MARKING SCHEME

NAME:……………………………………………………………CLASS:……….ADM NO:……
ANESTAR SCHOOLS
MATHEMATICS FORM 4
PAPER 2
END OF TERM 2 2022
LANJET
TIME:

INSTRUCTIONS.
Answer all the questions in the spaces provided.
SECTION 1 (50mks)

1. The sum of n terms of the sequence: 3,9,15,21…. Is 7500
   a) Find the 20^{th} term of the sequence. (2mks)

$a = 3$
$d = 6$
$n^{th} = a+(n-1)d$
$\therefore 20^{th} = 3+(20-1)6$
$= 117$

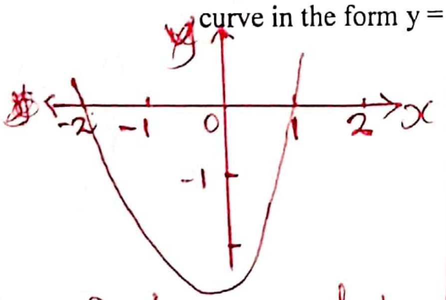
M1 – Equation
A1
2

   b) Determine the value of n. (2mks)

$S_n = \frac{n}{2}\{2a+(n-1)d\}$
$7500 = \frac{n}{2}\{2\times3+(n-1)6\}$
$15000 = 6n^2$
$\therefore n^2 = 2500$
$n = \pm 50$
$= 50$ terms

M1 – Equation
A1
2

2. A quadratic curve passes through the points (-2, 0) and (1, 0). Find the equation of the curve in the form $y = ax^2+bx+c$, where a, b and c are constants. (2mks)



Roots -2 and 1

$\therefore (x+2) = 0$
and
$(x-1) = 0$
let $y = (x+2)(x-1)$
$\Rightarrow y = x^2+x-2$

M1 – roots & Factors expressed.
A1 – Equation
2

3. Make h the subject of the formula. (2mks)

$$q = \frac{1+rh}{1-ht}$$

$q - qht = 1+rh$
$q-1 = rh+qht$
$q-1 = h(r+qt)$
$\therefore h = \frac{q-1}{r+qt}$

M1 - factorising
A1
2

4. P (1,2) and Q(9,8) are the points on the ends of the diameter of a circle. Write down in terms of x and y the equation of the circle in the form: $ax^2+by^2+x+y+c=0$. (3mks)

P(1,2) ——— Q(9,8)

$PQ = Q - P$

$\begin{pmatrix}9\\8\end{pmatrix} - \begin{pmatrix}1\\2\end{pmatrix}$

$= \begin{pmatrix}8\\6\end{pmatrix}$

$|PQ| = \sqrt{8^2+6^2}$

$= 10$ Units.

$\therefore$ Radius $= 5$ Units

Let Mid-Point be

$\left(\frac{1+9}{2}, \frac{2+8}{2}\right)$

$= (5, 5)$

$(x-5)^2 + (y-5)^2 = 5^2$

$\therefore x^2 + y^2 - 10x - 10y + 25 = 0$

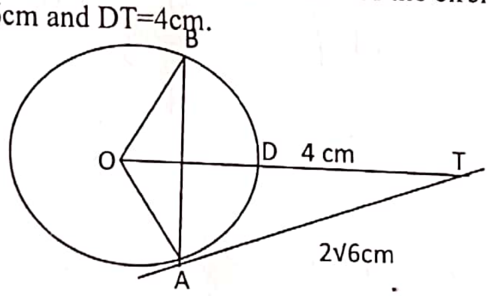
B1 – Radius got

M1 – Equation

A1 – Equation

3

5. In the figure below, O is the centre of the circle and AT is a tangent to the circle at A. AT $= 2\sqrt{6}$cm and DT=4cm.



Determine:

a) OA (2mks)

Let $OD = x$

$TO \times TD = TA^2$

$(4+x)4 = (2\sqrt{6})^2$

$16 + 4x = 24$

$4x = 8$

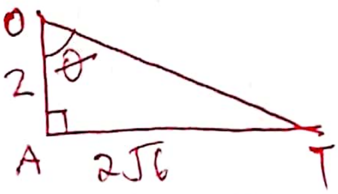
$\therefore x = 2$

M1 – Equation

A1

2

b) The value of angle AOB (2mks)



$\tan\theta = \frac{2\sqrt{6}}{2}$

$= 2.449$

$\therefore \theta = 67.8$

$\therefore \angle AOB = 2 \times 67.8$

$= 135.6°$

M1

A1

2

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6. In a transformation, an object with an area of $5cm^2$ is mapped onto an image whose area is $30cm^2$. Given that the matrix of the transformation is $\begin{pmatrix} x & x-1 \\ 2 & 4 \end{pmatrix}$

a) Find the value of x. (2mks)

Determinant $= 4x - 2(x-1)$
$= 2x + 2$

Area Scale factor $= \frac{30}{5}$

Det. = ASF

$\therefore 2x + 2 = \frac{30}{5}$

$\therefore x = 2$

M1 - Equation

A1

2

b) Hence, determine the inverse of the matrix $\begin{pmatrix} x & x-1 \\ 2 & 4 \end{pmatrix}$ (~~1~~2 marks)

$\Rightarrow$ Matrix $= \begin{pmatrix} 2 & 1 \\ 2 & 4 \end{pmatrix}$

$\therefore$ Det. $= 8 - 2$
$= 6$

Inverse $= \frac{1}{6}\begin{pmatrix} 4 & -1 \\ -2 & 2 \end{pmatrix}$

$= \begin{pmatrix} 2/3 & -1/6 \\ -1/3 & 1/3 \end{pmatrix}$

B1 ~~Determinant~~

A1

2

7. The co-ordinates of P are (0,7) and Q are (3.5, 1.4). A point S divides PQ externally in the ratio 9:2. Find the co-ordinates of S. (3mks)

Ratio $= 9 : -2$

$\Rightarrow -\frac{2}{7}\begin{pmatrix} 0 \\ 7 \end{pmatrix} + \frac{9}{7}\begin{pmatrix} 3.5 \\ 1.4 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 4.5 \\ 1.8 \end{pmatrix}$

$= \begin{pmatrix} 4.5 \\ -0.2 \end{pmatrix}$

$\therefore S = (4.5, -0.2)$

M1 - Ratio theorem expressed.

M1 - Simplifying

A1

3

8. The top of a coffee table is a regular hexagon. Each side of the hexagon measures 50.0cm, find the percentage error in calculating the perimeter of the top of the table. (3mks)

$n = 6$ Sides.

Actual Perimeter $= 50.0 \times 6$
$= 300$ cm

Maximum Per. $= 50.05 \times 6$
$= 300.3$

$\therefore$ Abs. error $= 300.3 - 300$
$= 0.3$

% Error $= \frac{0.3}{300} \times 100$

$= 0.1\%$

M1 - Absolute Error in Per.

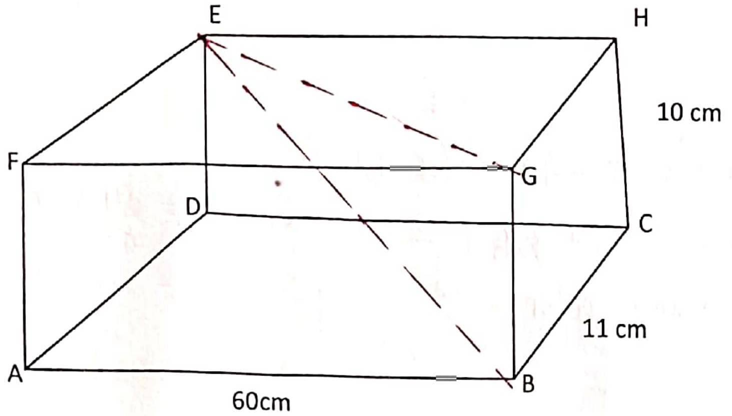
M1 - % error

A1

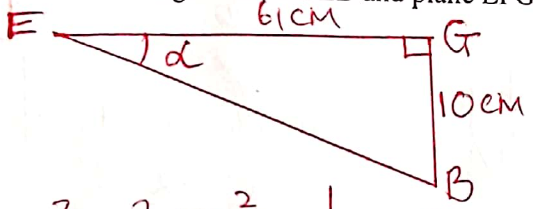
3

CamScanner

9. The figure below represents a cuboid ABCDEFGH. AB=60cm, BC=11cm and CH=10cm.



Calculate the angle between EB and plane EFGH. (3mks)



$EG^2 = 11^2 + 60^2$

$= 121 + 3600$

$EG = \sqrt{3721}$

$= 61CM$

$\tan\alpha = \frac{10}{61}$

$= 0.1639$

$\therefore \alpha = 9.3°$

M1 - EG got

M1 - Tan α or alternative

A1

3

10. Expand and simplify the expression $\left(4x - \frac{y}{2}\right)^5$ up to the third term. (2mks)

$(4x)^5 + (4x)^4\left(\frac{-y}{2}\right) + (4x)^3\left(\frac{-y}{2}\right)^2 + \ldots$

$1024x^5 + 5 \times 256\left(-\frac{y}{2}\right) + 10 \times 64x^3\left(-\frac{y^2}{4}\right) + \ldots$

$1024x^5 - 640x^4y + 160x^3y^2 + \ldots$

M1

A1

2

b. Hence use the expansion in (a) above to approximate the value of $(39.6)^5$ correct to 3 significant figures. (2mks)

Let $39.6 = 40 - 0.4$

$\Rightarrow 39.6^5 = (40 - 0.4)^5$

Comparing:

$4x = 40$

$\therefore x = 10$

$\frac{y}{2} = 0.4$

$\therefore y = 0.8$

Substituting:

$1024(10^5) - 640(10^4) \times 0.8 +$

$160(10^3)(0.8)^2 + \ldots$

$= 102400000 - 5120000$

$+ 102400$

$= 97382400$

M1 - values of X and Y substituted

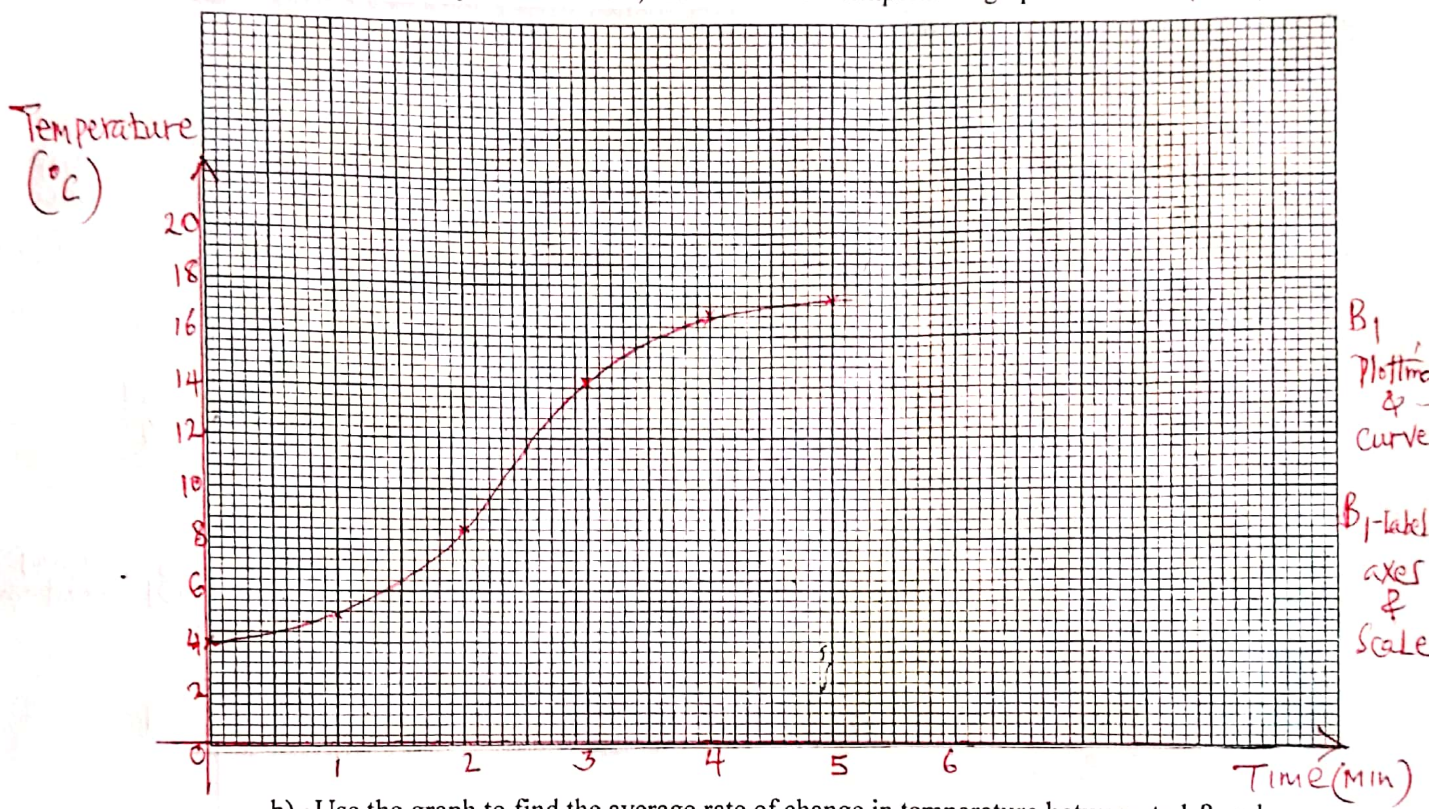
A1

2

11. A solution was gently heated, its temperature readings taken at intervals of 1 minute and recorded as shown in the table below:

Time (min)	0	1	2	3	4	5
Temperature (^oC)	4	5.2	8.4	14.3	16.8	17.5

a) On the grid provided below, draw the time – temperature graph. (2mks)



B_1 Plotting & Curve

B_1-Label axes & Scale

b) Use the graph to find the average rate of change in temperature between t=1.8 and t=3.4. (2mks)

$$\frac{15.6 - 7.6}{3.4 - 1.8} = \frac{8}{1.6}$$

$= 5^\circ C/\text{Min}$

M_1

A_1

4

12. The shortest distance between two points A(40^ON,20^OW) and B(θ^OS,20^OW) on the surface of the earth is 8008km. given that the radius of the earth is 6370km, determine the position of B. (Take $\pi = ^{22}/_{7}$). (3mks)

Let $\alpha = 40 + \theta$

$$\frac{(40+\theta)}{360} \times 2\pi \times 6370 = 8008$$

$\therefore \theta = 32.01^\circ$

$\therefore$ Position B $(32.01^\circ S, 20^\circ W)$

B_1-$(40+\theta)$

M_1

A_1-position must be given.

3

13. Simplify $\frac{\sqrt{3}}{\sqrt{3} - \sqrt{2}}$ (2mks)

$$\frac{\sqrt{3} \times (\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2}) \times (\sqrt{3} + \sqrt{2})}$$

$$\frac{3 + \sqrt{6}}{1} = 3 + \sqrt{6}$$

M_1-Conjugate

A_1

2

14. The table below shows income tax rates in a certain year.

Monthly income in Kshs.	Tax rate in each shilling
Up to 9680	10%
From 9681 to 18800	15%
From 18801 to 27920	20%
From 27921 to 37040	25%
Over 37040	30%

In that year, a monthly personal tax relief of ksh. 1056 was allowed. Calculate the monthly income tax paid by an employee who earned a monthly salary of kshs. 32,500. (4mks)

First $9680 \times \frac{10}{100} = 968$

Second $9120 \times \frac{15}{100} = 1368$

Third $9120 \times \frac{20}{100} = 1824$ +

Last $4580 \times \frac{25}{100} = 1145$

Gross tax $= 5305$

Income tax $= 5305 - 1056$

$= sh. 4249$

M1 – First & second

M1 – others

M1 – Less relief

A1

4

15. Three types of beverages are mixed in the ration 1:3:5 respectively. Type A costs sh 26, type B costs sh 28 and type C sh 32, per pocket. Find the cost of the mixture per packet. (3mks)

$1 + 3 + 5 = 9$

Type A → $26 \times 1 = sh. 26$

Type B → $28 \times 3 = sh. 84$ +

Type C → $32 \times 5 = sh. 160$

Total cost $= sh\ 270$

∴ cost per packet $= sh\ \frac{270}{9} = sh. 30$

B1 – Addition Ratios

M1

A1

3

16. The gradient of a curve is given by $\frac{dy}{dx} = x^2 - 4x + 3$. The curve passes through the point (1,0). Find the equation of the curve. (3mks)

$\int (x^2 - 4x + 3).dx$

$y = \frac{1}{3}x^3 - 2x^2 + 3x + C$

at $(1,0)$

$C = -\frac{4}{3}$

$\Rightarrow y = \frac{1}{3}x^3 - 2x^2 + 3x - \frac{4}{3}$

M1 - Integrating

M1 - Substitution of (1,0)

A1 - value of C must be given

3

SECTION II (50MKS)

17. The hire purchase (H.P) price of an electronic device was ksh. 276,000. A deposit of ksh 60,000 was paid followed by 18 equal monthly installments.

a) Calculate the monthly installment. (2mks)

276000 – 60,000

= 216000

$\frac{216000}{18}$

= sh 12000

M1

A1

2

b) The cash price of the electronic device was 10% less than the hire purchase (H.P) price. Calculate the cash price. (2mks)

$\frac{90}{100} \times 276,000$

= sh 248,000

M1

A1

2

c) Madam Kanini decided to buy the electronic device in cash. She was allowed a 5% discount on the cash price, she took a bank loan to buy the device. The bank charged compound interest on the loan at the rate of 20% p.a. the loan was repaid in 2 years.

i. Calculate the amount repaid to the bank by the end of the second year. (3mks)

$\frac{95}{100} \times 248000$

= sh 235,980

Let Principal = sh 235,980

Rate = 20%

n = 2 yrs.

$A = P(1 + \frac{r}{100})^n$

$= 235980(1 + \frac{20}{100})^2$

= sh 339,811.20

M1

M1 – Substituting

A1

3

ii. Express as a percentage of the hire purchase (HP) price, the difference between the amount repaid to the bank and the hire purchase price. (3mks)

339,811.20 – 276,000

= sh 63,811.20

$\frac{63811.20}{276000} \times 100$

= 23.12%

M1

M1

A1

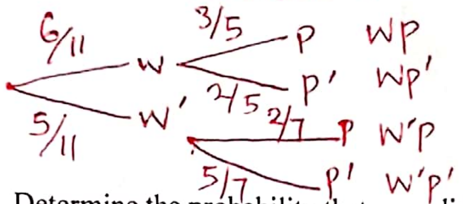
3

18. An examination involves a written test and a practical test. The probability that a candidate passes the written test is $^{6}/_{11}$. If the candidate passes the written test, then the probability of passing the practical test is $^{3}/_{5}$, otherwise it would be $^{2}/_{7}$.

a) Illustrate this information on a tree diagram. (2mks)

Let W → Written test

" P → Practical "



M1 – branch

M1 – branch

2

b) Determine the probability that a candidate is awarded:

i. For passing both tests. (2mks)

$P(WP)$

$= \frac{6}{11} \times \frac{3}{5}$

$= \frac{18}{55}$

M1

A1

2

ii. For passing the written test. (2mks)

$P(WP)$ or $P(WP')$

$\frac{6}{11} \times \frac{3}{5} + \frac{6}{11} \times \frac{2}{5}$

$= \frac{6}{11}$

~~Accept~~

M1

A1

2

c) Determine the probability that the candidate;

i. Passes one test. (2mks)

$P(WP')$ or $P(W'P)$

$\frac{6}{11} \times \frac{2}{5} + \frac{5}{11} \times \frac{2}{7}$

$= \frac{1474}{4235}$

~~Accept 0.3481~~

M1

A1

2

ii. Fails for not passing the written test. (2mks)

$P(W'P')$

$\frac{5}{11} \times \frac{5}{7}$

$= \frac{25}{77}$

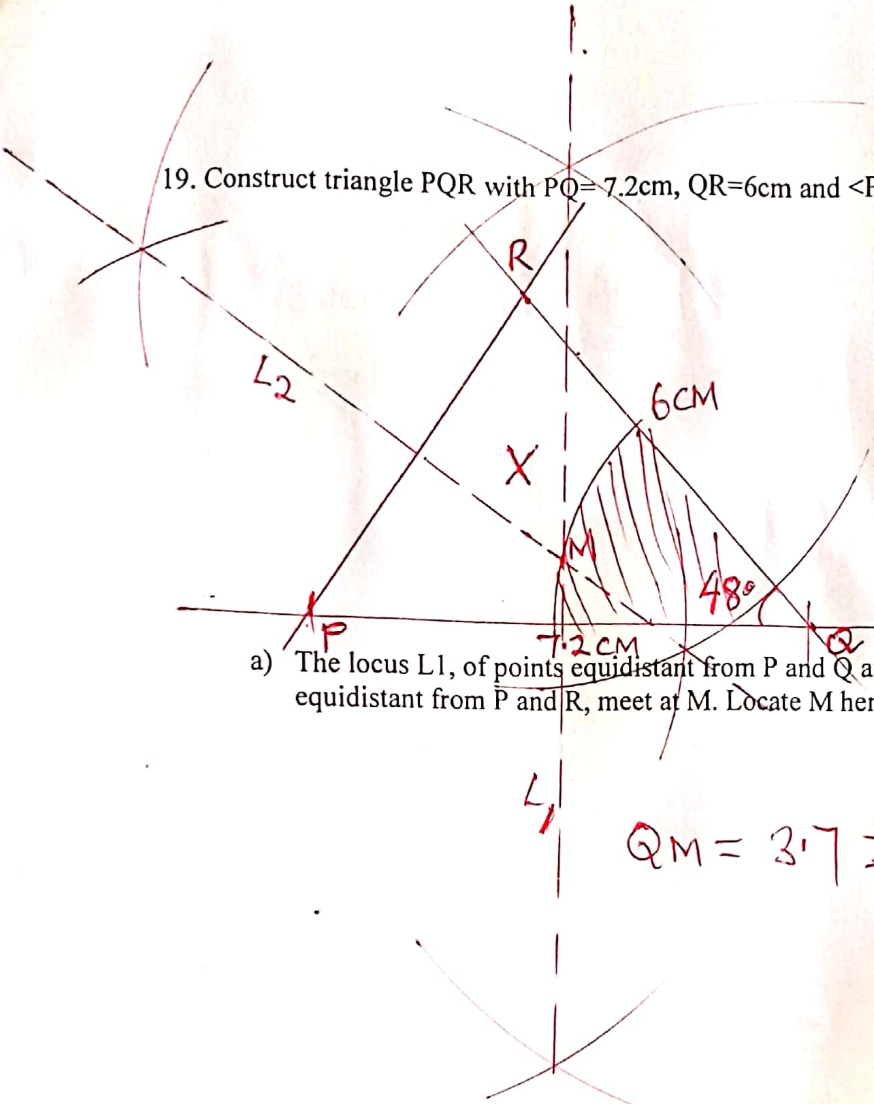
M1

A1

2

19. Construct triangle PQR with PQ= 7.2cm, QR=6cm and <PQR=48^0. (3mks)

B_1	Sides PQ, QR
B_1	∠PQR
B_1	Δ PQR
3	



a) The locus L1, of points equidistant from P and Q and Locus L2 of points equidistant from P and R, meet at M. Locate M hence measure QM. (4mks)

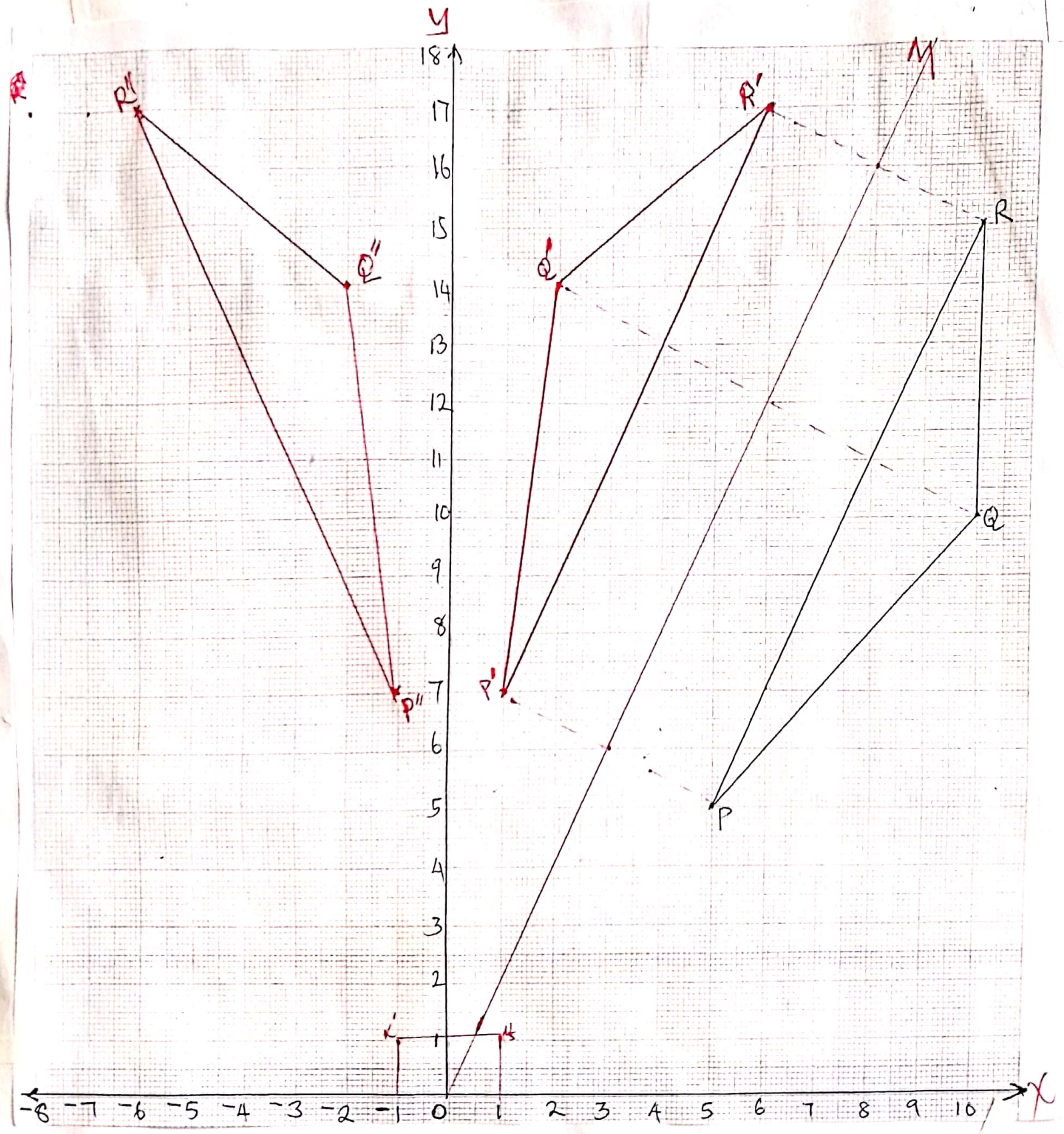
QM = 3.7 ± 0.1

B_1	– L1
B_1	– L2
B_1	– point M located
B_1	– QM given
4	

b) A point X moves within triangle PQR such that QX ≥QM. Shade and label the locus of X. (3mks)

B_1	– Arc QM drawn (must be continuous).
B_1	– Unwanted region shaded.
B_1	– Region X determined
3	

20. Triangle PQR shown on the grid below has vertices P(5,5), Q(10,10) and R(10,15)



(a)
$$\begin{pmatrix} -0.6 & 0.8 \\ 0.8 & 0.6 \end{pmatrix} \begin{array}{c} \begin{array}{ccc} P & Q & R \end{array} \\ \begin{pmatrix} 5 & 10 & 10 \\ 5 & 10 & 5 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} P' & Q' & R' \end{array} \\ \begin{pmatrix} -3+4 & -6+8 & -6+12 \\ 4+3 & 8+6 & 8+9 \end{pmatrix} \end{array}$$

$$= \begin{array}{c} \begin{array}{ccc} P' & Q' & R' \end{array} \\ \begin{pmatrix} 1 & 2 & 6 \\ 7 & 14 & 17 \end{pmatrix} \end{array}$$

M1 - multiplying

M1 - Simplifying

A1

3

a) Given that M is a reflection:

i. Draw triangle P′Q′R′ and the mirror line of the reflection. (2mks)

Line $y = 2x$ drawn/identified

B_1 – P′Q′R′ drawn

B_1 – mirror Line ~~equation~~ drawn.

2

ii. Determine the equation of the mirror line of the reflection. (2mks)

Gradient of M = 2 units

Let $y = mx + c$

$y = 2x + 0$

$\therefore y = 2x$

M_1 – Gradient given

A_1

2

b) Triangle P″Q″R″ is the image of triangle P′Q′R′ under reflection N, where N is a reflection in the Y-axis.

i. Determine triangle P″Q″R″ (1mk)

In the graph.

B_1

1

ii. Determine a 2x2 matrix equivalent to the transformation NM. (2mks)

$$N = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\therefore NM = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -0.6 & 0.8 \\ 0.8 & 0.6 \end{pmatrix}$$

$$= \begin{pmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \end{pmatrix}$$

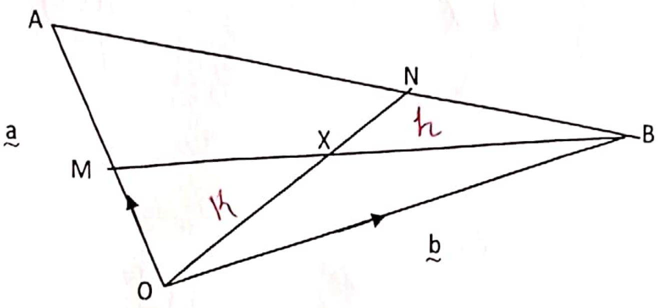
B_1 – N determined

Accept if unit square shown in graph

B_1 – NM determined.

2

21. In the figure below $OA=\underset{\sim}{a}$ and $OB=\underset{\sim}{b}$. M is the mid-point of OA and AN:NB=2:1



a) Express in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

i. BA (1mk)

$BA = BO + OA$
$= \underset{\sim}{a} - \underset{\sim}{b}$

B1

1

ii. BN (1mk)

$BN = \frac{1}{3}BA$
$= \frac{1}{3}\underset{\sim}{a} - \frac{1}{3}\underset{\sim}{b}$

B1

1

iii. ON (2mks)

$ON = OB + BN$
$= \underset{\sim}{b} + \frac{1}{3}\underset{\sim}{a} - \frac{1}{3}\underset{\sim}{b}$
$= \frac{1}{3}\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b}$

B1

B1 Accept alt.

2

b) Given that BX=hBM and OX=KoN, determine the values of h and k. (6mks)

$BM = BO + OM$
$= -\underset{\sim}{b} + \frac{1}{2}\underset{\sim}{a}$
$= \frac{1}{2}\underset{\sim}{a} - \underset{\sim}{b}$

$BX = hBM$
$= h\left(\frac{1}{2}\underset{\sim}{a} - \underset{\sim}{b}\right)$
$= \frac{1}{2}h\underset{\sim}{a} - h\underset{\sim}{b}$

✓ M1 – BX given

$OX = KON$
$= K\left(\frac{1}{3}\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b}\right)$
$= \frac{1}{3}K\underset{\sim}{a} + \frac{2}{3}K\underset{\sim}{b}$

✓ M1 – OX given

Also;
$OX = OB + BX$
$= \underset{\sim}{b} + \frac{1}{2}h\underset{\sim}{a} - h\underset{\sim}{b}$
$= \underset{\sim}{b}(1-h) + \frac{1}{2}h\underset{\sim}{a}$

✓ M1 – OX given

$\therefore$ (i) $\frac{2}{3}k = 1 - h$
and
(ii) $\frac{1}{3}K = \frac{1}{2}h$
$\Rightarrow K = \frac{3}{2}h$

✓ M1 – simultaneous equations

$\frac{2}{3} \times \frac{3}{2}h = 1 - h$
$\therefore h = \frac{1}{2}$

✓ M1 – substitution

$\Rightarrow K = \frac{3}{2} \times \frac{1}{2}$
$= \frac{3}{4}$

✓ A1 – both h and K

6

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22. Three quantities R,S and T are such that R varies directly as S and inversely as the square of T.

a) Given that R= 480 when S=150 and T=5, write an equation connecting R, S and T. (4mks)

Let $R \propto \frac{S}{T^2}$

$\therefore R = \frac{KS}{T^2}$ ✓

$480 = \frac{150K}{25}$ ✓

$\therefore K = 80$ ✓

Equation: $R = \frac{80S}{T^2}$ ✓

B1 - Expression
M1 - substitutions
A1 - value of K
B1 - Equation
4

b) (i) Find the value of R when S=360 and T=1.5. (2mks)

$R = \frac{80 \times 360}{1.5^2}$

$= 12,800$

M1 - substitution
A1
2

(ii) Find the percentage change in R if s increase by 5% and T decreases by 20%. (4mks)

Let $R' = \left[\frac{105}{100} \div \left(\frac{80}{100}\right)^2\right] R$

$= \frac{1.05}{0.64} R$

$= 1.640625R$

$1.640625 - 1$

$= 0.640625$

$\Rightarrow 0.640625 \times 100$

$= 64.06\%$ increase

M1 - Expression
M1 - Change determined
M1 - % convert
A1
4

23. For a C B C inservice training course for teachers, at least four (4) but not more that nine(9) teachers are to be chosen per school. The ratio of the number of male teachers to the number of female teachers must be less than 2:1 and there must be more males than females. If x and y represent the number of male teachers and female teachers respectively:

a) Write down in their simplest form the inequalities that x and y must satisfy. (4mks)

(i) $x+y \geq 4$ (at least 4 teachers) B1

(ii) $x+y \leq 9$ (not more than 9 teachers) B1

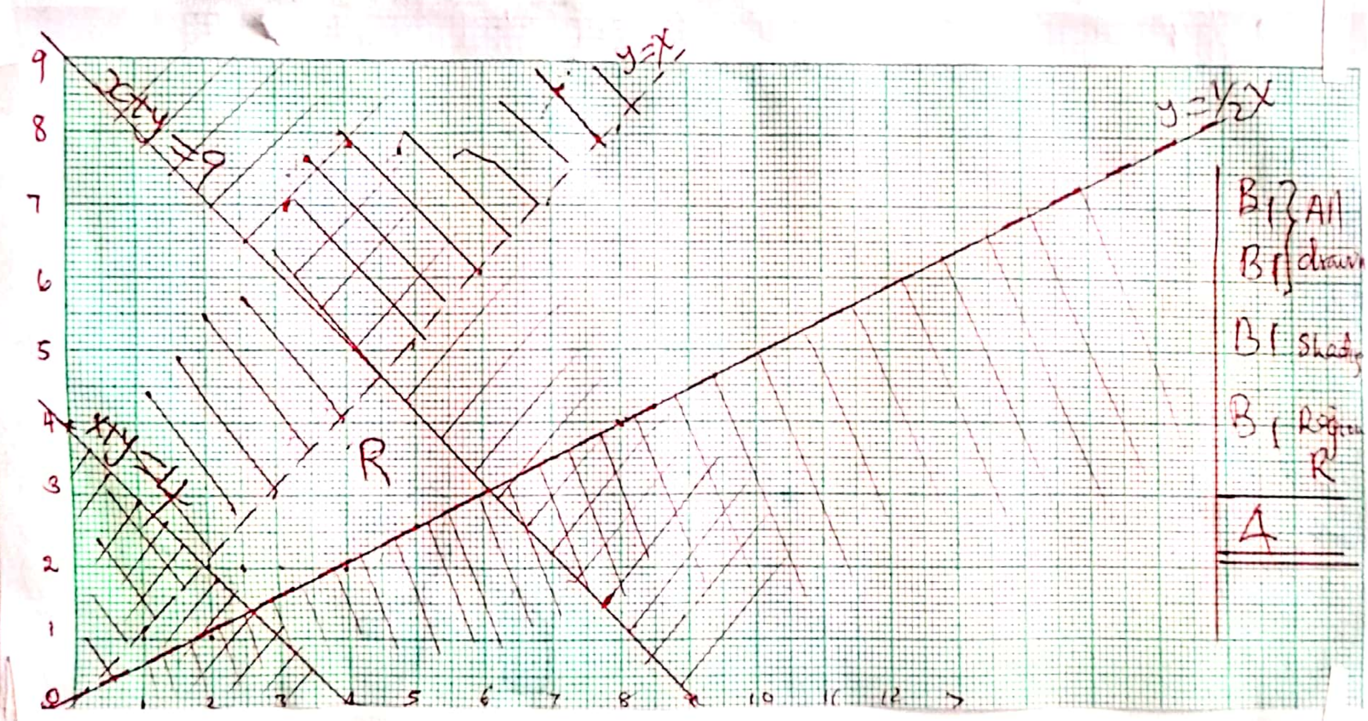
(iii) $x:y < 2:1$ B1

$x < 2y$

(iv) $x > y$ (More males than females) B1

4

b) On the gird provided below, represent the inequalities on the graph. (4mks)



c) Use the graph to determine the composition of the training group of:

i. The largest size. (1mk)

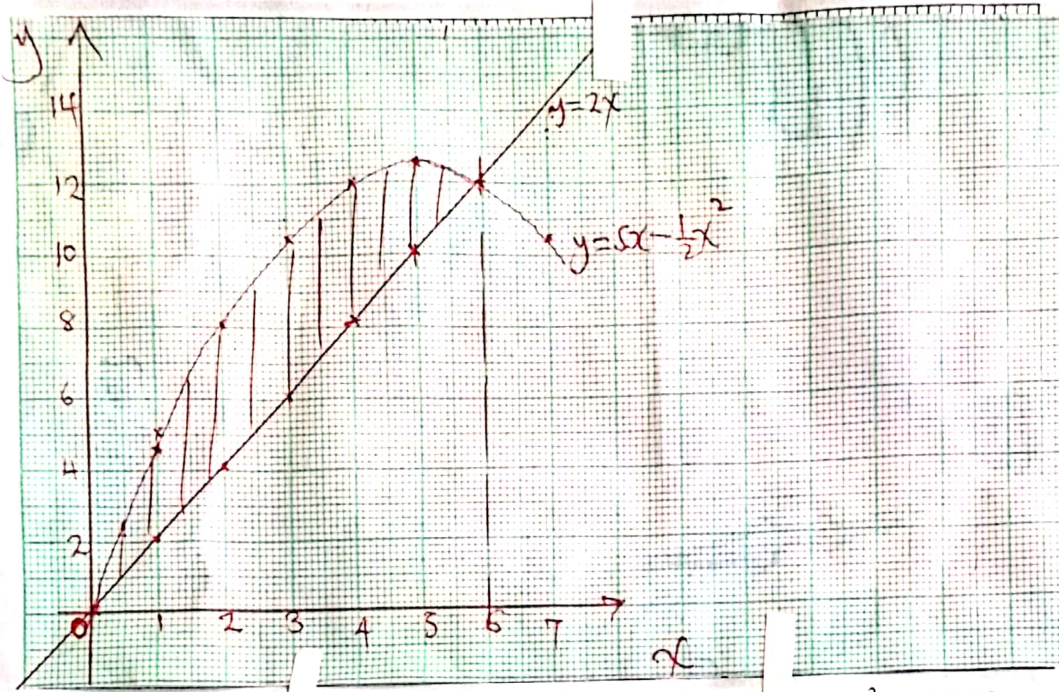
Integral values (5, 4)
5+4 = 9 teachers B1

ii. The smallest size. (1mk)

Integral values (3, 2)
3+2 = 5 teachers. B1

2

24. The equation of a curve is given by $y=5x-\frac{1}{2}x^2$.



P1 - values plotted

C1 - smooth curve

2

a) On the grid provided below, draw the curve of $y=5x-\frac{1}{2}x^2$ for $0\leq x\leq 6$. (3mks)

x	0	1	2	3	4	5	6
y	0	4.5	8	10.5	12	12.5	12

B1 - Table

6 b) By integration, find the area bounded by the curve, the line x=6 and the x-axis. (3mks)

$$\int_0^6 \left(5x-\frac{1}{2}x^2\right)dx$$

$$\left[\frac{5}{2}x^2-\frac{x^3}{6}\right]_0^6$$

$$90-36$$

$$=54 \text{ sq. units}$$

M1 - Expression limits must be given

M1 - simplifying

A1

3

c) On the same grid, draw the line y=2x. (1mk)

Line drawn (must be straight)

B1

d) Determine the area bounded by the curve and the line y=2x. (3mks)

$$A=54-\text{Area of triangle}$$

$$=54-\frac{1}{2}\times 6\times 12$$

$$=54-36$$

$$=18 \text{ sq. units}$$

M1 - Area expressed

M1

A1

3