NAME______________________ ADM NO________________ CLASS___________

ALLIANCE HIGH SCHOOL

TRIAL EXAMS 2022

MATHEMATICS PAPER 2

TIME: 2 ½ HOURS

INSTRUCTIONS TO CANDIDATES:

1) Write your NAME, ADMISSION NUMBER and your CLASS in the spaces provided above.
2) Sign and write the date of examination in the spaces provided above.
3) This paper consists of two section I and II.
4) Answer ALL questions in section I and only five questions from section II.
5) Answers and working must be written on the question paper in the spaces provided below each question.
6) Marks may be given for correct working even if the answer is wrong.
7) Non-programmable electronic calculators may be used.

FOR EXAMINERS' USE ONLY.

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL
Q	Q	Q	R	R	R	G	G	G	G	M	M	M	K	K	K	

SECTION II

17	18	19	20	21	22	23	24	TOTAL
A	A	B	B	C	C	D	D	

GRAND TOTAL

SECTION 1-50 MARKS (ANSWER ALL THE QUESTIONS)

1.A stop watch reads correct to $\frac{1}{5}$ seconds. Two races are timed as $49\frac{3}{5}$ seconds and $49\frac{4}{5}$ seconds. Calculate the maximum percentage error in sum of these two timings (3 marks)

$AE = \frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$

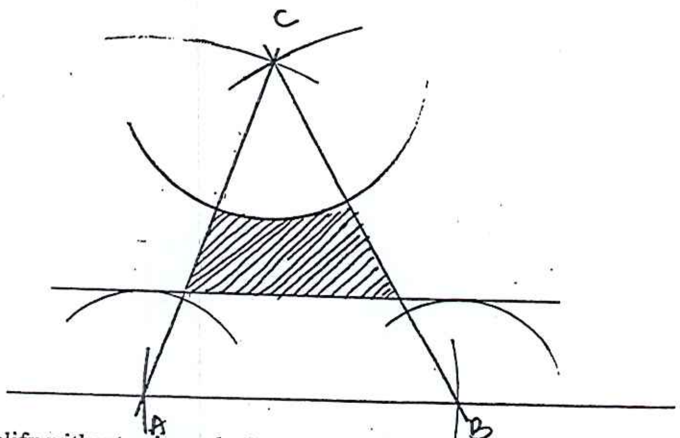
AE in Sum =

$\frac{1}{10} \times 2 = \frac{1}{5}$

$\%\ \text{error} = \left[\frac{1/5}{49\,3/5 + 49\,4/5}\right] \times 100$

$= \underline{0.2012\%}$

2.Construct, using a scale of 1:100 construct a triangular plot ABC where AB=6m, AC=7m and BC=7.5m using AB as the base. Cows are allowed to graze inside the plot provided that they are at least 2 meters from AB and more than 3 meters from C. Indicate by shading the area available for grazing. (4 marks)



3.Simplify without using calculators or mathematical tables, the value of (3 marks)

$$8 - \frac{\sqrt{60}}{2} + \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$\frac{\sqrt{60}}{2} = \frac{\sqrt{15} \times \sqrt{4}}{2}$

$= \frac{2\sqrt{15}}{2}$

$= \sqrt{15}$

$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \cdot \frac{(\sqrt{5}+\sqrt{3})}{\sqrt{5}+\sqrt{3}} = \frac{8+2\sqrt{15}}{2}$

$= 4+\sqrt{15}.$

$8-\sqrt{15} + 4 + \sqrt{15}$

$= \underline{12}$

7.Given that $S = \frac{(1-r^n)}{1-r}$ make **n** the subject of the formula. (3 marks)

$$S - Sr = 1 - r^n$$
$$r^n = 1 - S + Sr$$
$$n \log r = \log(1 - S + Sr)$$
$$n = \frac{\log(1 - S + Sr)}{\log r}$$

8. State the amplitude and the period of the following function (2 marks)
y=tan 3x

a= Undefined

Period = 180°

9.In a Geometric Progression, the first term is 2 and the common ratio is 2. Given that the product of the last two terms of the GP is 8192, find the sum of the last two terms. (3 marks)

$$(ar^{n-1})(ar^{n-2}) = 8192$$
$$2 \times 2^{n-1} \times 2 \times 2^{n-2} = 2^{13}$$
$$2^{2n-1} = 2^{13}$$
$$n = 7$$

last $= 2 \times 2^6 = 128$

2nd $= 2 \times 2^5 = 64$

Sum $= 128 + 64$
$= 192$

10. Given that x= m+ n, and m varies directly as y while n varies directly as the square of y. If x=16, y=2 and when x=33, y=3. Find x when y=8. (3 marks)

$m \propto y \Rightarrow m = ky$

$n \propto y^2 \Rightarrow n = ay^2$

$x = ky + ay^2$

$16 = 2k + 4a$

$33 = 3k + 9a$

$k = 2$

$a = 3$

$x = 2y + 3y^2$

$x = 2(8) + 3(64)$

$x = 208$

4.Use binomial expansion to determine the value of $(1\frac{1}{2})^5$ (3 marks)

1 5 10 10 5 1

$$(1)^5 + 5(1^4)\left(\tfrac{1}{2}\right) + 10(1^3)\left(\tfrac{1}{2}\right)^2 + 10(1^2)\left(\tfrac{1}{2}\right)^3 + 5(1)\left(\tfrac{1}{2}\right)^4 + \left(\tfrac{1}{2}\right)^5$$
$$= 1 + 5/2 + \tfrac{5}{2} + 5/4 + 5/16 + 1/32$$
$$= 7.59375$$

5.The mean and standard deviation of the marks scored by a group of 10 students was found to be 47 and 11 respectively. An eleventh student had a score of 58 marks. Calculate the mean and standard deviation of the 11 students. (4 marks)

$\Sigma fx = 47 \times 10 = 470$

New $(\bar{X}) = \frac{470 + 58}{11}$

$= 48.$

$sd = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$

$11^2 = \frac{\Sigma fx^2}{10} - 47^2$

$\Sigma fx^2 = 23{,}300$

New sd =

$\sqrt{\frac{23{,}300}{11} - 48^2}$

$= 10.95$

6. Two pipes fill a swimming bath in 12 hours. The larger pipe is $33\frac{1}{3}$% more efficient than the smaller pipe. How long does the larger pipe take to fill the bath? (3 marks)

Bigger	Smaller
$133\frac{1}{3}\% x$	x

$1\text{hr} \Rightarrow \frac{1}{x} + \frac{4}{3x} = \frac{1}{12}$

$\frac{7}{3x} = \frac{1}{12}$

$x = 28$

larger pipe =

$\frac{3}{4} \times 28$

$= 21$ hours

15. Find the value of p if $\int_0^3 (px^2 + 2x + 3)dx = 54$ (3 marks)

$$\left[\frac{px^3}{3} + x^2 + 3x\right]_0^3 = 54$$

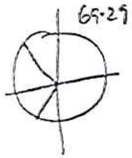
$$9p + 18 = 54$$

$$9p = 36$$

$$\underline{p = 4}$$

16. Solve for x in the domain $0^c \le x \le 2\pi^c$ (4 marks)

$2\cos 2x = -0.7071$



$$\cos 2x = -0.35355$$

$$2x = 110.71, 249.29, 470.71, 609.29$$

$$x = 55.355, 124.645, 235.355, 304.645$$

$$x = 0.3075\pi^c, 0.6923\pi^c, 1.3075\pi^c, 1.6925\pi^c$$

11. A curve is such that $\frac{dy}{dx} = 4 - x$ and the point (2,9) lies on the curve. Find the equation of the curve. (3 marks)

$$y = \int (4 - x)dx$$

$$y = 4x - \frac{x^2}{2} + c$$

$$9 = 8 - 2 + c$$

$$c = 3$$

$$y = 4x - \tfrac{1}{2}x^2 + 3$$

12. Given that p= 2i – 3j +k and q= 3i – 4j – 3k, a point R divides a line PQ externally in the ratio of 4:1. Find the coordinates of R. (3 marks)

m : n
4 : −1

$$-\frac{1}{3}\begin{pmatrix}2\\-3\\1\end{pmatrix} + \frac{4}{3}\begin{pmatrix}3\\-4\\-3\end{pmatrix}$$

$$\begin{pmatrix}-2/3\\1\\-1/3\end{pmatrix} + \begin{pmatrix}4\\-16/3\\-4\end{pmatrix}$$

$$\underline{R = \left(3\tfrac{1}{3}, -4\tfrac{1}{3}, -4\tfrac{1}{3}\right)}$$

13. Given that x and y are both positive, solve the equations (3marks)

$\log(xy) = 7$ and $\log\left(\frac{x}{y}\right) = 1$

$$\log x + \log y = 7$$

$$\log x - \log y = 1$$

$$2\log y = 6$$

$$\log y = 3$$

$$\underline{y = 1000}$$

$$\cancel{\log x = 4} \quad \log x = 4$$

$$\underline{x = 10{,}000}$$

14. Use the mid-ordinate rule to estimate the area enclosed by the curve $y = x^2 - 9$, x-axis and the lines x=2 and x=5 using six strips (3marks)

X	2.25	2.75	3.25	3.75	4.25	4.75
y	−3.9375	−1.4375	1.5625	5.0625	9.0625	13.5625

$$A = 0.5(3.9375 + 1.4375 + 1.5625 + 5.0625 + 9.0625 + 13.5625)$$

$$\underline{A = 17.3125} \text{ sq. units}$$

18.A triangle ABC with vertices at A (1, -1), B (3, -1) and C (1, 3) is mapped onto triangle $A^1B^1C^1$ by a transformation whose matrix is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.Triangle $A^1B^1C^1$ is then mapped onto $A^{11}B^{11}C^{11}$ with vertices at A^{11} (2, 2), B^{11} (6, 2) and C^{11} (2, -6) by a second transformation.

(i) Find the coordinates of $A^1B^1C^1$ (3 marks)

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{array}{ccc} A & B & C \\ \end{array} \begin{bmatrix} 1 & 3 & 1 \\ -1 & -1 & 3 \end{bmatrix} = \begin{array}{ccc} A' & B' & C' \\ \end{array} \begin{bmatrix} -1 & -3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

$A'(-1,-1)$ $B'(-3,-1)$ $C'(-1,3)$

(ii) Find the matrix which maps $A^1B^1C^1$ onto $A^{11}B^{11}C^{11}$. (3 marks)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$-a-b=2$

$-c-d=2$

$-3a-b=6$

$-3c-d=2$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -3 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

$-a-b=2$

$-3a-b=6$

$2a=-4$

$a=-2$

$b=0$

$-c-d=2$

$-3c-d=2$

$2c=0$

$c=0$

$d=-2$

$$\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

(iii) Determine the ratio of the area of triangle $A^1B^1C^1$ to triangle $A^{11}B^{11}C^{11}$. (1 mark)

ASF= 4

(iv) Find the transformation matrix which maps $A^{11}B^{11}C^{11}$ onto ABC (3 marks)

$$\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

det= −4

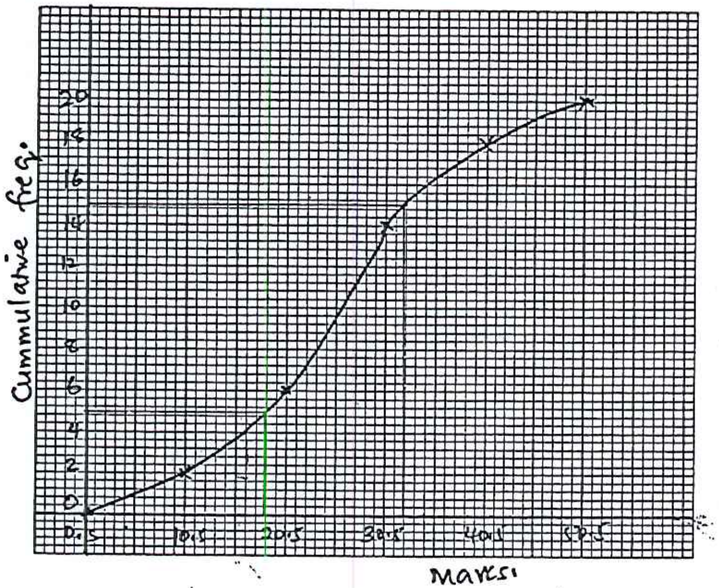
$$\Rightarrow -\frac{1}{4} \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$$

17. The table below shows the frequency distribution of marks scored by students in a test.

Marks	1-10	11-20	21-30	31-40	41-50
Frequency	2	4	8	4	2
CF	2	6	14	18	20

a). On the grid provided, draw an ogive for the data. (3 marks)



b). Use your graph to determine;

(i). The pass mark if only 6 students passed the exam. (1 mark)

30.5 marks

(ii). The quartile deviation (3 marks)

$0.25 \times 20 = 5^{th}$

18.5

$0.75 \times 20 = 15^{th}$

32.5

$\frac{32.5 - 18.5}{2} = 7$

c). Range of marks scored by the middle 60 % of the students (3 marks)

$0.2 \times 20 = 4^{th} \rightarrow 16.5$

$0.8 \times 20 = 16^{th} \Rightarrow 34.5$

range $\Rightarrow$ 16.5 to 34.5

20. The acceleration of a particle, t seconds after passing a fixed-point P is given by a = 4t-7. Given that the initial velocity of the particle is 5m/s, find;

a) Its acceleration when t = 4 seconds (1 Mark)

$a = 4(4) - 7$

$= 9ms^{-2}$

b) Its velocity when t=3 seconds (3 Marks)

$V = \int (4t-7)\,dt$

$V = 2t^2 - 7t + c$

$5 = c$

$V = 2t^2 - 7t + 5$

$V = 2(9) - 7(3) + 5$

$V = 2\,m/s$

c) Values of t when the particle is momentarily at rest (3 Marks)

$V = 0$

$2t^2 - 7t + 5 = 0$

$t^2 - 3.5t = -2.5$

$t^2 - 3.5t + (-1.75)^2 = -2.5 + (-1.75)^2$

$(t - 1.75)^2 = 0.5625$

$t - 1.75 = \pm 0.75$

$t = 1.75 \pm 0.75$

$t = 1\ sec$ or $t = 2.5 sec$

d) Its maximum velocity (3 marks)

$a = 0$

$0 = 4t - 7$

$t = 1.75\ sec$

$V = 2(3.0625) - 7(1.75) + 5$

$V = -1.125\,m/s$

19. The table below shows the taxation rates for income earned.

Income in ksh pm	Tax rates (%)
1 – 9680	10
9681 – 18800	15
18801 – 27920	20
27921 – 37040	25
Excess over 37041	30

In that year, Mr. Hamisi paid a net tax of KSh. 5,512 per month. He is entitled to the following monthly allowances:

House Allowance -Shs. 10,000
Medical Allowance -Shs. 2400
Acting Allowance -Shs. 2820.

He is entitled to a monthly personal relief of KShs. 1162 while 7.5% of his basic salary is tax-exempted.

Calculate Mr. Hamisi's monthly basic salary in KSh. (7 Marks)

Gross tax $= 5512 + 1162$

$= 6674$

$9680 \times 0.1 = 968$

$9120 \times 0.15 = 1368$

$9120 \times 0.2 = 1824$

$9120 \times 0.25 = 2280$

$0.3 \times y = 234$

$y = 780$

$TI = 9680 + 3(9120) + 780$

$= 37,820$

$BS = 37820 - (10,000 + 2400 + 282$

$= 22,600$

$\frac{22600 \times 100}{92.5}$

$= 24,432.43$

(b) The following deductions also made every month.

(i) N.H.I.F. KSh. 320
(ii) Co-operative society shares KSh. 6000
(iii) Union dues KSh. 200

Calculate his net monthly salary. (3 Marks)

Deductions $= 320 + 6000 + 200 + 5512 = 12032$

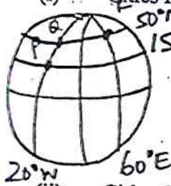
Net Sal $= 39,652.43 - 12032$

$= 27,620.43$

22. The position of 3 cities P, Q and R are (15°N, 20°W) (50°N, 20°W) and (50°N, 60°E) respectively.

a) Find the distance in nautical miles between:

(i) Cities P and Q (2 marks)



$D = 60 \times 35$

$= \underline{2100\ nm}$

(ii) Cities Q and R along a circle of latitudes (2 marks)

$D = 60 \times 80 \cos 50$

$D = \underline{3085.38\ nm}$

b) A plane left city P at 0250h and flew to city Q where it stopped for 3 hours then flew on to city R, maintaining a ground speed of 900 knots throughout. Calculate:

(i) The local time at city R when the plane left city P (3 marks)

$80 \times 4 = 320$ min

$= \underline{5\text{hrs } 20\text{ min}}$

$$\begin{array}{r} 0250 \\ +\ 5\ 20 \\ \hline 8{:}10 \end{array}$$

$= \underline{0810\text{hrs}}$

(ii) The local time (to the nearest minute) at city R when the plane landed at R. (3 marks)

Time from P to Q =

$= \dfrac{2100}{900} = 2\frac{1}{3}$ hrs

Q to R =

$\dfrac{3085.38}{900} = 3$hrs 26min

Total time =

3hrs 26 + 2hr 20m + 3hrs

= 8hrs. 46 min
+ 8 10

$= \underline{\underline{1656\text{hrs}}}$

21. Mr. Mairura has two lorries A and B used to transport at least 42 tons of potatoes to the market. Lorry A carries 4 tons of potatoes per trip while lorry B carries 6 tons of potatoes per trip. Lorry A uses 2 liters of fuel per trip while lorry uses 4 liters of fuel per trip. The two lorries are to less than 24 liters of fuel. The number of trips made by lorry A should be less than the number of trips made by lorry B. Lorry A should make more than 4 trips.

a) Taking X to represent the number of trips made by lorry A and Y to represent the number of trips made by lorry B, write the inequalities that represent the above information (4marks)

i) $4x+6y \leq 42$

ii) $2x+4y \leq 24$

iii) $x < y$

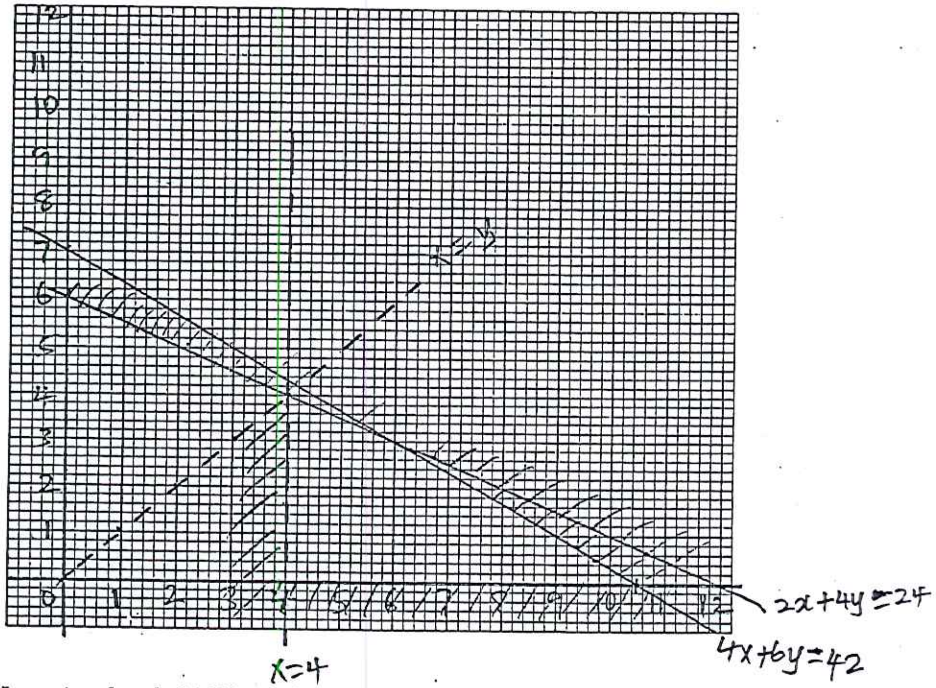
iv) $x > 4$ v) $x \geq 0$

x	0	10.5
y	7	0

x	0	12
y	6	0

x	1	2	3
y	1	2	3

a) Plot the inequalities above in the graph provided below (4 marks)



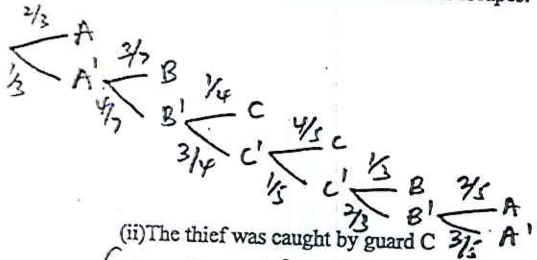
c) If Lorry A makes sh. 35,000 per trip and Lorry B makes sh.28,000 per trip, use the graph above to determine the number of trips made by lorry A and by lorry B to deliver the greatest number of potatoes and hence find the maximum profit. (2marks)

$(6, 3) \rightarrow (6 \times 35000) + (3 \times 28000)$

$= \underline{\underline{294{,}000}}$

24. (a) A jewel is guarded by three guards A, B and C in that order. On his way in, the probability of a thief getting caught by guard A is $\frac{2}{3}$, by B is $\frac{3}{7}$ and by C is $\frac{1}{4}$. On his way out, the probability of being caught by guard C is $\frac{4}{5}$, by B is $\frac{1}{3}$ and by guard A is $\frac{2}{5}$. Calculate the probability that:

(i) The jewel is stolen and the thief escapes. (2 marks)



$$\frac{1}{3} \times \frac{4}{7} \times \frac{3}{4} \times \frac{1}{5} \times \frac{2}{3} \times \frac{3}{5}$$

$$= \frac{2}{175}$$

(ii) The thief was caught by guard C (2 marks)

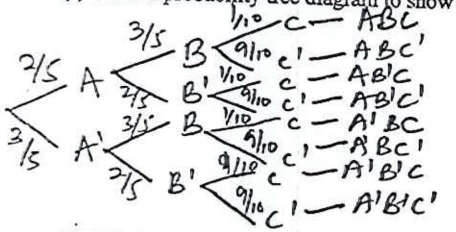
$$\left(\frac{1}{3} \times \frac{4}{7} \times \frac{1}{4}\right) + \left(\frac{1}{3} \times \frac{4}{7} \times \frac{3}{4} \times \frac{4}{5}\right)$$

$$= \frac{4}{84} + \frac{4}{35}$$

$$\underline{\underline{\frac{17}{105}}}$$

(b) Albert, Bonny and Charles competed in a game of chess. Their probabilities of winning the game are $^2/_5$, $^3/_5$ and $^1/_{10}$ respectively.

(a) Draw a probability tree diagram to show all the possible outcomes. (2 marks)



(b) Calculate the probability that;

(i) No one loses the game. (2 marks)

$P(ABC)$

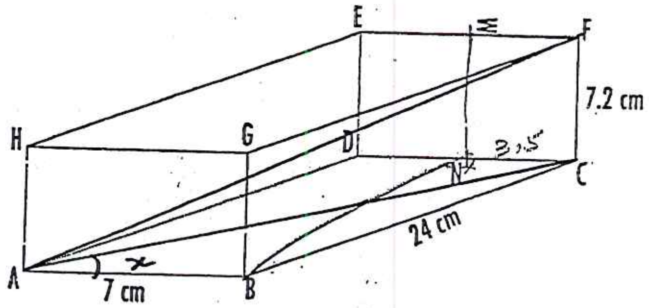
$$= \frac{2}{5} \times \frac{3}{5} \times \frac{1}{10} \quad = \frac{3}{125}$$

(ii) Only one of them wins the game. (2 marks)

$$\left(\frac{2}{5} \times \frac{2}{5} \times \frac{9}{10}\right) + \left(\frac{3}{5} \times \frac{3}{5} \times \frac{9}{10}\right) + \left(\frac{3}{5} \times \frac{2}{5} \times \frac{1}{10}\right)$$

$$= \frac{123}{250}$$

23. The figure below represents a cuboid ABCDEFGH, with AB = 7 cm, BC = 24 cm and CF = 7.2 cm. M and N are the mid-points of EF and DC respectively.



Calculate to 2 decimal places the:

a) Angle AF makes with the plane ABCD (3 marks)

$AC = \sqrt{24^2 + 7^2}$

$= \underline{25 cm}$

$\tan\theta = \frac{7.2}{25}$

$\theta = \underline{16.07^\circ}$

b) Angle between the lines HF and AB. (2 marks)

$\tan x = \frac{24}{7}$

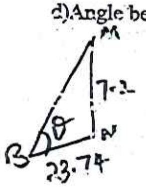
$x = \underline{\underline{73.74^\circ}}$

c) Angle between the planes GHEF and ABFE (2 marks)

$\tan\theta = \frac{7.2}{24}$

$\theta = \underline{\underline{16.70^\circ}}$

d) Angle between BM and the plane ABCD (3 marks)



$\tan\theta = \frac{7.2}{23.74}$

$BN = \sqrt{24^2 - 3.5^2}$

$\theta = \underline{\underline{16.87^\circ}}$