MANGU HIGH SCHOOL

121/2
MATHEMATICS
PAPER 2
MOCK EXAMS - 2022
TIME: 2½ HOURS

NAME: Marking Scheme

ADM.NO____________ CLASS: ____________

Kenya Certificate of Secondary Education
Mathematics
Paper 2
2½ Hours.

Instructions to Candidates

(i) Write your **Name, Adm. No., Class** and **Index No.** in the spaces provided above.
(ii) This paper contains **TWO** sections: section **I** and section **II.**
(iii) Answer **ALL** the questions in section **I.** In section **II** choose **FIVE** questions only.
(iv) Show all the steps in your calculations, giving your answers at each stage in the spaces below each question.
(v) Marks may be given for correct working even if the answer is wrong.
(vi) Negligent and slovenly work will be penalized.
(vii) Non programmable silent electronic calculators and KNEC mathematical tables may be used, except where stated otherwise.

For Examiner's Use only

Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

Section II

17	18	19	20	21	22	23	24	Total

GRAND TOTAL =

This paper consists of ***16 printed pages.*** Candidates should check the question paper to ensure that all the pages are printed as indicated and no questions are missing.

Turn Over

SECTION I: 50 MARKS

Answer all questions in this section

1. Solve the following simultaneous equations. (4mks)

$2x - y = 3$

$x^2 - xy = -4$

$2x - 3 = y$

$x^2 - x(2x-3) = -4$

$x^2 - 2x^2 + 3x = -4$

$-x^2 + 3x + 4 = 0$

$x^2 - 3x - 4 = 0$

$x^2 - 4x + x - 4 = 0$

$x(x-4)\ (x-4)$

$x(x-4) + 1(x-4)$

$(x+1) = 0 \quad x - 4 = 0$

$x = -1 \quad x = 4$

When $x = -1$, $y = 2(-1) - 3$
$= -2 - 3 = -5$

$x = 4$, $y = 2(4) - 3$
$= 8 - 3 = 5$

2. Calculate the standard deviation of the following set of numbers to two decimal places. (4mks)

10, 13, 14, 16, 17, 20

$\bar{x} = \frac{16+17+20+14+13+10}{6} = 15$ $\quad \frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right.$

X	$d = x - 15$	d^2
10	-5	25
13	-2	4
14	-1	1
16	1	1
17	2	4
20	5	25
		60

$V = \frac{\Sigma d^2}{\Sigma f}$

$= \frac{60}{6}$

$= 10.00$

$\sqrt{10} = 3.162$ $\quad 3.16$

3. The base and perpendicular height of a triangle measured to the nearest cm are 6cm and 4cm respectively. Find the absolute error in calculating the area of the triangle. (2mks)

$A = \frac{1}{2}bh.$

Max. Area $= \frac{1}{2} \times 6.5 \times 4.5 = 14.625$

min. Area $= \frac{1}{2} \times 5.5 \times 3.5 = 9.625$

Actual $= \frac{1}{2} \times \overset{2}{\cancel{4}} \times 6 = 12\,cm^2$

$AE = \frac{max - min}{2}$

$= \frac{14.625 - 9.625}{2}$

$= \frac{5}{2}$

$= 2.5$

4. Solve for x in the following equation (4mks)

$4\sin x\cos x - \sin x = 0$ for $-90^\circ \le x \le 90^\circ$

$\theta = 75.52$

$\sin x(4\cos x - 1) = 0$

$\sin x = 0$

$4\cos x = 1$

$\cos x = 1/4$

$\sin x = 0^\circ, 360^\circ$

$\cos x = 0.25$

$x = 75.52^\circ, 284.48$

$= -75.52^\circ$

$x = 0^\circ, 75.52^\circ, -75.52^\circ$

5. Simplify (3mks)

$$\frac{(\sqrt{x}+2)(\sqrt{x}-1)}{\sqrt{x}}$$

$\frac{(\sqrt{x}+2)(\sqrt{x}-1)}{\sqrt{x}}$

$\sqrt{x}(\sqrt{x}-1)+2(\sqrt{x}-1)$

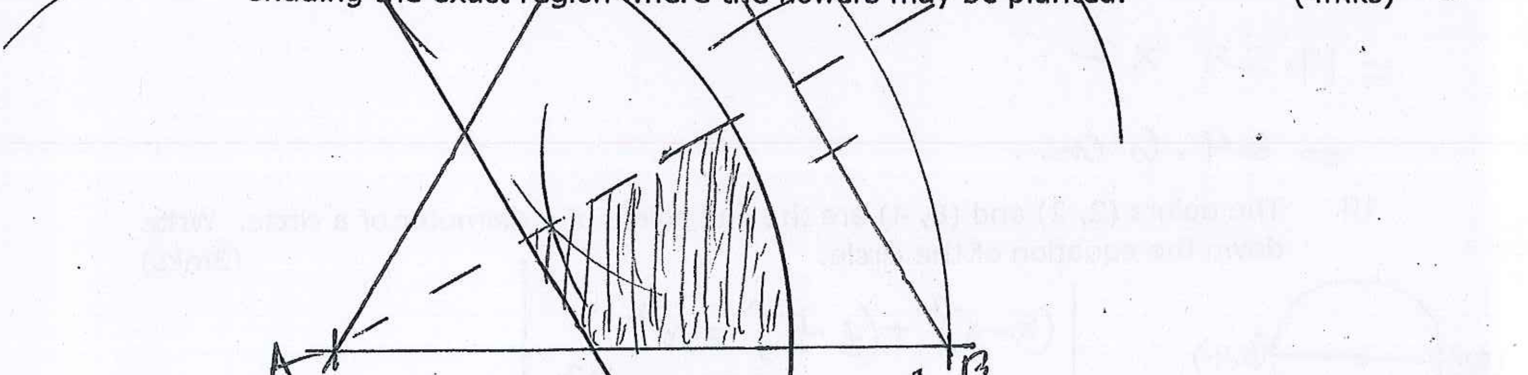
$\frac{x-\sqrt{x}+2\sqrt{x}-2}{\sqrt{x}}$

$\frac{(x+\sqrt{x}-2)\sqrt{x}}{\sqrt{x}\cdot\sqrt{x}} = \frac{x\sqrt{x}+x-2\sqrt{x}}{x}$

$= \frac{x\sqrt{x}}{x}+\frac{x}{x}-\frac{2\sqrt{x}}{x}$

$= \sqrt{x}+1-\frac{2\sqrt{x}}{x}$

6. A field is in the shape of an equilateral triangle ABC of sides 80m. The owner wishes to plant some flowers in the field. The flowers must be at most 60m from A and in the half of the field nearer to B than to C. If no flower is to be more than 40m from BC, using a scale of 1cm to represent 10m, show by shading the exact region where the flowers may be planted. (4mks)



7. Solve for x in the equation $\log_5 x + \frac{1}{\log_5 x} = 2$ (3mks)

Let $\log_5 x$ be v

$v + \frac{1}{v} = 2$

$v^2+1 = 2v$

$v^2-2v+1 = 0$

$v^2-v-v+1 = 0$

$(v-1) = 0$

$v = 1$

$\log_5 x = v$, $v = 1$

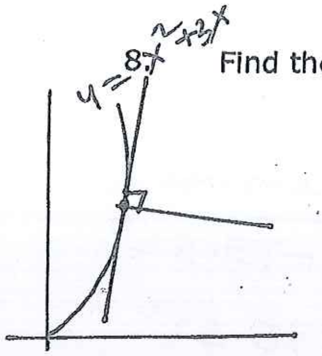
$\log_5 x = 1$

$5^1 = x$

$x = 5$

8. Find the equation of the normal to the curve $y=x^2 + 3x$ at the point where $x=1$ (3mks)

$y = x^2+3x$

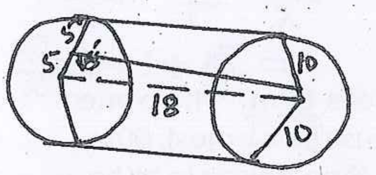


When $x=1$, $y = 1^2+3(1)$
$= 1+3$
$= 4$
$P(1,4)$

$\frac{dy}{dx} = 2x+3$,
when $x=1$,
$g = 2(1)+3 = 5$
g of normal $= -\frac{1}{5}$
$\frac{y-4}{x-1} = -\frac{1}{5}$
$5(y-4) = -1(x-1)$

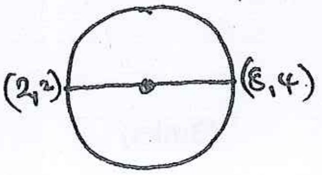
$5y-20 = -x+1$
$5y = -x+1+20$
$\frac{5y}{5} = \frac{-x}{5} + \frac{21}{5}$
$y = -\frac{1}{5}x + \frac{21}{5}$

9. A belt is passing round two pulleys of radii 5cm and 10cm respectively making two exterior common tangents and two arcs. If the distance between the centers of the two pulleys is 18cm, find the total length of the exterior common tangents to 1 decimal place. (2mks)



$\sqrt{18^2-5^2}$
$= 17.29 \times 2$
$= 34.6$ cm.

10. The points (2, 2) and (8, 4) are the end points of a diameter of a circle. Write down the equation of the circle. (3mks)



centre $= \left(\frac{2+8}{2}, \frac{2+4}{2}\right)$
$= (5,3)$

$r = \begin{pmatrix}8\\4\end{pmatrix} - \begin{pmatrix}5\\3\end{pmatrix} = \begin{pmatrix}3\\+1\end{pmatrix}$

$r = \sqrt{3^2+1^2}$

$(x-a)^2+(y-b)^2 = r^2$
$(x-5)^2+(y-3)^2 = (\sqrt{10})^2$
$x^2-10x+25+y^2-6y+9 = (\sqrt{10})^2$
$x^2+y^2-10x-6y = 10-25-9$
$x^2+y^2-10x-6y = -24$
or
$x^2+y^2-10x-6y+24=0$

$\frac{6}{4} - \frac{2}{4} =$

11. Use the matrix method to solve the following simultaneous equations. (4mks)

2x + 2y = 1

x + 3y = 0

$Mm^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\det = 6 - 2$
$= 4$

$\frac{1}{4}\begin{pmatrix} 3 & -2 \\ -1 & 2 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 3 & -2 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$x = 3/4$

$y = -1/4$

12. The variable y varies partly as x and partly as the inverse of x. If y=13 when x=3 and y=12 when x=2, find the value of y when x = 6. (4mks)

$y = kx + \frac{c}{x}$

$13 = 3k + \frac{c}{3}$

$39 = 9k + c$ —(i)

$12 = 2k + \frac{c}{2}$

$24 = 4k + c$ —(ii)

$39 = 9k + c$
$24 = 4k + c$

$\frac{15}{5} = \frac{5k}{5}$

$k = 3$

$24 = 4(3) + c$
$c = 24 - 12$
$= 12$

$y = 3x + \frac{12}{x}$,

when $x = 6$

$y = 3(6) + \frac{12}{6}$
$= 18 + 2$
$= 20$

13. The difference between the fourth and the seventh terms of an increasing arithmetic progression is 12. Calculate the sum of the first five terms of the progression if the first term is 9. (3mks)

$a + 6d - (a + 3d) = 12$

$a + 6d - a - 3d = 12$

$3d = 12$

$d = 4$

$a = 9$

Terms: 9, 13, 17, 21, 25

$S_5 = \underline{\underline{85}}$

14. Find the constant term in the expansion $(2x + \frac{1}{x})^6$ (2mks)

$(2x)^6, \quad (2x)^5\left(\frac{1}{x}\right)^1, \quad (2x)^4\left(\frac{1}{x}\right)^2, \quad (2x)^3\left(\frac{1}{x}\right)^3,$

$1, \quad 6, \quad 15, \quad 20$

$$20\,\frac{(8\cancel{x^3})}{\cancel{x^3}}$$

$$=160$$

15. Given that $4\mathbf{p} - 3\mathbf{q} = \binom{10}{5}$ and $\mathbf{p} + 2\mathbf{q} = \binom{-14}{15}$ find $/3\mathbf{p} + \mathbf{q}/$ (3mks)

Let $p = \begin{pmatrix} x \\ y \end{pmatrix}, \; q = \begin{pmatrix} a \\ b \end{pmatrix}$

$4\begin{pmatrix} x \\ y \end{pmatrix} - 3\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$

$4x - 3a = 10$

$4y - 3b = 5$

$\begin{pmatrix} x \\ y \end{pmatrix} + 2\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -14 \\ 15 \end{pmatrix}$

$x + 2a = -14$

$y + 2b = 15$

$4x - 3a = 10$

$(x + 2a = -14)4$

$4x + 8a = -56$

$-\;4x - 3a = 10$

$11a = -66$

$a = -6$

$x = -14 + 12$

$x = -2$

$4y + 3b = 5$

$y + 2b = 15, \; y = 15 - 2$

$4(15 - 2b) + 3b = 5$

$60 - 8b + 3b = 5$

$-5b = -55$

$b = 5$

$y = 15 - 2(5) = 5$

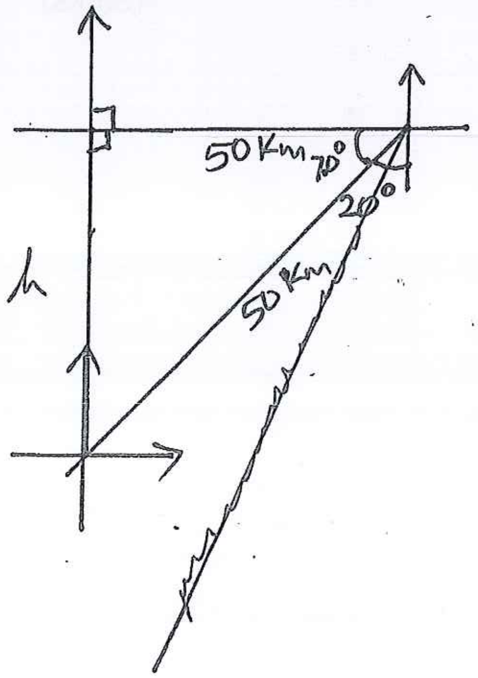
$p = \begin{pmatrix} -2 \\ 5 \end{pmatrix}, \; q = \begin{pmatrix} - \end{pmatrix}$

$3\begin{pmatrix} -2 \\ 5 \end{pmatrix} + \begin{pmatrix} -6 \\ 5 \end{pmatrix} = \begin{pmatrix} -12 \\ 20 \end{pmatrix}$

16. A boat leaves a port and sails 50km due east. It then changes direction and sails another 50km on a bearing of 200°. Calculate how far from the port the boat is. (3mks)

$|m| = \sqrt{-12^2 + 20^2}$

$= 23.32$



$\tan \cancel{50}70^\circ = \frac{h}{50}$

$h = 50 \tan 70$

$= 137.37$ Km

SECTION II: 50 MARKS

Choose **FIVE** questions only in this section

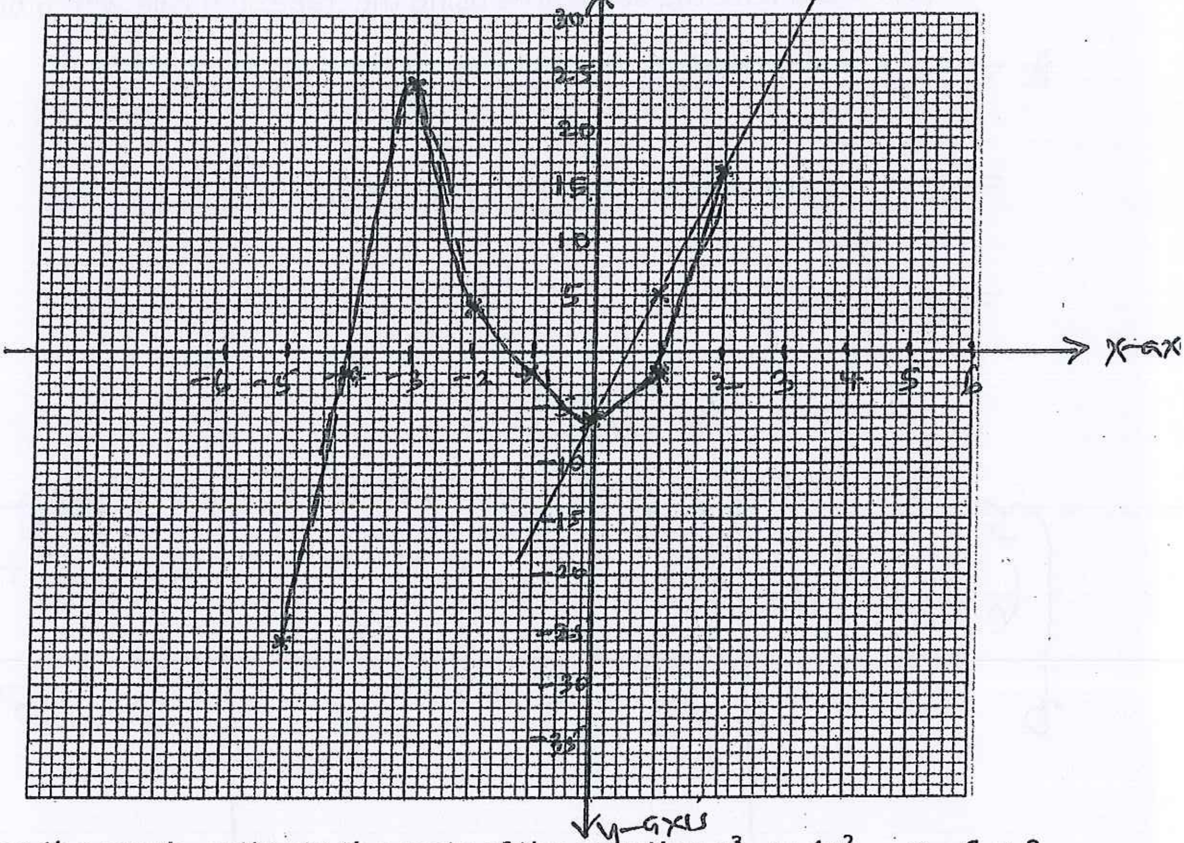
17. (a) Complete the following table for the equation $y = x^3 + 4x^2 - x - 6$ (2mks)

x	-5	-4	-3	-2	-1	0	1	2
x^3	-125	-64	-9	-8	-1	0	1	8
$4x^2$	100	64	36	16	4	0	4	16
-x	5	4	3	2	1	0	-1	-2
-6	-6	-6	-6	-6	-6	-6	-6	-6
y	-26	-2	24	4	-2	-6	-2	16

B1 B1

$y=11x-6$

(b) Draw the graph of $y = x^3 + 4x^2 - x - 6$ for $-5 \leq x \leq 2$ (3mks)



(c) Using the graph, estimate the roots of the equation $x^3 + 4x^2 - x - 6 = 0$ (2mks)

$x = -4, -1.4, 1.1$

$-x - -12x$
$-x + 12x$

(d) Using the same graph, estimate the solution to the equation

$x^3 + 4x^2 - 12x = 0$ (3mks)

$y = x^3 + 4x^2 - x - 6$
$0 = x^3 + 4x^2 - 12x + 0$
$y = 11x - 6$

x	0	1	2
y	-6	5	16

$x_1 = 0$
$x_2 = 2$

18. (a) Use the mid-ordinate rule with 5 strips to estimate the area bounded by the curve $y = x^2 + x + 1$, the y axis, x axis and the line x=5. (3mks)

x	0	1	2	3	4	5
y	1	3	7	13	21	35

x	0.5	1.5	2.5	3.5	4.5
y	1.75	4.75	9.75	16.75	25.75

$A = h(y_1 + y_2 + y_3 + y_4 \ldots) = 58.75$ sq. units

(b) Estimate the same area using the trapezium rule with 6 ordinates. (3mks)

$A = \frac{1}{2}h\left[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \ldots)\right]$

$= \frac{1}{2} \times 1\left[(1 + 35) + 2(3 + 7 + 13 + 21)\right]$

$= \frac{1}{2}[124]$

$= 62$ sq. units.

(c) Calculate the exact area covered by the region (a) above. (2mks)

$\int_0^5 (x^2 + x + 1)\,dx$

$\left[\frac{x^{2+1}}{3} + \frac{x^{1+1}}{2} + 1x^0 + c\right]$

$\left[\frac{x^3}{3} + \frac{x^2}{2} + x + c\right]_0^5$

$A = \left[\frac{5^3}{3} + \frac{5^2}{2} + 5 + c\right] - [0$

$= 59\frac{1}{6}$ sq. units

(d) Find the percentage error in (a) and (b) above. (2mks)

% in (a)

$\frac{59\frac{1}{6} - 58.75}{59\frac{1}{6}} \times 100\%$

$= 0.7042\%$

% in (b)

$\frac{62 - 59\frac{1}{6}}{59\frac{1}{2}} \times 100\%$

$= 4.789\%$

$\frac{22-76}{}$ $\frac{3+6}{3} = \frac{9}{3} =$

$a, a+6d \quad a+24d$

19. The 1st, 7th and 25th terms of an arithmetic progression are the first three consecutive terms of a geometric progression. The 20th term of the arithmetic progression is 22. Find:-

$a+19d=22, \quad a=22-19d$

(a) (i) The first term and common difference of the arithmetic progression. (4mks)

$$\frac{a+6d}{a} = \frac{a+24d}{a+6d}$$

$$a^2+12ad+36d^2 = a^2+24da$$

$$12ad+36d^2 = 24da$$

$$36d^2+12ad-24da=0$$

$$36d^2-12ad=0$$

$$12d(3d-a)=0$$

$$a = 22-19d$$

$$3d = a, \quad a=22-19d$$

$$3d = 22-19d$$

$$22d=22$$

$$d=1$$

$$a = 3d = 3\times 1 = 3$$

(ii) The sum of the first 40 terms of the arithmetic progression. (2mks)

$$S_{40} = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$= \frac{40}{2}\left[2(3)+39\right]$$

$$= 20\left[6+39\right] = 900$$

(b) (i) The 10th term of the geometric progression. (2mks)

$$r = \frac{3+6(1)}{3} = 9/3 = 3$$

$$a=3$$

$$r=3$$

$$10^{th} \text{ term} = ar^9 = 3(3)^9 = 59049$$

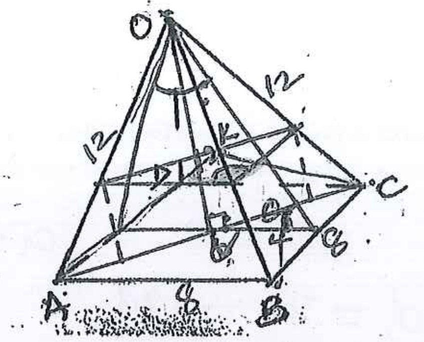
(ii) The sum of the first 10 terms of the geometric progression. (2mks)

$$S_{10} = \frac{a(r^n-1)}{r-1}$$

$$= \frac{3(3^{10}-1)}{3-1}$$

$$= 88572$$

20. OABCD is a pyramid in which the horizontal base ABCD is a square of sides 8cm. If OA=OB=OC=OD=12cm.



Find to four significant figures where applicable;

(a) The vertical height of the pyramid (2mks)

$AC = \sqrt{8^2+8^2}$

$= 11.3137$

$Vc = 5.657$

$VO = \sqrt{12^2 - 5.657^2}$

$= \sqrt{112}$

$= 10.58$

(b) The angle between planes OAB and ABCD (2mks)

$\operatorname{Tan} \theta = \frac{10.58}{4}$

$= 69.30^\circ$

(c) The angle between planes OBC and OAD (2mks)

20.7048×2

$= 41.41^\circ$

(d) The angle between planes OAB and OBC (4mks)

Area of OAB =

$\frac{12+12+8}{2} = \frac{32}{2} = 16$

$A = \sqrt{16(16-12)(16-12)(16-8)}$

$= \sqrt{16(4)(4)(8)}$

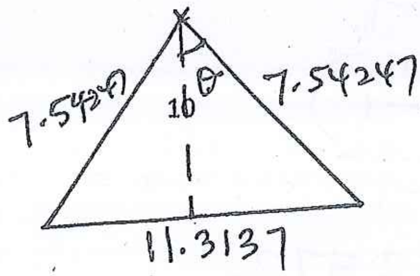
$= \sqrt{2048}$

$= 45.2548$

$\frac{1}{2} \times 12 \times h = 45.2548$

$h = \frac{45.2548 \times 2}{12}$

$= 7.54247$



$\operatorname{Sin} \theta = \frac{5.657}{7.54247}$

$\operatorname{Sin} \theta = 0.7500$

$\theta = 48.59^\circ$

$\angle AKC = 48.59 \times 2$

$= 97.18^\circ$

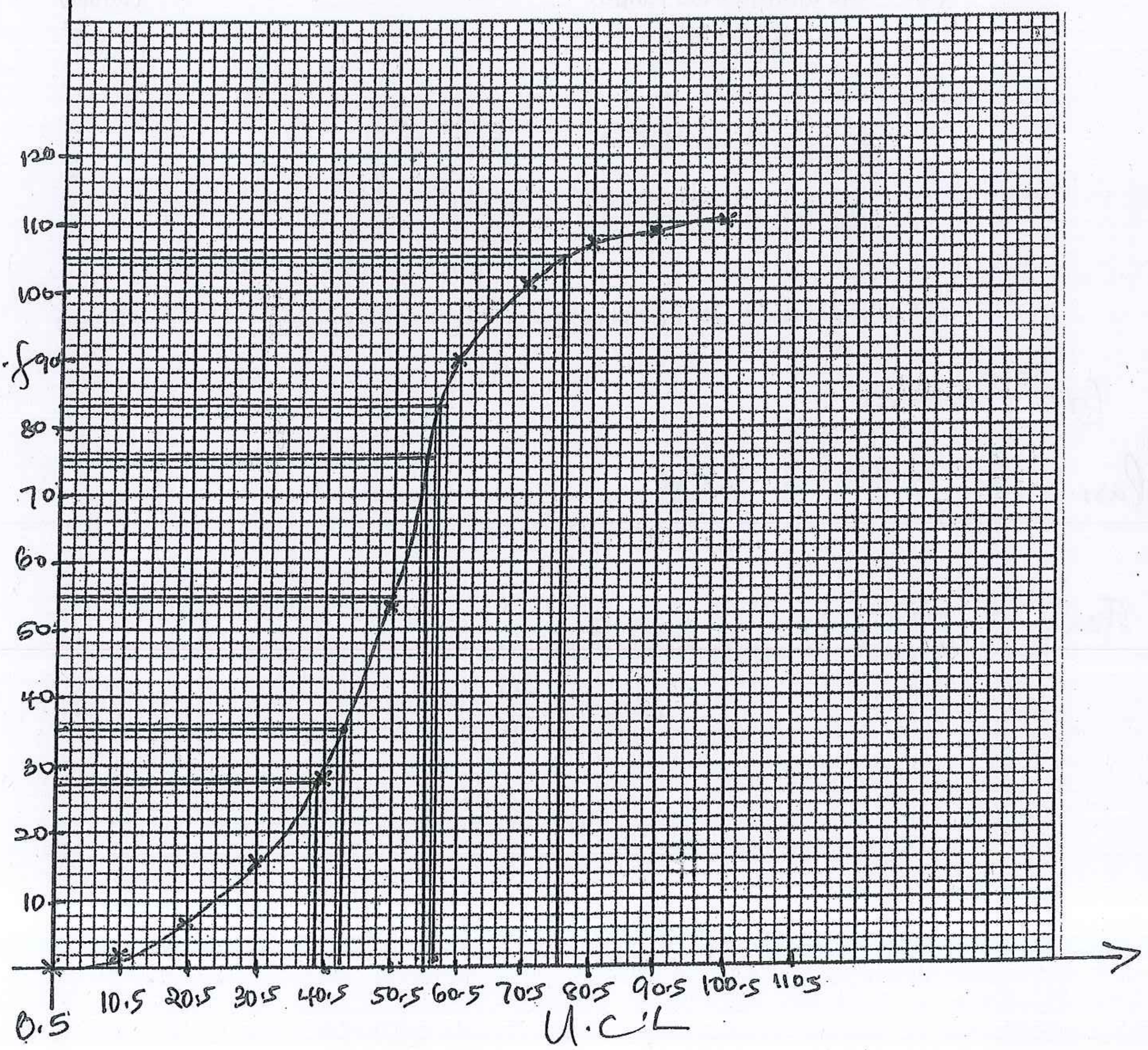
21. The frequency distribution of marks of 110 students is given in the table below.

0.5 10.5 20.5 30.5 40.5

Marks	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90-	91-100
Frequency	2	5	9	12	25	37	11	6	2	1

c.f 2 7 16 28 53 90 101 107 109 110

(a) Draw an o-give curve to illustrate the data (4mks)



(b) From your graph estimate

(i) The median mark (1mk)

$\frac{110}{2} = 55$

$= 50.5$

(ii) The interquartile range (3mks)

$\frac{Q_3 - Q_1}{2}$ -

$\frac{3}{4}$ of $110 = 82.5 = 55.5$

$\frac{1}{4}$ of $110 = 27.5 = 39.5$

$16 \div 2 = 8.$

(iii) The pass mark if 68% of the students are to pass. (2mks)

68^{th} Percentile

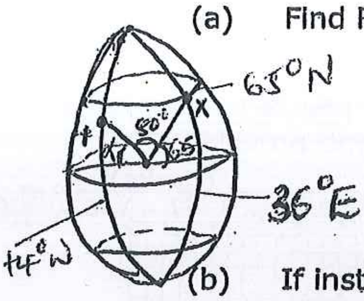
Pass $\frac{68}{100} \times 110 = 74.8$

Fail $= \frac{32}{100} \times 110 = 35.2$

43.5%

22. An aircraft takes off from the airport X(65°N, 36°E) and flies by the most direct route to another airport Y (R°N, 144°W) covering a distance of 4800nm.

(a) Find R (2mks)



$\frac{60\theta}{60} = \frac{4800\text{nm}}{60}$

$\theta = 80°$

$\alpha = 180 - (80 + 65)$

$= 35°$, $R = 35°N$

(b) If instead the aircraft had flown along the meridian 144°W to point Y, find how much further it would have flown. (4mks)

$360 - 80 = 280$

$60 \times 280 = 16800$

$$\begin{array}{r} 16800 \\ -\ 4800 \\ \hline 12000 \text{ nm} \end{array}$$

(c) Two air crafts take off from X to Y at the same time. Given that both fly at the same speed and one flies on the direct route and the other takes the route described in (b) above, state the position of the second aircraft when the first is landing at Y. (3mks)

$\frac{4800}{60} = \frac{60\theta}{60}$

$80° = \theta$

$80 - 65 = 15°$

position of the second aircraft

(15°S, 36°E)

(d) If the local time at X was 8.00am on Sunday, find the local time at Y. (1mk)

$\frac{180 \times 4}{60} = 12\text{hrs}$

8.00 pm on Saturday

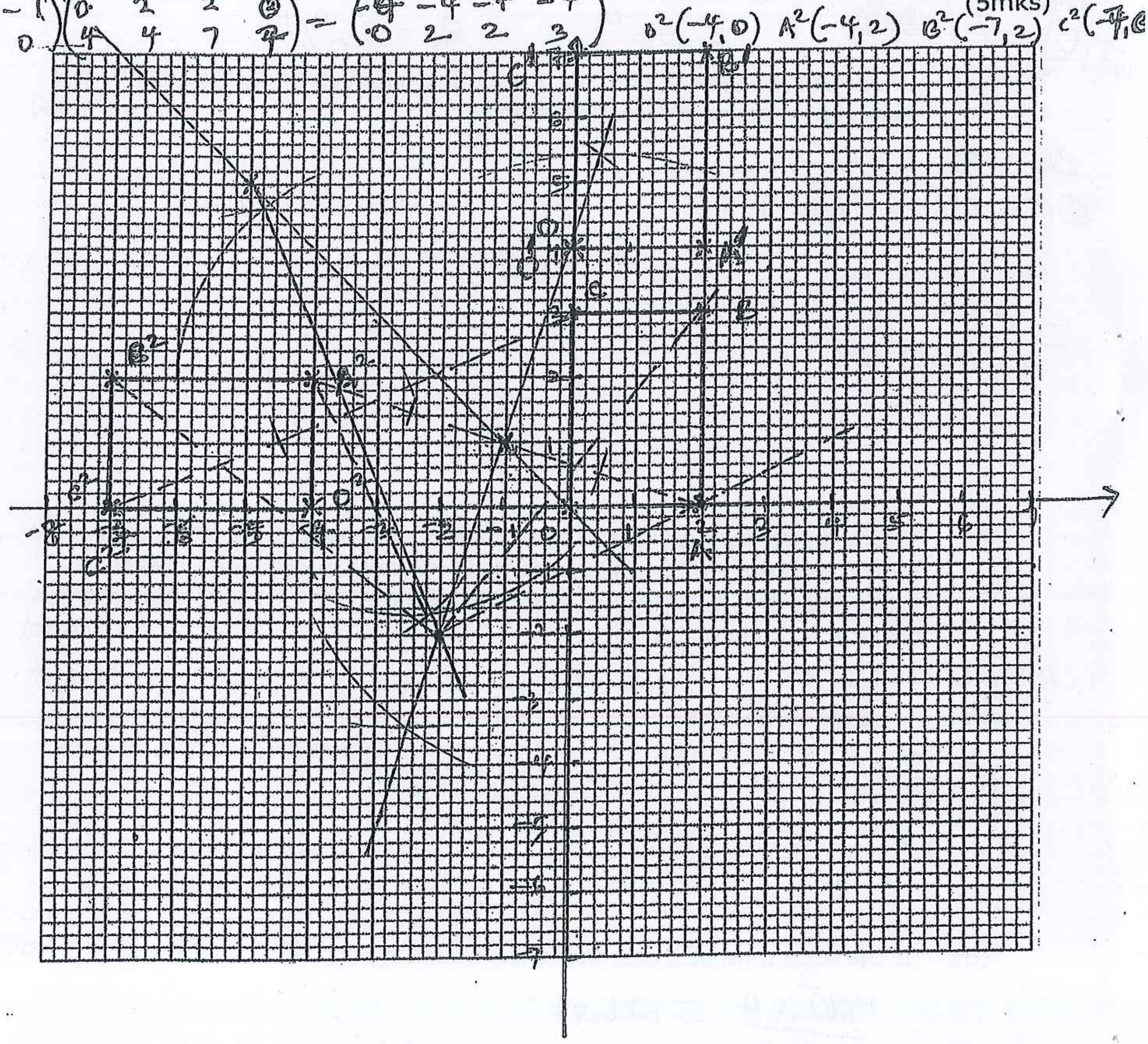
23. A rectangle OABC has vertices O(0, 0), A(2, 0), B(2, 3) and C(0, 3).

$O^1A^1B^1C^1$ is the image of OABC under a translation $T=\begin{pmatrix}0\\4\end{pmatrix}$. $O^2A^2B^2C^2$ is the image of $O^1A^1B^1C^1$ under a transformation given by the matrix $m=\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$

$O \begin{pmatrix}0\\0\end{pmatrix}+\begin{pmatrix}0\\4\end{pmatrix} = \begin{matrix}0\\4\end{matrix}$ $(0,4)O^1$ $A^1 = \begin{pmatrix}2\\0\end{pmatrix}+\begin{pmatrix}0\\4\end{pmatrix}$ $A^1(2,4)$ $B^1\begin{pmatrix}2\\3\end{pmatrix}+\begin{pmatrix}0\\4\end{pmatrix}$ $B^1(2,7)$ $C^1\begin{pmatrix}0\\3\end{pmatrix}+\begin{pmatrix}0\\4\end{pmatrix}$ $C^1(0,7)$

(a) Plot the rectangles OABC, $O^1A^1B^1C^1$ and $O^2A^2B^2C^2$ on the grid provided. (5mks)

$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}0 & 2 & 2 & 0\\4 & 4 & 7 & 7\end{pmatrix}=\begin{pmatrix}-4 & -4 & -7 & -4\\0 & 2 & 2 & 3\end{pmatrix}$ $O^2(-4,0)$ $A^2(-4,2)$ $B^2(-7,2)$ $C^2(-7,0$



(b) Describe fully the transformation that maps OABC onto $O^2A^2B^2C^2$ (3mks)

Rotation, centre (-2, -2) angle of rotation +90°.

(c) Find the coordinates of $O^3A^3B^3C^3$ the image of $O^1A^1B^1C^1$ under a reflection in the line y=-x. (2mks)

interchange x and y coordinate then negate.

$O^1(0, 4) \rightarrow (O^3(-4, 0))$

$A^1(2, 4) \rightarrow (A^3(-4, -2))$

$B^1(2, 7) \rightarrow (B^3(-7, -2))$

$C^1(0, 7) \rightarrow (C^3(-7, 0))$

$O^3(4, 0)$ $A^3(-4, -2)$, $B^3(-7, -2)$ $C^3(-7, 0$

24. In this question, use a ruler and a pair of compasses only.

(a) Construct triangle ABC in which AB=6cm, BC=5.5cm and angle ABC=60°. Measure AC. (3mks)

(b) On the same side of AB as C

(i) Determine all the locus of a point P such that angle APB=60° and APB=90° (4mks)

(ii) Construct the locus of R such that AR =3cm (1mk)

(c) By shading the unwanted region show the region T such that AR ≥ 3cm, angle APB ≥ 60° and angle APB ≤ 90° (2mks)