Name: MARKING SCHEME .... Class: .... Adm.No................

School: .... Date: ....

Sign:..

121/1
MATHEMATICS
PAPER 1
DECEMBER 2021
TIME: 2 ½ HOURS

MOKASA II JOINT EXAMINATION - 2021

Kenya Certificate to Secondary Education

MATHEMATICS (PAPER 1)

TIME: 2 ½ HOURS

Instructions

- *Write your name, class, admission number, school, date and signature in spaces provided above.*
- *The paper contains* ***two*** *sections* ***A*** *and* ***B.***
- *Answer* ***all*** *questions in section* ***A*** *and* ***any five*** *questions from section* ***B*** *in the spaces provided below each question.*
- *Show all the steps in your calculations giving your answers at each stage in the spaces below each question.*
- *Non-programmable silent electronic calculator and mathematical tables may be used except where stated otherwise.*

For Examiner's Use Only

SECTION A

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

SECTION B

17	18	19	20	21	22	23	24	TOTAL

PERCENTAGE SCORE	

MARKING SCHEME PAPER 1
MOKASA 2

SECTION A

*Answer **all** questions in this section in the spaces provided*

1. The sum of two numbers exceeds their product by one. Their difference is equal to their product less five. Find the two numbers. **(3 marks)**

(i) $x+y = xy-1$
(ii) $x-y = xy-5$ — M1 for Both

M1 for elimination

$2y = 4$
$y = 2$ — A1 for Both.
$x = 3$

3

2. Musa has twenty shillings more than Aisha. After he spends a quarter of his money and Aisha ⅕ of hers, they find that Aisha has 10 shillings more than Musa. How much money did both have? **(4 marks)**

Let Aisha have x
Musa $x+20$ — B1

$\frac{4}{5}x - 10 = \frac{3}{4}(x+20)$ — M1

$\frac{4}{5}x - \frac{3}{4}x = 15+10$

$\frac{x}{20} = 25$ — A1

$x = 500$ — A1

3

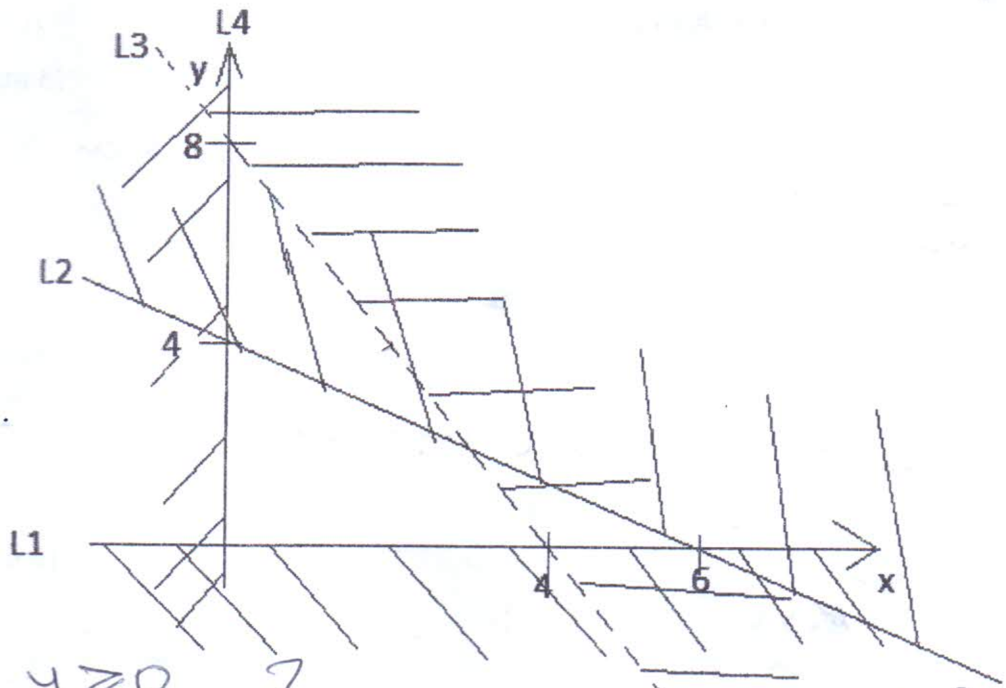
∴ Aisha have Ksh. 500, Musa Ksh 520
Both Ksh 1020 Accept.

3. The number 2942m08 is divisible by 11. Find the least value of m and the square of m. **(3 marks)**

$9+2+0 = 11$
$2+4+m+8 = 14+m$
$(14+m) - 11 = 0$ — M1 or equivalent
$14+m = 22$
$m = 22-14$
$m = 8$ — A1
$m^2 = 64$ — B1

3

4. Give the inequalities L2, L2, L3 and L4 which define the region R in the inequalities shown below. **(3 marks)**



$L1 \Rightarrow y \geq 0$
$L4 \Rightarrow x \geq 0$ — B1 for Both.

$L2 \Rightarrow 12\left(\frac{x}{6}\right) + 12\left(\frac{y}{4}\right) = 1 \times 12$

$2x + 3y \leq 12$ — B1

$L3 \Rightarrow 8\left(\frac{x}{4}\right) + 8\left(\frac{y}{8}\right) = 1 \times 8$

$2x + y < 8$ — B1

3

5. Given $P = \begin{pmatrix} 2 & 0 \\ -2 & 1 \end{pmatrix}$, $Q = \begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix}$ and $R = \begin{pmatrix} 2 & -1 \\ -4 & 5 \end{pmatrix}$, find PQ + R. **(3 marks)**

$\begin{pmatrix} 2 & 0 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 8 & -2 \\ -5 & 4 \end{pmatrix}$ — M1

$\begin{pmatrix} 8 & -2 \\ -5 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ -4 & 5 \end{pmatrix} = \begin{pmatrix} 10 & -3 \\ -9 & 9 \end{pmatrix}$ M1 A1

3

6. A Kenyan businessman bought goods from Japan worth 2,900,000 Japanese Yen. On arrival in Kenya, custom duty 10% was charged on the value of the goods. If the exchange rates were as follows:
1 US dollar = 118 Japanese Yen
1 US dollar = 78 Kenyan Shillings.

Calculate the duty paid in Kenya Shillings. **(3 marks)**

$\frac{10}{100} \times 2900,000 = 290,000$ J. yen M1

$\frac{290,000}{118} \times 78$ M1

$= $ Ksh $191,694.92$ A1

3

7. Solve the equation; **(3 marks)**
$4^x + 2^{2x+1} = 36$

$(2^2)^x + 2^{2x} \times 2^1 = 36$

$2^{2x} + 2^{2x} \times 2^1 = 36$

$2^{2x}(1+2) = \frac{36}{3}$

$2^{2x} = 12$

$\log 2^{2x} = \log 12$ M1

$2x \log 2 = \log 12$ M1

$2x = \frac{\log 12}{\log 2}$

$2x = 3.585$

$x = 1.7925$ A1

3

8. Line AB is perpendicular to a line whose equation is $y - 2x + 7 = 0$ and passes through point $(-4, 5)$. Determine the equation of AB in the form y = mx + c. **(3 marks)**

$y = 2x - 7$

Gradient of AB $= -\frac{1}{2}$ B1

$\frac{y-5}{x+4} = \frac{-1}{2}$ M1

$2(y-5) = -1(x+4)$

$2y - 10 = -x - 4$

$2y = -x + 6.$ A1

$y = -\frac{1}{2}x + 3$

3

3

9. Simplify the following expression. ~~(3 marks)~~ (2 Marks)

$$\frac{\cos^2\theta - 1}{\sin\theta}$$

$$\frac{\sin^2\theta}{\sin\theta}$$ —— M1

$= \sin\theta$ —— A1

2

10. Without using a calculator evaluate using squares, square roots and reciprocal tables the following:- (3 marks)

$$\frac{2}{30.16^2} + \frac{10}{\sqrt{588.3}}$$

$\frac{2}{(3.016\times10)^2} + \frac{10}{\sqrt{5.883\times10^2}}$ M1 Both Sq. & Root/1

$\frac{2}{9.097\times10^2} + \frac{10}{2.425\times10}$ —— M1 for Recip ✓

$(2\times0.001099)+(10\times0.0412)$

$0.002198+0.412$

0.414198 —— A1

3

11. Two of the exterior angles of a polygon are 63^0 each. The remaining exterior angles are each 26^0. Determine the number of sides of the polygon. (3 marks)

Let the sides be n

$(63\times2)+26(n-2)=360$ M1 or equiv.

$126+26n-52=360$

$26n+74=360$

$26n=286$ ← M1

$n=11$ — A1

3

12. A number when divided by 10, 15 and 18, the reminders are 7, 12 and 15 respectively. Find the lowest number. **(3 marks)**

2	10	15	18
3	5	15	9
3	5	5	3
3	5	5	1
5	1	1	1

$2 \times 3^3 \times 5 = 270$ M1

$270 - 3 = \underline{267}$ M1A1

3

13. The figure below shows part of a circle. Complete the circle and determine the radius and the centre of the circle. **(3 marks)**

Radius = 3~~4~~0·1cm

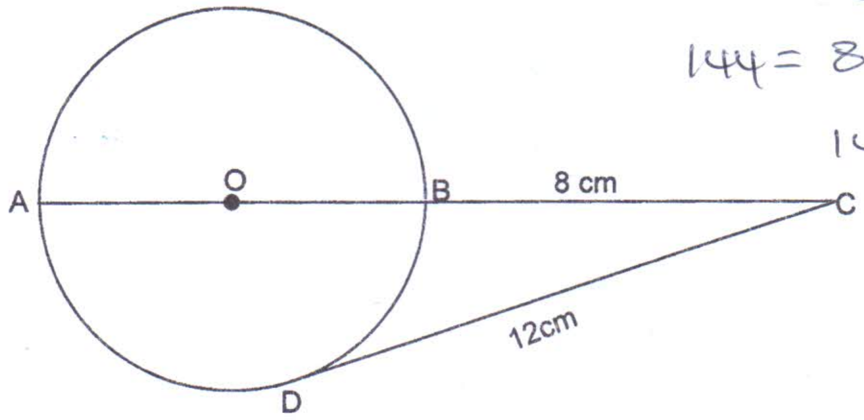
B1 ✓ Const 2 chords and bisecting

B1 Complete Circle

B1 Radius

3

14. In the figure below, DC is a tangent to the circle centre O at D. AOBC is a straight line meeting DC at C. DC = 12 and BC = 8. Find the radius of the circle. **(3 marks)**



$12^2 = (AB+8)8$ — M1

$144 = 8AB + 64$

$144 - 64 = 8AB$

$AB = \frac{80}{8}$ — M1

$AB = 10$

Radius = 5cm. — A1

3

15. x varies directly as the cube of y and inversely as the square root of z. When x=24, y=2 and z=16. Find z in terms of x and y. ~~(3 marks)~~ (4 Marks)

$x \propto \frac{y^3}{\sqrt{z}}$

$x = \frac{ky^3}{\sqrt{z}}$

$24 = \frac{k(2)^3}{\sqrt{16}}$

$24 = \frac{8k}{4}$

$k = 12$

$x^2 = \left(\frac{12y^3}{\sqrt{z}}\right)^2$ M1

$z = 144y^6$

16. Evaluate; $\int_{-1}^{2}(-x^3 + 5x - 2)\,dx$ **(4 marks)**

$\left[-\frac{x^4}{4} + \frac{5x^2}{2} - 2x\right]_{-1}^{2}$ M1

$\left[-\frac{(2)^4}{4} + \frac{5(2)^2}{2} - 2(2)\right] - \left[-\frac{(-1)^4}{4} + \frac{5(-1)^2}{2} - 2(-1)\right]$ M1

$\left[-\frac{16}{4} + \frac{20}{2} - 4\right] - \left[-\frac{1}{4} + \frac{5}{2} + 2\right]$ — M1

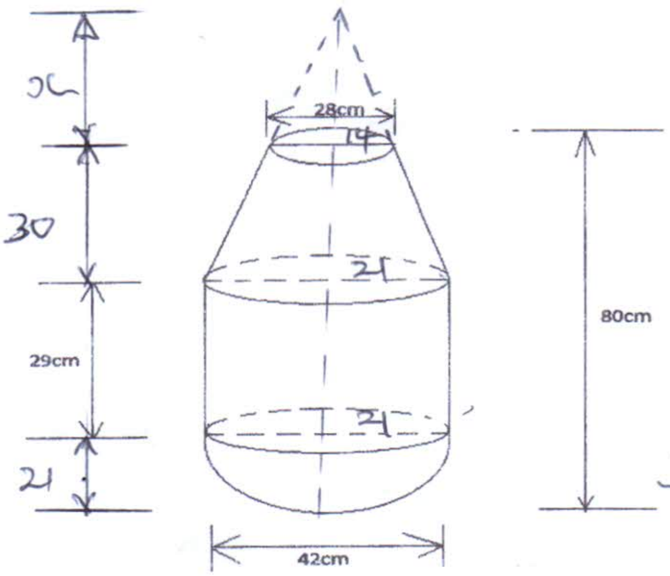
$[2] - \left[\frac{17}{4}\right]$

$-\frac{9}{4} = \underline{\underline{-2\frac{1}{4}}}$ — A1

SECTION B

*Answer any **five** questions in this section*

17. The figure below is a model representing an open storage container. The model whose total height is 80 cm is made up of a frustum top, a hemispherical bottom and the middle part is cylindrical. The diameter of the top of the frustum is 28 cm, the base of the frustum diameter of the cylindrical and hemispherical part is 42 cm. The height of the cylindrical part is 29 cm. (take $\pi = \frac{22}{7}$)



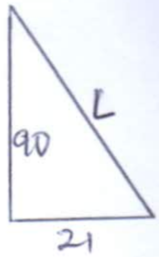
LSf = 21 : 14
3 : 2

$\frac{30+x}{x} = \frac{3}{2}$

$2(30+x) = 3x$

$60+2x = 3x$

$60 = x$ B1



SA of Cone $L = \sqrt{(9^2+21^2)}$ 92.42 M1

$\frac{22}{7} \times 21 \times 92.42$

$= 6099.72$

(a) Calculate the surface area of the ....Model. ASF := 9:4 (~~5~~ marks) 8

SA of Smaller Cone $= \frac{6099.72 \times 4}{9} = 2710.99$ — M1

SA of Frustum $= 6099.72 - 2710.99 = 3388.73$

SA of Cylinder $= \frac{22}{7} \times 2 \times 21 \times 29 = 3828$ + M1

SA of Hemisfere $= 2 \times \frac{22}{7} \times 21^2 = 2772$

Total SA of Model $= 9983.73$ ~~cm²~~ A1

(b) The actual height of the container is 8 metres. Calculate the capacity of the container to the nearest litre. (5 marks)

Volume of Cone $\frac{1}{3} \times \frac{22}{7} \times 21^2 \times 90 = 41580$ cm³

" of Smaller, $\frac{1}{3} \times \frac{22}{7} \times 14^2 \times 60 = 12320$

Volume of Frustum $= (41580 - 12320) = 29260$ M1

Volume of Cylinder, $\frac{22}{7} \times 21^2 \times 29 = 40194$ + M1

Volume of Hemisphere $= \frac{1}{2} \times \frac{4}{3} \times \frac{22}{7} \times 21^3 = 19404$ M1

Total Vol. $= 88858$ cm³.

LSf 8 : 80 = 1 : 10

VSf = 1 : 1000

10

Volume of Container $\frac{88858}{1000} = 88.858$ m³ M1

$= 88.858 \times 1000$

$= 88858$ Litres — A1

18. The table below shows marks scored by 40 students in a Mathematics test.

Marks	30-39	40-49	50-59	60-69	70-79
No. of Students	2	10	13	8	7

(a) Using an assumed mean of 54.5, calculate the mean mark. **(5 marks)**

Marks	f	x	d=x−A	t=d/10	ft	ft²
30-39	2	34.5	−20	−2	−4	8
40-49	10	44.5	−10	−1	−10	10
50-59	13	54.5	0	0	13	0
60-69	8	64.5	10	1	8	8
70-79	7	74.5	20	2	14	28
	$\Sigma f=40$				$\Sigma ft=21$	$\Sigma ft^2=62$

B1 for x

B1 for d

B1 for ft

$\bar{x} = \left(\frac{21}{40} \times 10\right) + 54.5$ — M1

$\bar{x} = 5.25 + 54.5$

$\bar{x} = 59.75$ — A1

(b) Calculate the variance. **(3 marks)**

B1 ft²

$S^2 = \left[\frac{62}{40} - \left(\frac{21}{40}\right)^2\right] \times 10 \times 10$ — M1

$= 127.4375$ — A1

(c) Calculate the standard deviation. **(2 marks)**

$S = \sqrt{127.4375}$ — M1

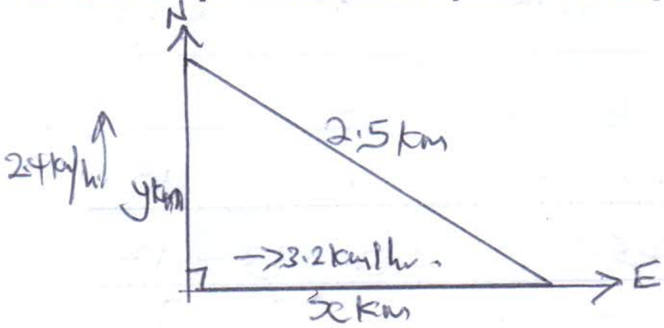
$= 11.2888$ — A1

$= 11.29$

10

19. Two policemen were together at a road junction. Each had a walkie talkie. The maximum distance at which one could communicate with the other was 2.5km. One of the policemen walked due East at 3.2km/h while the other walked due North at 2.4km/h. The policeman who headed east travelled for x km while the one who headed North travelled for y km before they were unable to communicate.

(a) Draw a sketch to represent the relative positions of the policemen. **(1 mark)**



B1

(b) (i) From the information above form two simultaneous equations in form of x and y. **(2 marks)**

(i) $y^2+x^2=2.5^2 \Longrightarrow y^2+x^2=6.25$ — B1

(ii) $\frac{y}{2.4}=\frac{x}{3.2} \Longrightarrow 3.2y=2.4x$ — B1

$y=\frac{2.4x}{3.2}$

(ii) Find the value of x and y. **(5 marks)** M1

$y^2+x^2=6.25$

$\left(\frac{2.4x}{3.2}\right)^2+x^2=6.25$

$\frac{5.76x^2}{10.24}+x^2=6.25$

$1.5625x^2=6.25$

$x^2=\frac{6.25}{1.5625}=4$

$x=\sqrt{4}=2$

$y^2+2^2=6.25$ M1 M1

$y^2=6.25-4$

$y=\sqrt{2.25}$

$y=1.5$ A1 A1

(iii) Calculate the time taken before the police were unable to communicate. **(2 marks)**

$\frac{2}{3.2}=0.625$ hrs. M1

$=37.5$ min. A1

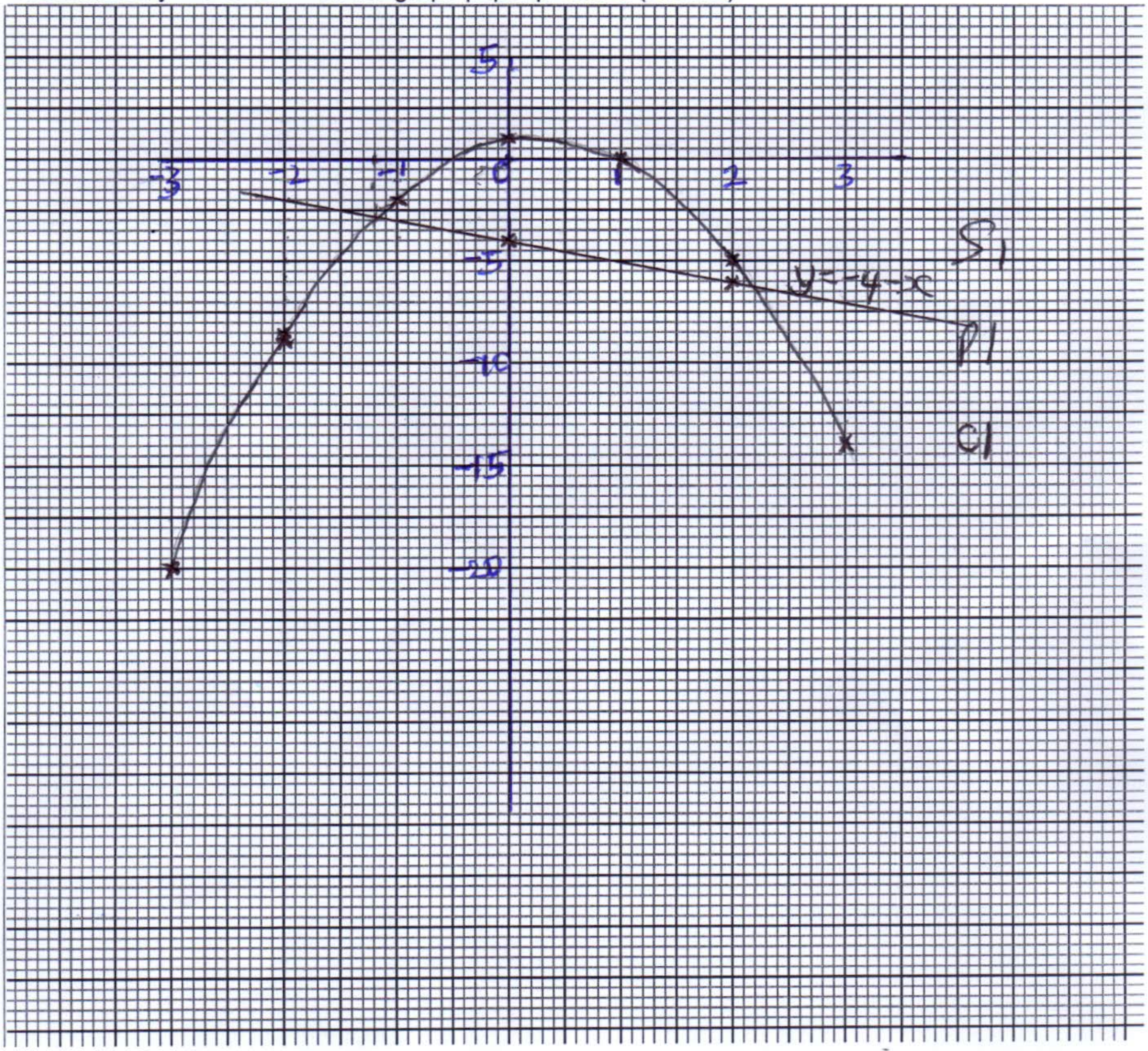
10

20. Complete the table of the functions $y = 1+x-2x^2$ (2marks)

x	-3	-2	-1	0	1	2	3
$-2x^2$	-18	-8	-2	0	-2	-8	-18
1	1	1	1	1	1	1	1
y	-20	-9	-2	1	0	-5	-14

B2 All ✓
B1 1 wrong.

b) Draw the graph of the function $y = 1+x-2x^2$ on the graph paper provided (3marks)



c) Use your graph to find the value for x in the equations i) $1 + x - 2x^2 = 0$ (1 mark)

$x = -0.5$ and $x = 1$ B1 for Both.

$1 + x - 2x^2 = y$

ii) $5 + 2x - 2x^2 = 0$ L1 ✓ line drawn (3marks)

$-4 - x = y$

$x = -1.2 \pm 0.1$ — B1

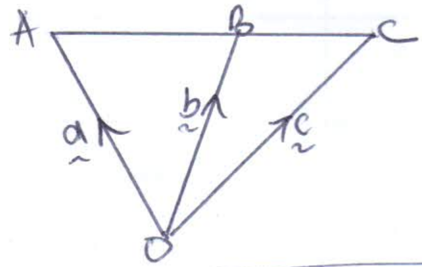
iii) State the maximum point of the function $y = 1+x-2x^2$ $x = 2.2 \pm 0.1$ — B1 (1mark)

Maximum Point (0,1) — B1

10

21. (a) The position vectors of points A and B are **a** and **b** respectively. C is another point with position vectors $\boldsymbol{c} = \frac{6}{4}\boldsymbol{b} - \frac{2}{4}\boldsymbol{a}$. Express in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ vectors.

(i) $\overrightarrow{AB}$ **(1 mark)**



$\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$ ———— B1

(ii) $\overrightarrow{CB}$ **(1 mark)**

$-\frac{6}{4}\underset{\sim}{b} + \frac{2}{4}\underset{\sim}{a} + \underset{\sim}{b}$

$\frac{1}{2}\underset{\sim}{a} - \frac{1}{2}\underset{\sim}{b}$ ———— B1

(iii) $\overrightarrow{CA}$ **(1 mark)**

$-\frac{6}{4}\underset{\sim}{b} + \frac{2}{4}\underset{\sim}{a} + \underset{\sim}{a}$

$\frac{3}{2}\underset{\sim}{a} - \frac{3}{2}\underset{\sim}{b}$ ———— B1

(iv) Show that A, B and C are collinear. **(3 marks)**

$\underset{\sim}{AB} = k\underset{\sim}{BC}$

$\underset{\sim}{b} - \underset{\sim}{a} = k(\frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{c})$

$k = 2$ ———— or equivalent — B1

$AB = 2BC$ ———— B1

$AB /\!/ BC$

B is in Common hence they are Collinear. B1

(v) Determine the ratio AB:BC **(1 mark)**

$AB : BC = 2 : 1$ —— B1

(b) Given that $\overrightarrow{OP} = 3i + 8j - 3k$ *and* $\overrightarrow{OP} = 3i + 2j + 2k$. Find $|PQ|$ correct to 2dp. **(3 marks)**

$PQ = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 8 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ 5 \end{pmatrix}$ —— M1 A1

$|PQ| = \sqrt{(0)^2 + (-6)^2 + (5)^2}$

$= \sqrt{61} = 7.81$ B1

10

22. (a) Given a curve $y = 10 + 3x - x^2$, use the trapezoidal rule with 5 trapezia to estimate the area bounded by the curve from $x = -1$ *to* $x = 4$. **(4 marks)**

x	-1	0	1	2	3	4
y	6	10	12	12	10	6

$y = 10 + 3x - x^2$ ——— B2

$A = \frac{1}{2} \times 1 \left[(6+6) + 2(10+12+12+10) \right]$ ——— M1

$= \frac{1}{2} \left[(12) + 88 \right]$

$= 50$ sq. unit ——— A1

(b) Find the actual area under the curve by integration method from $x = -1$ *to* $x = 4$. **(4 marks)**

$\int_{-1}^{4} (10 + 3x - x^2)\,dx = \left[10x + \frac{3x^2}{2} - \frac{x^3}{3} \right]_{-1}^{4}$ — M1

$\left[10(4) + \frac{3(4)^2}{2} - \frac{(4)^3}{3} \right] - \left[10(-1) + \frac{3(-1)^2}{2} - \frac{(-1)^3}{3} \right]$ ——— M1

$\left(42\frac{2}{3}\right) - \left(-8\frac{1}{6}\right)$ ——— M1

$\frac{305}{6} = 50\frac{5}{6}$ sq unit. A1

(do not Accept decimal.)

(c) Find the percentage error introduced by the approximation. **(2 marks)**

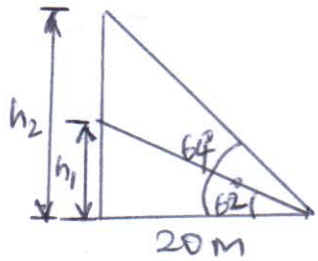
$AE = 50\frac{5}{6} - 50 = \frac{5}{6}$

% Error $\frac{\frac{5}{6}}{50\frac{5}{6}} \times 100 = \frac{100}{61} = 1\frac{39}{61}\%$ M1 A1

(Accept. 1.639%)

10

23. (a) A man standing 20m away from a building notices that the angles of elevation of the top and bottom of a flagpole mounted at the top of the building are 64^0 and 62^0 respectively. Calculate the height of the flagpole. **(4 marks)**



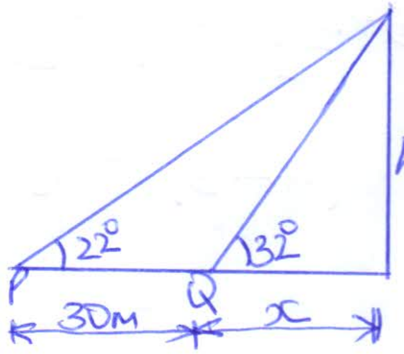
$\tan 62^0 = \frac{h_1}{20} \Rightarrow h_1 = 37.61m$ M1

$\tan 64^0 = \frac{h_2}{20} \Rightarrow h_2 = 41.01m$ M1

$h_2 - h_1 = 41.01 - 37.61$ M1

$= 3.4M$ A1

(b) The angles of elevation of the top of a tree from P and Q which are 30m apart are 22^0 and 32^0 respectively. Given that the two points are on the same side of the tree and on a straight line, determine the height of the tree. **(6 marks)**



(i) $x \tan 32^0 = h$ M1

(ii) $(x+30)\tan 22^0 = h$ M1

$x \tan 32 = (x+30)\tan 22^0$ M1

$0.6249x = 0.404x + 12.12$

$0.2209x = 12.12$ A1

$x = 54.87$

$h = 54.87 \tan 32$ M1

$= 34.29m$ A1

10

24. The displacement s metres after t seconds is given as $s = -t^3 + 3t^2 + 4$.

(a) Find its initial acceleration. **(3 marks)**

$V = \frac{ds}{dt} = -3t^2 + 6t \Rightarrow v = -3t^2 + 6t$ M1

$a = \frac{dv}{dt} = -6t + 6 \rightarrow a = -6t + 6$

Initial acc. when $t = 0$ M1

$a = -6(0) + 6$

$a = 6\,ms^{-2}$ A1

(b) . Calculate;

(i) The time when the particle was momentarily at rest. **(3 marks)**

at rest $v = 0$

$-3t^2 + 6t = 0$ M1

$3t(-t + 2) = 0$ M1

$t = 0$

$t = 2$ sec. A1

(ii) The acceleration in m/s^2 *when* $t = 3s$ **(2 marks)**

$a = -6(3) + 6$ M1

$= -18 + 6$

$= -12\,m/s^2$ A1

(c) Find the maximum velocity attained by the particle. **(2 marks)**

when $a = 0$

$-6t + 6 = 0$

$6 = 6t$

$t = 1$ sec.

$V = -3(1)^2 + 6(1)$ M1

$V = 3\,ms^{-1}$ A1

10