{3}}{2}\right)2}$$

$$\frac{2\sqrt{3}}{(2-\sqrt{3})}\frac{(2+\sqrt{3})}{(2+\sqrt{3})} \checkmark M1$$

$$= \frac{4\sqrt{3}+6}{4-3}$$

$$= \frac{4\sqrt{3}+6}{1}$$

$$= 4\sqrt{3}+6 \checkmark A1$$

5. Expand and simplify $(3x-y)^4$ hence use the first three terms of the expansion to approximate the value of $(6-0.2)^4$. (3mks)

$1(3x)^4(-y)^0 + 4(3x)^3(-y)^1 + 6(3x)^2(-y)^2 + 4(3x)^1(-y)^3 + 1(3x)^0(-y)^4$

$81x^4 + -108x^3y + 54x^2y^2 - 12xy^3 + y^4$

$81x^4 - 108x^3y + 54x^2y^2 - 12xy^3 + y^4$ ✓B1

$3x = 6 \quad -y = -0.2$

$x = 2 \quad y = 0.2$

$= 81x^4 - 108x^3y + 54x^2y^2$

$= 81(2)^4 - (108 \cdot 8 \cdot 0.2) + (54 \cdot 4 \cdot 0.04)$ ✓M1

$= 324 - 172.8 + 8.64$

$= 159.84$ ✓ A1

6. Find x without using tables if $3 + \log_2 3 + \log_2 x = \log_2 5 + 2$ (3mks)

$3(\log_2 2) + \log_2 3 + \log_2 x = \log_2 5 + 2(\log_2 2)$ M1 all terms in logs

$\log_2 8 + \log_2 3 + \log_2 x = \log_2 5 + \log_2 4$ M1 Single logs

$\log_2(8 \times 3 \times x) = \log_2(5 \times 4)$

$\frac{24x}{24} = \frac{20}{24}$ A1

$x = \frac{5}{6}$

7. Find the value of m for which the matrix transforms an object into a straight line. (3mks)

$$\begin{pmatrix} m^2 & 1 \\ 2m-1 & 1 \end{pmatrix} \Rightarrow \text{Singular Matrix}$$

$(M^2 \times 1) - (2M-1)1 = 0$ ✓

$M^2 - (2M-1) = 0$ $\quad S = -2$

$M^2 - 2M + 1 = 0$ $\quad P = 1$

$(M^2 \cancel{-M})(-M+1) = 0$ $\quad (-1, -1)$

$M(M-1) - 1(M-1) = 0$ ✓

$(M-1)(M-1) = 0$

$M - 1 = 0$

$M = 1$

$M - 1 = 0$

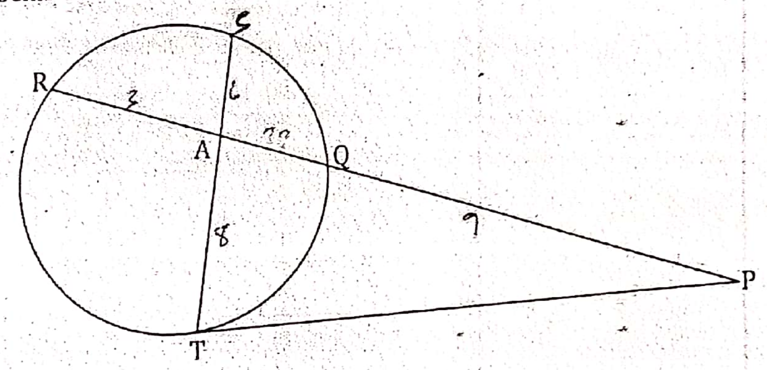
$M = 1$

M1 2 eqn

M1 attempt to solve

A1 both answers

8. In the figure below PT is a tangent to the circle at T, PQ = 9cm, SA = 6cm, AT = 8cm and AR = 3cm. Calculate the length of;



(a) AQ (2mks)

$\frac{8 \times 6}{3} = \frac{3 \times Q}{3}$ ✓

$AQ = 16cm$ ✓

M1

A1

(b) PT (1mk)

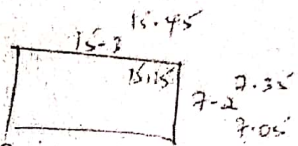
$9 \times 28 = PT^2$

$PT = \sqrt{9 \times 28}$

$= \sqrt{252}$

$= 15.8745cm$ ✓

B1



9. A right angled triangle has a base of 15.3 cm and height 7.2 cm, each measured to the nearest 3 mm. Determine the percentage error in finding the area of the triangle, giving your answer to 2 decimal places. (3mks)

Actual Area $= 15.3 \times 7.2 = 110.16$

Max. Area $= 15.45 \times 7.35 = 113.5575$

Min. Area $= 15.15 \times 7.05 = 106.8075$

$|E| = \frac{106.8075 - 113.5575}{2}$ ✓ M1

$= \frac{6.75}{2} = 3.375$

$\% E = \frac{|E|}{A.A} \times 100$

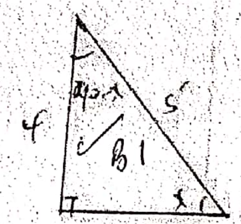
$= \frac{3.375}{110.16} \times 100$ ✓ M1

$= 3.063725490196078$

$= 3.06$ ✓ A1

10. Given that sin x=0.8, without using a mathematical table and calculator find tan(90-x) (3mks)

$\sin x = \frac{8}{10} = \frac{4}{5}$



$\sqrt{25-16} = 3$ ✓ B1

$\therefore \tan(90-x) = \frac{o}{a}$

$= \frac{3}{4}$ ✓ B1

11. The point B(3,2) maps onto B^1(7,1) under a translation T_1. Find T_1 (2mks)

$T_1 = T' - T$

$= \begin{pmatrix} 7 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ ✓ M1

$= \begin{pmatrix} 7-3 \\ 1-2 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ ✓ B1

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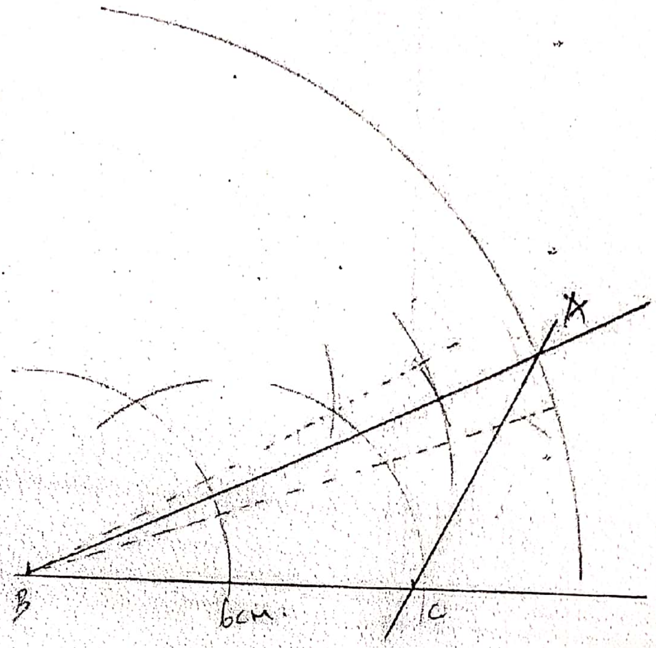
12. Using a ruler and a pair of compasses only, construct triangle ABC in which BC=6cm, AB= 8.8cm and angle ABC= 22.5°. (3mks)



✓ ∠22.5 B1

✓ BC & AB B1

✓ △ B1

13. Two grades of tea A and B, costing sh 100 and 150 per kg respectively are mixed in the ratio 3:5 by mass. The mixture is then sold at sh 160 per kg. Find the percentage profit on the cost price. (3mks)

$\frac{3}{8}(100) + \frac{5}{8}(150) \Rightarrow$ cost price

$37.5 + 93.75 = $ Sh. 131.25 B1

Profit = 160 − 131.25

= Sh 28.75

% Profit = $\frac{\text{Profit}}{\text{CP}} \times 100$

$= \frac{28.75}{131.25} \times 100$ M1

= 21.9047619

= 21.9048% A1

14. The first, the third and the ninth term of an increasing AP, makes, the first three terms of a G.P. If the first term of the AP is 3, find ~~the common ratio of the GP~~ difference of the AP and common ratio of GP. (4mks)

$a, a+2d, a+8d.$

$3, 3+2d, 3+8d$

$\frac{3+8d}{3+2d} = \frac{3+2d}{3}$ ✓M1

$9+24d = 9+12d+4d^2$

$0 = 4d^2 - 12d$

$0 = 4d(d-3)$

$4d = 0 \quad d-3=0$

$d=0 \quad d=3$ ✓A1

$r = \frac{3+2(3)}{3}$ ✓M1

$= \frac{3+6}{3}$

$= \frac{9}{3}$

$r = 3$ ✓A1

15. The matrix M=$\begin{pmatrix} 3 & -2 \\ -5 & y \end{pmatrix}$ maps a triangular object of area 7 square units onto one with area of 35 square units. Find the value of x. (4mks)

$|\det| = A.s.f$

$A.s.f = \frac{I_A}{O_A}$

$= \frac{35}{7}$ ✓M1

$= 5$ ✓A1

$5 = 3y - 10$ ✓M1

$15 = 3y$

$\therefore y = 5$ ✓A1

16. The equation of a circle is given by $x^2+4x+y^2-2y-4=0$. Determine the centre and radius of the circle (3mks)

$x^2+4x+\left(\frac{4}{2}\right)^2+y^2-2y+\left(\frac{-2}{2}\right)^2 = 4+4+1$

$(x+2)^2+(y-1)^2 = 3^2$ B1

$(x-a)^2+(y-b)^2 = r^2$

$\therefore (a,b) = (-2, 1)$ B1 & $r = 3$ unit B1

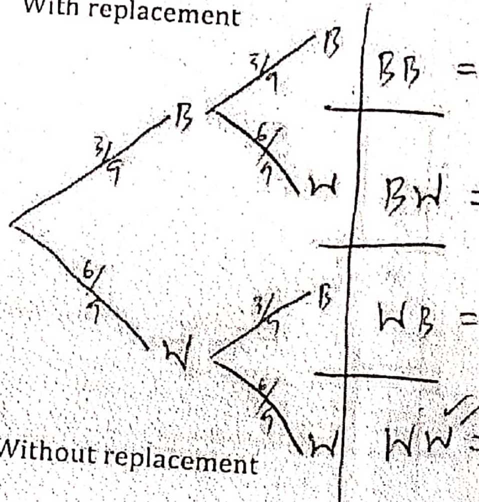
SECTION B (50 MARKS)

*(Answer any **five** questions in this section)*

17. A bag contains 3 black balls and 6 white balls. If two balls are drawn from the bag one at a time, find the:

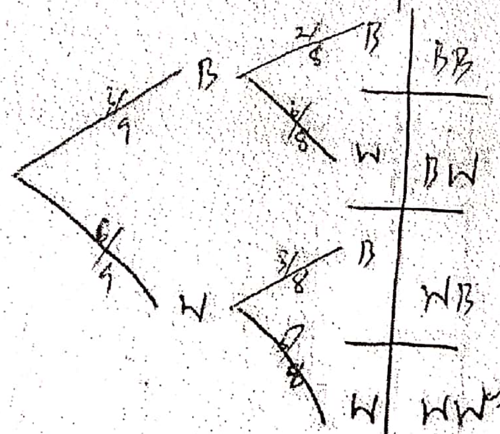
a) Probability of drawing two white balls:

i) With replacement (2mks)



$WW = \frac{6}{9} \times \frac{6}{9} = \frac{4}{9}$ M1 A1

ii) Without replacement (2mks)



$WW = \frac{6}{9} \times \frac{5}{8} = \frac{5}{12}$ M1 A1

b) Probability of drawing a black ball and white ball:

i) With replacement (3mks)

$= P(BW)$ or $P(WB)$

$= \left(\frac{3}{9} \times \frac{6}{9}\right) + \left(\frac{6}{9} \times \frac{3}{9}\right) = \frac{2}{9} + \frac{2}{9} = \frac{4}{9}$

B1 ✓ tree diag
M1 A1

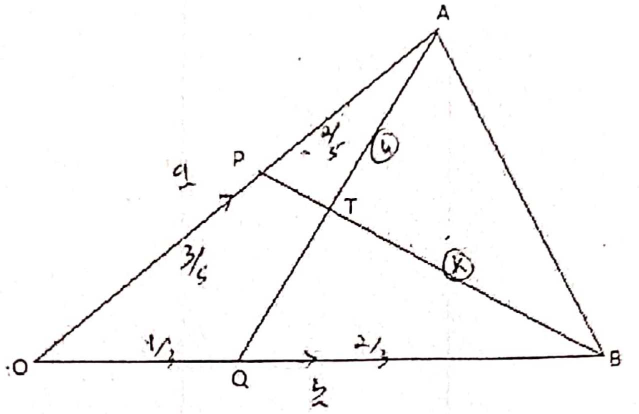
ii) Without replacement. (3mks)

$= \left(\frac{3}{9} \times \frac{6}{8}\right) + \left(\frac{6}{9} \times \frac{3}{8}\right) = \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$

B1 ✓ tree diag
M1 A1

18. In the triangle below P and Q are points on OA and OB respectively such that $OP:PA = 3:2$ and $OQ:QB = 1:2$. AQ and PQ intersect at T. Given that $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$.



$BT = -b + \frac{3}{5}a$

$= \frac{3}{5}a - b.$

$\frac{3-k}{5} = -\frac{k}{5}$

(a) Express AQ and PQ in terms of **a** and **b**. (2mks)

$\vec{AQ} = \vec{AO} + \vec{OQ}$

$= -a + \frac{1}{3}b$

$= \frac{1}{3}b - a$ ✓B1

$PQ = -\frac{3}{5}a + \frac{1}{3}b$

$= \frac{1}{3}b - \frac{3}{5}a$ ✓B1

(b) Taking $BT = kBP$ and $AT = hAQ$ where h and k are real numbers.

(i) Find two expressions for OT in terms of **a** and **b**. (2mks)

$OT = OA + AT$

$= a + h(\frac{1}{3}b - a)$

$= a + \frac{1}{3}hb - ha$

$= (1-h)a + \frac{1}{3}hb$ ✓B1

$OT = OB + BT.$

$= b + k(\frac{3}{5}a - b).$

$= b + \frac{3}{5}ka - kb$

$= (1-k)b + \frac{3}{5}ka$ ✓B1

(ii) Use the expression in b(i) above to find the values of h and k. (4mks)

a: $(1-h) = \frac{3}{5}k$

b: $\frac{1}{3}h = 1-k$

$h = 3-3k$

B1 for any eqn in a or b

$1-(3-3k) = \frac{3}{5}k$ M1

$1-3+3k = \frac{3}{5}k$

$-2 = -3k + \frac{3}{5}k$

$\frac{5}{12} \times 2 = -\frac{12}{5}k \times -\frac{5}{12}$

$k = \frac{5}{6}$ ✓A1

$h = 3-3k$

$= 3 - 3(\frac{5}{6})$

$= 3 - \frac{5}{2}$

$h = \frac{1}{2}$ ✓B1

(c) Give the ratio $BT:TP$. (2mks)

$BT:TP$

$k : 1-k$

$\frac{5}{6} : 1-\frac{5}{6}$

$6 \times \frac{5}{6} : \frac{1}{6} \times 6$

✓M1

$\therefore BT:TP = 5:1$ ✓A1

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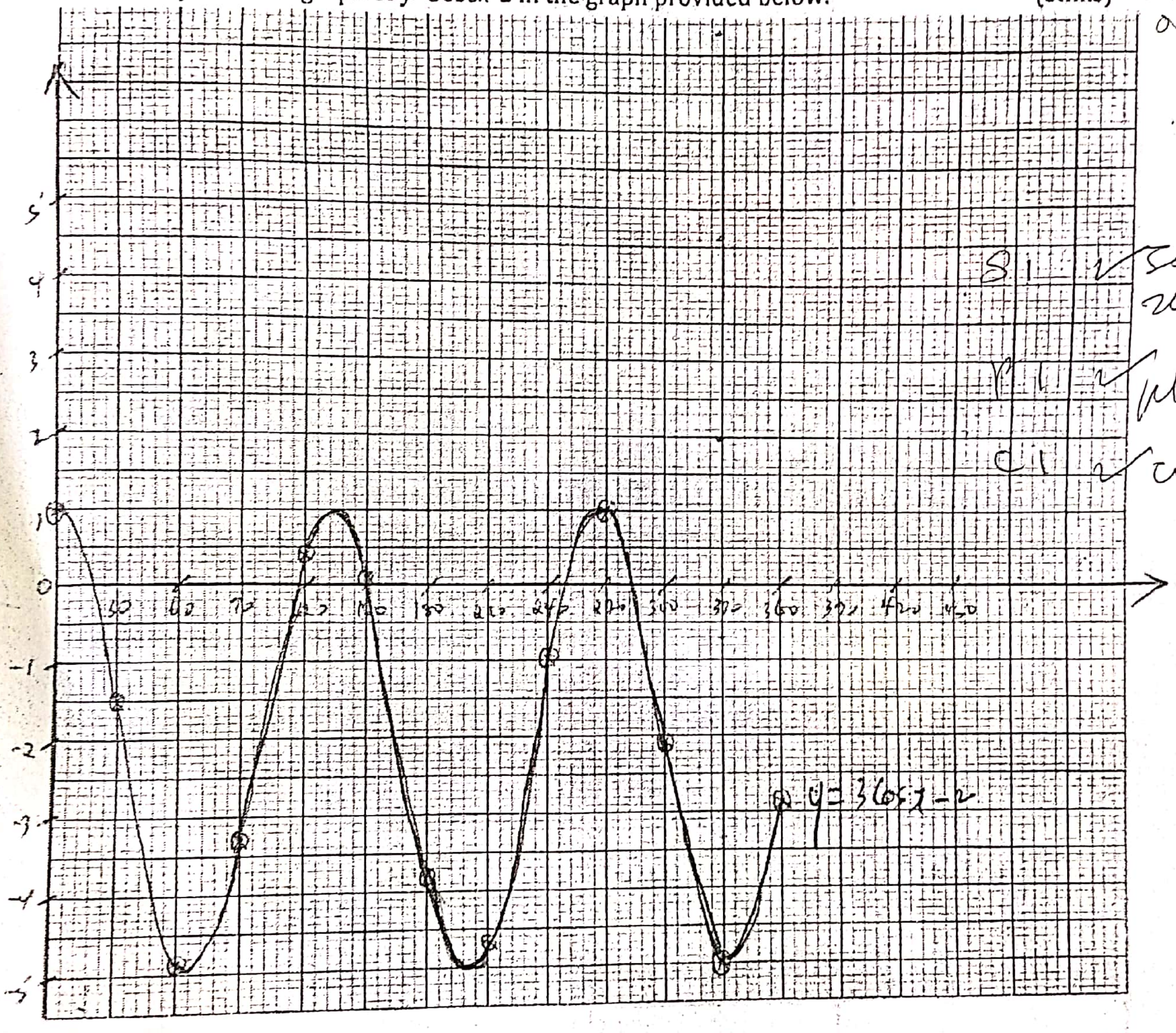
19. Complete the table below for the functions y=3cosx-2 for $0^0 \le x \le 360^0$ (2mks)

x	0	30	60	90	120	150	180	210	240	270	300	330	360
y=3cosx-2	1.0	-1.5	-4.9	-3.3	0.4	0.1	-3.8	-4.7	-1.0	1.0	-2.1	-5.0	-2.9

B2 all
B1 any 7 or more

a) Plot the graph of y=3cosx-2 in the graph provided below. (3mks)



scale used
plotting
curve

b) From the graph

i. Find the amplitude of the wave. $\frac{1 - -5}{2} = 3$ units (2mks)

M1 A1

ii. The period of the wave. (1mk)

270° B1

iii. Find the solution to 3cosx=2 (2mks)

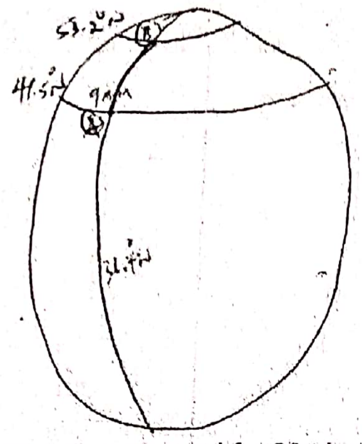
$3\cos x - 2 = 0$

18°, 117°, 150°, 249, 282

B2 all
10
B1 atleast 3

20. A plane leaves an airport A (41.5°N, 36.4°W) at 9:00am and flies due north to airport B on latitude 53.2°N. Taking π as $\frac{22}{7}$ and the radius of the earth as 6370Km,

a) Calculate the distance covered by the plane in km (4mks)



✓ angle diff B1

$$\text{Distance} = \frac{11.7}{360} \times 2 \times \frac{22}{7} \times 6370 \quad ✓ \text{M1}$$

$$= 1{,}301.3 \text{ KM}. \quad ✓ \text{A1}$$

b) The plane stopped for 30minutes to refuel at B and flew due east to C, 2500km from B.

Calculate:

i) position of C (3mks)

$$\frac{\theta}{360} \times 2 \times \frac{22}{7} \times 6370 \cos 53.2 = 2500 \quad ✓ \text{M1}$$

$$-108.\cancel{5461207841558} = 2500$$

$$\theta = \cancel{22.77°}$$

$$\frac{66.6247\theta}{66.6247} = \frac{2500}{66.6247}$$

$$\theta = 37.52 \quad ✓ \text{A1}$$

$\cancel{36.4 - 22.77 = 13.63°}$

$\cancel{C(53.2°N, 13.63°W)}$

$37.52 - 36.4 = 1.12°$ ✓ A1

$\therefore C(53.2°N, 1.12°E)$. ✓ B1

ii) The time the plane lands at C if its speed is 500km/h (3mks)

$$t = \frac{D}{S}$$

$$= \frac{1{,}301.3}{500} + \frac{2500}{500}$$

$= (2\text{hrs } 36\text{Min}) + 5\text{hrs}$

$= 7\text{hr } 36\text{ Min}$ ✓ B1

7hr 36 ~~Min~~
\+ ~~30 Min~~
~~8hrs 06 Mins~~

$37.52° \times 4 = 150.08$ Min
= 2hrs 30 Min

0900 hrs
0236
1136 hrs

0900 hrs
0736
1636 hrs ✓ M1

1636
230 +
1906 hrs
11

7.06 P.M. ✓ B1

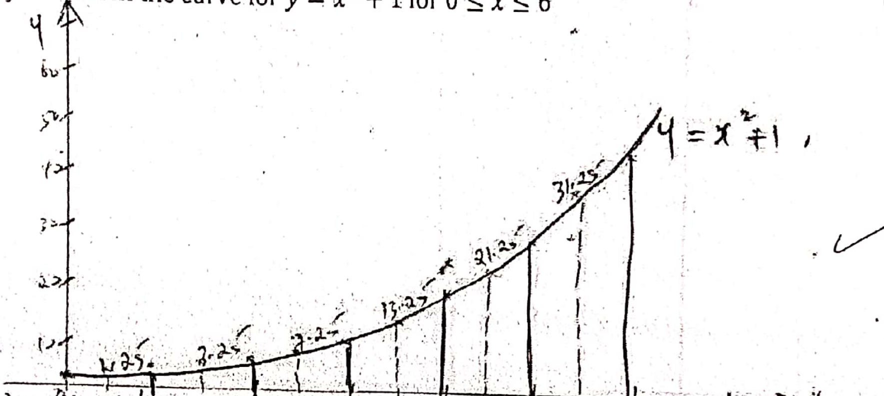
21. The curve given by the equation $y = x^2 + 1$ is defined by the values in the table below.

(a) Complete the table by filling in the missing values. (2mks)

X	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0
Y	1.0	1.25	2.0	3.25	5.0	7.25	10.0	13.25	17.0	21.25	26.0	31.25	37.0

B2 all ✓
B1 others

(b) Sketch the curve for $y = x^2 + 1$ for $0 \le x \le 6$ (2mks)



✓ B2

(c) Use the mid-ordinate rule with 5 ordinates to estimate the area of the region bounded by the curve $y = x^2 + 1$, the x-axis, the lines x = 0 and x = 6. (2mks)

$A = 1(1.25 + 3.25 + 7.25 + 13.25 + 21.25 + 31.25)$ M1

$= 1(77.5)$

$= 77.5$ sqr units A1

(d) Use method of integration to find the exact value of the area of the region in (c) above. (2mks)

$A = \int_0^6 (x^2 + 1)\,dx$

$= \left[\frac{x^3}{3} + x + c\right]_0^6$ ✓

$= \left(\frac{216}{3} + 6 + c\right) - (0 + c)$ M1 integrating with limits

$= 72 + 6 + c - c$

$= 78$ sqr units ✓ A1

(e) Calculate the percentage error involved in using the mid-ordinate rule to find the area. (2mks)

$|E| = \text{Appr.}A - \text{Actual}.A$

$= 77.5 - 78$

$= 0.5$ sqr units

$\% E = \frac{|E|}{A.A} \times 100$

$= \frac{0.5}{78} \times 100$ ✓ M1

12

$= 0.6410256410256\ldots$

$= 0.6410\ \%$ ✓ A1

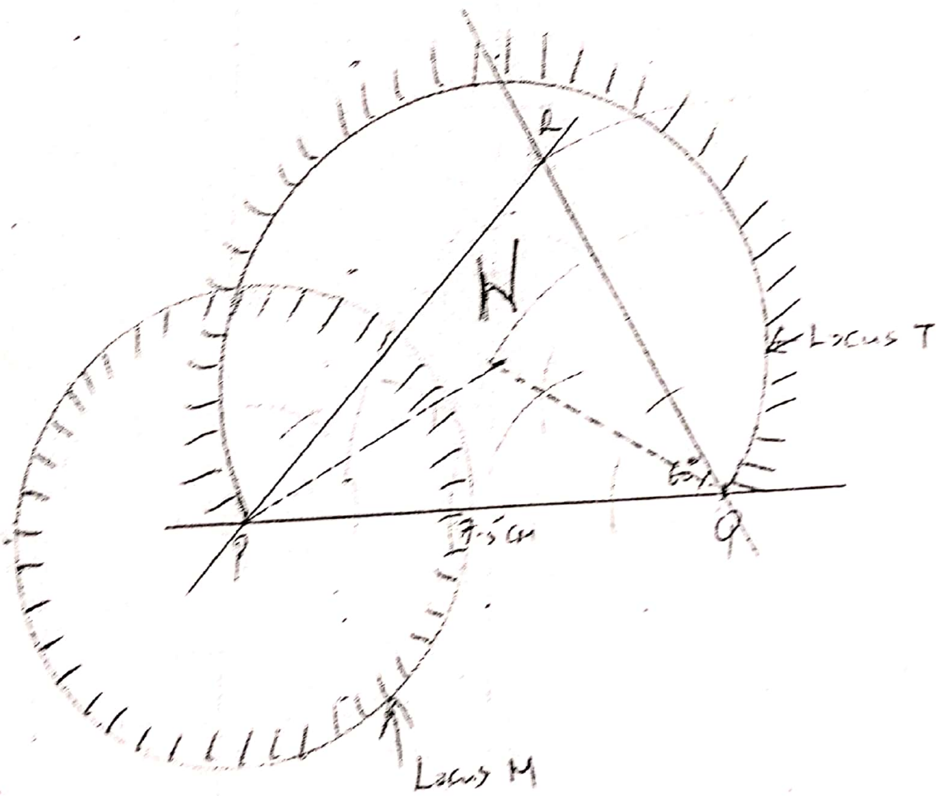
22. (a) Using a ruler and pair of compasses only construct triangle PQR in which $PQ = 7.5cm$ $QR = 6.0cm$ and angle $PQR = 60^0$. Measure PR (3mks)

(b) On same side of PQ as R

(i) Determine the locus of a point T such that angle $PTQ = 60^0$ (3mks)

(ii) Construct the locus of M, such that $PM \geq 3.5cm$. (2mks)

(iii) Identify the region W such that $PM \geq 3$ and angle $PTQ \geq 60^0$ by shading the unwanted part. (2mks)



PR = 6.9cm

∠60° ✓ B1

PQ, QR ✓ B1

PR = 6.9±0.1cm B1

Centre ✓ located B1

arc ✓ drawn B1

✓ Locus of T shaded B1

P identified ✓ B1

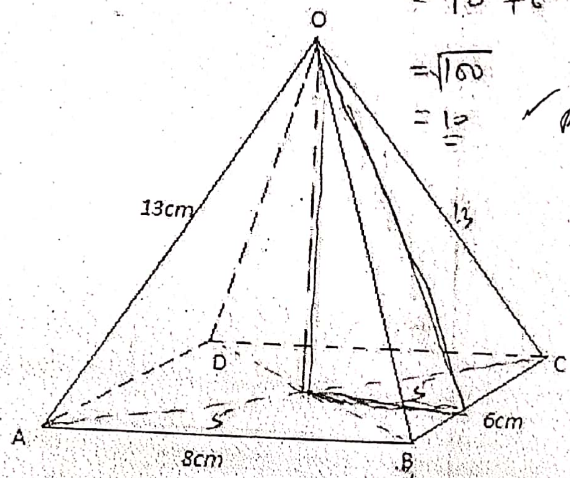
✓ locus of PM ≥ 3.5 B1

✓ identify W B2

13

23. OABCD is a right pyramid on a rectangular base with AB = 8 cm, BC = 6 cm, OA = OB = OC = OD = 13 cm. Calculate;

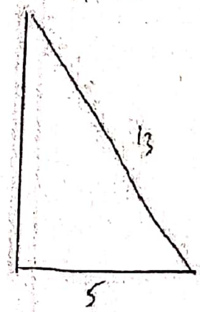
(a) the height of the pyramid. (3mks)



$= \sqrt{8^2+6^2}$

$= \sqrt{100}$

$= 10$ B1



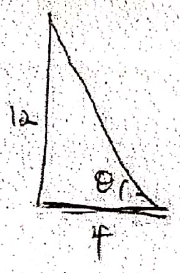
$= \sqrt{13^2 - 5^2}$ M1

$= \sqrt{169 - 25}$ A1

$= \sqrt{144}$

$= 12$ cm

(b) the inclination of OBC to the horizontal. (2mks)



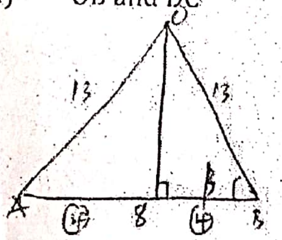
$\tan\theta = \frac{12}{4}$ M1

$\theta = \tan^{-1} 3$ A1

$= \cancel{1.250} \; 71.57°$

(c) the angle between;

(i) OB and DC (3mks)



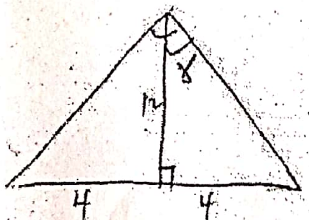
$\cos\beta = \frac{4}{13}$ B1

$\beta = \cos^{-1}\frac{4}{13}$ M1

$= \cancel{1.26°} \; 72.08°$ A1

$= \cancel{0.250}$

(ii) the planes OBC and OAD (2mks)



$\tan\gamma = \frac{4}{12}$ M1

$\gamma = \tan^{-1}\frac{4}{12}$ A1

$= \cancel{0.32°} = 18.43°$

$\therefore 2\gamma = \cancel{0.64°} = 36.87°$

24. The games master wishes to hire two matatus for a trip. The operators have a Toyota which carries 10 passengers and a Kombi which carries 20 passengers. Altogether 120 people have to travel. The operators have only 20litres of fuel and the Toyota consumes 4 litres on each round trip and the Kombi 1 litre on each round trip. If the Toyota makes x round trips and the kombi y round trips;

(a) write down four inequalities in x and y which must be satisfied . (2mks)

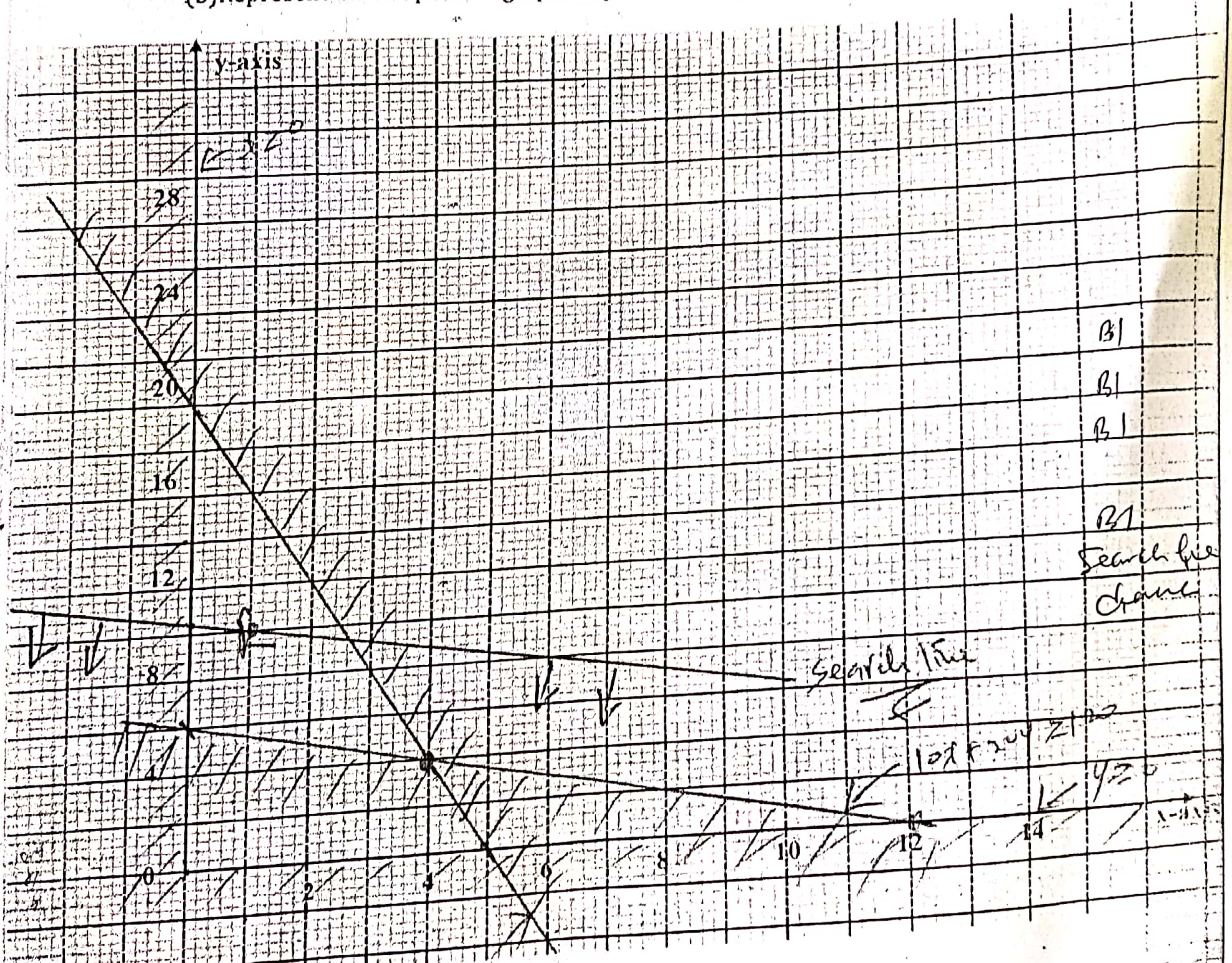
$10x + 20y \geq 120$ B1

$4x + y \leq 20$ B1

$x \geq 0$

$y \geq 0$

(b)Represent the inequalities graphically on the grid provided. (3mks)



$10x + 20y = 120$

$\frac{x}{12} + \frac{y}{6} = 1$

$4x + y = 20$

$\frac{x}{5} + \frac{y}{20} = 1$

15

(c) The operators charge shs.100 for each round trip in the Toyota and shs.300 for each round trip in the kombi;

(i) determine the number of trips made by each vehicle so as to make the total cost a minimum. (4mks)

$100x + 300y = K$ (1,10)

$100(1) + 300(10) = K$

$100 + 3000 = K$

$K = 3100$ ✓ B1

$\frac{100x}{3100} + \frac{300y}{3100} = \frac{3100}{3100}$

$\frac{x}{31} + \frac{y}{10.3} = 1$ ✓ B1

Search line

Minimum cost (4,4) ✓ B1

> 4 Toyota trips

> 4 Kombi trips

(ii) find the minimum cost. (1mk)

$100x + 300y \Rightarrow$ Cost

$100(4) + 300(4) = 400 + 1200$ B1

$=$ sh. 1600