Marking scheme paper 2

1. Max Area $= 6.95 \times 3.065 = 21.30175$
Min Area $= 6.85 \times 3.055 = 20.92675$ } M1 for both
Actual Area $= 6.9 \times 3.06 = 21.114$

$\frac{21.30175 - 20.92675}{2} \times 100\%$ M1

$= 0.8880$ A1

3

2. $x^2 = \frac{yp - y^2}{p - i}$ M1 removing $\sqrt{}$

$Px^2 - x^2 i = yp - y^2$

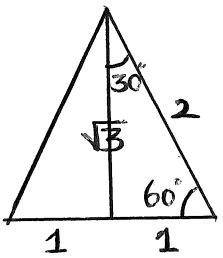
$Px^2 - yp = x^2 i - y^2$

$P(x^2 - y) = x^2 i - y^2$ m1 Factoring out P

$P = \frac{x^2 i - y^2}{x^2 - y}$ A1

3

3.



$\cos 30 = \frac{\sqrt{3}}{2}$

$\sin 60 = \frac{\sqrt{3}}{2}$

$\frac{1 + \frac{\sqrt{3}}{2}}{1 - \frac{\sqrt{3}}{2}}$ m1 expression in surd form

$\frac{\left(1 + \frac{\sqrt{3}}{2}\right)\left(1 - \frac{\sqrt{3}}{2}\right)}{\left(1 - \frac{\sqrt{3}}{2}\right)\left(1 + \frac{\sqrt{3}}{2}\right)}$ m1 Multiplication by conjugate

$\frac{\frac{7}{4} + \sqrt{3}}{\frac{1}{4}} = 7 + 4\sqrt{3}$ A1

4. $x^2 + \frac{3}{4}x - \frac{9}{8} = 0$

$x^2 + \frac{3}{4}x = \frac{9}{8}$

$x^2 + \frac{3}{4}x + \frac{9}{64} = \frac{9}{8} + \frac{9}{64}$ — M1

$\left(x + \frac{3}{8}\right)^2 = \frac{9}{8} + \frac{9}{64}$

$x + \frac{3}{8} = \pm\frac{9}{8}$ — M1

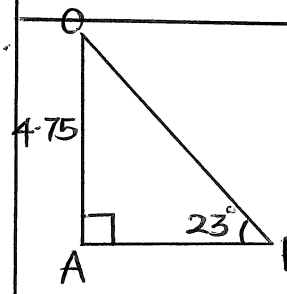
$x = -\frac{3}{8} + \frac{9}{8} = \frac{3}{4}$

Or

$x = -\frac{3}{8} - \frac{9}{8} = -1\frac{1}{2}$ — A1

3

5.



M1 Find XA

M1 Find OA

M1

$5 \times 3 = 2 \times x$ ✓

$x = 7.5$

$\frac{7.5 + 2}{2} = 4.75$ ✓ M1

Radius = 4·75

$AP = \frac{4.75}{\tan 23}$ — M1

$= 11.19\text{cm}$ — A1

4

6 $\log_8(x+5) - \log_8(x-3) = \frac{2}{3}\log_8 8$

$\log_8\left(\frac{x+5}{x-3}\right) = \frac{2}{3}$ — M1

$\frac{x+5}{x-3} = 8^{\frac{2}{3}}$ — M1 index notation

$\frac{x+5}{x-3} = 4$

$x + 5 = 4(x-3)$

$x + 5 = 4x - 12$

$5 + 12 = 4x - x$

$17 = 3x$

$x = 5^2/_3$ — A1

4

7. $N \propto \frac{r^2}{P}$

$N = \frac{kr^2}{P}$

New $r = \frac{115}{100} = 1.15$ ——— m_1

New $P = \frac{64}{100} = 0.64$

$N = \left(\frac{1.15}{\sqrt{0.64}} - 1\right) \times 100$ ——— m_1

$0.6531 \times 100 = 65.31\%$ ——— A_1

3

8. $x^2 + 6x + y^2 - 10y = 2$

Completing Square.....

$x^2 + 6x + \left(\frac{6}{2}\right)^2 + y^2 - 10y + \left(\frac{-10}{2}\right)^2 = 2 + \left(\frac{6}{2}\right)^2 + \left(\frac{-10}{2}\right)^2$ ——— m_1

$(x+3)^2 + (y-5)^2 = 36$ ——— m_1

$(x-h) + (y-k) = r^2$

$h = -3$

$k = 5$

Centre $= (-3, 5)$ ——— A_1 For centre and radius

Radius $= 6$

9. $y = x^2 - 4x + 4$

$\frac{dy}{dx} = 2x - 4$ ——— m_1

At $x = 3$

Gradient $= 2(3) - 4$

$= 2$

At $(3, 1)$

$\frac{y-1}{x-3} = 2$ ——— M_1

$y - 1 = 2(x - 3)$

$y = 2x - 5$ ——— A_1

3

10. $6(1-\sin^2\theta) - \sin\theta - 4 = 0$ M1

$6 - 6\sin^2\theta - \sin\theta - 4 = 0$

$-6\sin^2\theta - \sin\theta + 2 = 0$

$6\sin^2\theta + \sin\theta - 2 = 0$

let $\sin\theta$ be y

$6y^2 + y - 2 = 0$

$6y^2 + 4y - 3y - 2 = 0$

$2y(3y+2) - 1(3y+2) = 0$

$(2y-1)(3y+2) = 0$ M1

$y = \frac{1}{2}$ or $\frac{-2}{3}$ $30^\circ, 150^\circ$ A1

3

11. (a) $1 + 6(2x) + 15(2x)^2 + 20(2x)^3 + 15(2x)^4 + 6(2x)^5 + (2x)^6$ M1

$1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + 64x^6$ A1

(b) $(1.02)^6 = (1+0.02)^6$

$1 + 12(0.01) + 60(0.01)^2 + 160(0.01)^3$ M1

$= 1.12616$ A1

4

12 $\frac{68x + 53y}{x + y} = 62$ M1

$68x + 53y = 62(x+y)$

$68x + 53y = 62x + 62y$

$6x = 9y$

$\frac{x}{y} = \frac{3}{2}$

$x : y = 3 : 2$ A1

2

13. $A = P\left(1 + \frac{r}{100}\right)^n$

$A = 21000\left(1 + \frac{10}{100}\right)^6$ ——— M1

$A = 21000(1.1)^6 = 37202.78$

Compound interest $= 37202.78 - 21000$ ——— M1

Sh. 16202.78 ——— A1

3

14. Tap B takes x hrs

Tap A takes $(x+3)$ hrs

Both $\left(\frac{18}{5}\right)$ hrs

$\frac{1}{x+3} + \frac{1}{x} = \frac{5}{18}$ ——— M1

$\frac{x+x+3}{x(x+3)} = \frac{5}{18}$

$36x + 54 = 5x^2 + 15x$ ——— M1

$5x^2 - 21x - 54 = 0$

$x = \frac{21 \pm \sqrt{441 + 1080}}{10}$

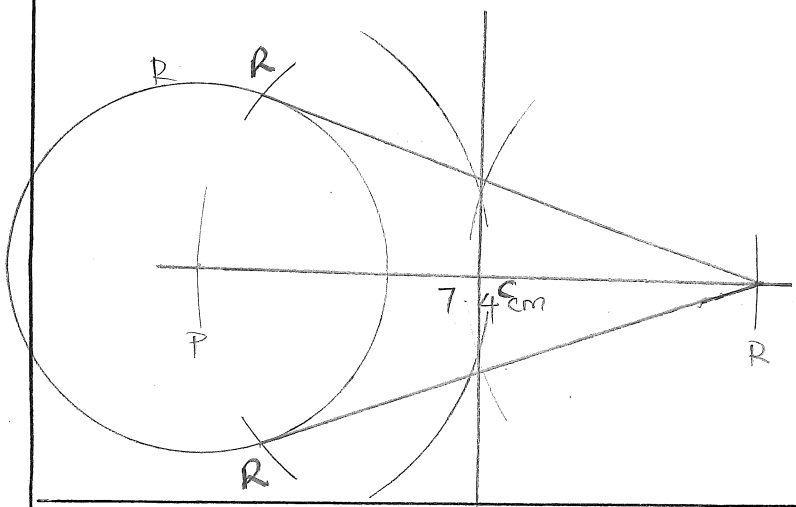
$= 6$ or -1.8 ——— M1 root removed

$x = 6$

$A = 6 + 3 = 9$ hrs ——— A1

4

15.



B1 Length 7.4cm

B1 radius 2.5cm from P

B1 Two tangents

3

16 $y = 2x - 3$

$x^2 - x(2x-3) = -4$ — M1 correct substitution

$x^2 - 2x^2 + 3x + 4 = 0$

$-x^2 + 3x + 4 = 0$

$x^2 - 3x - 4 = 0$

$x^2 - 4x + x - 4 = 0$

$x(x-4) + 1(x-4) = 0$

$(x+1)$ or $(x-4) = 0$ — M1

$x = -1$ or 4

$y = 2(-1) - 3$
$= -2 - 3$
$= -5$

or

$y = 2(4) - 3$
$= 8 - 3$
$= 5$

A1 for the two sets

3

17.a $S_n = \frac{n}{2}(2a+(n-1)d)$

$n = 5$

$560 = \frac{5}{2}(2a+4d)$ ——— M1

$\frac{2}{5} \times 560 = 2a+4d$

$224 = 2a+4d$

$112 = a+2d$ ..... (i)

Cast 5 terms: 41^{st} term $= a+40d$
40^{th} term $= a+39d$
39^{th} term $= a+38d$
38^{th} term $= a+37d$
37^{th} term $= a+36d$
total $= 5a+190d$

$5a+190d = -250$ ..... (ii) ——— M1

Solve (i) and (ii) simultaneously

$a+2d = 112$

$a+38d = -50$

$-36d = 162$ ——— M1

$d = -4.5$ ——— A1

$a+2(4.5) = 112$

$a-9 = 112$

$a = 121$ ——— B1

b) Last term $= a+40d$

$= 121+40(-4.5)$ ——— M1

$= 121-180$

$= -59$ ——— A1

c) $S_n = \frac{n}{2}(2a+(n-1)d)$

$S_{41} = \frac{41}{2}(2\times 21+(41-1)-4.5)$ ——— M1

$S_{41} = \frac{41}{2}(242-180)$ ——— M1

$S_{41} = \frac{41}{2}(62)$

$S_{41} = 41 \times 31$

$S_{41} = 1271$ ——— A1

10

18. a) Taxable income = Basic salary + all allowances.

= 45000 + 13000 + 6000 — M1

= 64000. — A1

b) 1st tax band = $8000 \times \frac{10}{100} = 800$

2nd tax band = $8000 \times \frac{15}{100} = 1200$ } — M1

3rd tax band = $8000 \times \frac{20}{100} = 1600$

4th tax band = $8000 \times \frac{25}{100} = 2000$ } — M1

5th tax band = $8000 \times \frac{30}{100} = 2400$

6th tax band = $24000 \times \frac{35}{100} = 8400$. } — M1

Gross tax = 16400.

Net tax = 16400 − 1166 — M1

= Ksh 15234. — A1

c) Net pay = Taxable income − all deductions

= 64000 − (15234 + 1500 + 900) — M1 sum of deductions

= 64000 − 17634 — M1 subtraction

= Ksh. 46366. — A1

10

19 a) $VO = \sqrt{8^2 - 7.071^2}$ M1

$= \sqrt{64 - 50}$

$= 3.74\text{cm}$. A1

b) i) $\cos\theta = \frac{7.071}{8}$ M1

$= 27.89^\circ$ A1

ii) $\cos\theta = \frac{6}{8}$ M1

$= 51.32^\circ$ A1

iii) $\tan\theta = \frac{3.74}{5}$ M1

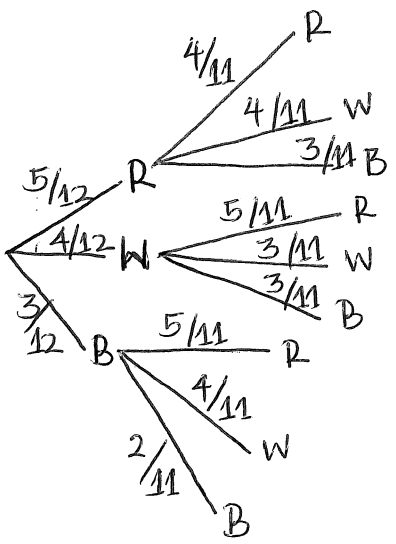
$= 36.80^\circ$ A1

c) $\frac{1}{3} \times 100 \times 3.74$ M1

$= 124.7$ A1

10

20(



RR
RW
RB
WR
WW
WB
BR
BW
BB

B1 1st selection

B1 2nd selection

B1 All possible outcomes

b) i) P(RR) or p(WR) or p(BR)

$\left(\frac{5}{12} \times \frac{4}{11}\right) + \left(\frac{4}{12} \times \frac{5}{11}\right) + \left(\frac{3}{12} \times \frac{5}{11}\right)$ M1

$= \frac{5}{33} + \frac{5}{33} + \frac{5}{44}$

$= \frac{5}{12}$ A1

ii) P(RR) or p(WW) or p(BB)

$\left(\frac{5}{12} \times \frac{4}{11}\right) + \left(\frac{4}{12} \times \frac{3}{11}\right) + \left(\frac{3}{12} \times \frac{2}{11}\right)$ M1

$= \frac{5}{33} + \frac{1}{11} + \frac{1}{22}$

$= \frac{19}{66}$ A1

iii) P(RB) or p(WB) or p(BR) or p(BW) or p(BB)

$\left(\frac{5}{12} \times \frac{3}{11}\right) + \left(\frac{4}{12} \times \frac{3}{11}\right) + \left(\frac{3}{12} \times \frac{5}{11}\right) + \left(\frac{3}{12} \times \frac{4}{11}\right) + \left(\frac{3}{12} \times \frac{2}{11}\right)$ M1

$= \frac{5}{44} + \frac{1}{11} + \frac{5}{44} + \frac{1}{11} + \frac{1}{22}$ m1

$= \frac{5}{11}$ A1

10

21.

X	0	30	60	90	120	150	180	240	240	270	300	330	360
$\sin(x+30)$	0·50	0·87	1·00	0·87	0·50	0	-0·50	-0·87	-1·00	-0·87	-0·50	0	0·50
$2\cos x$	2·00	1·74	1·00	0·00	-1·00	-1·74	-2·00	-1·74	-1·00	0·00	1·00	1·74	2·00

B_1 1st row

B_1 2nd row

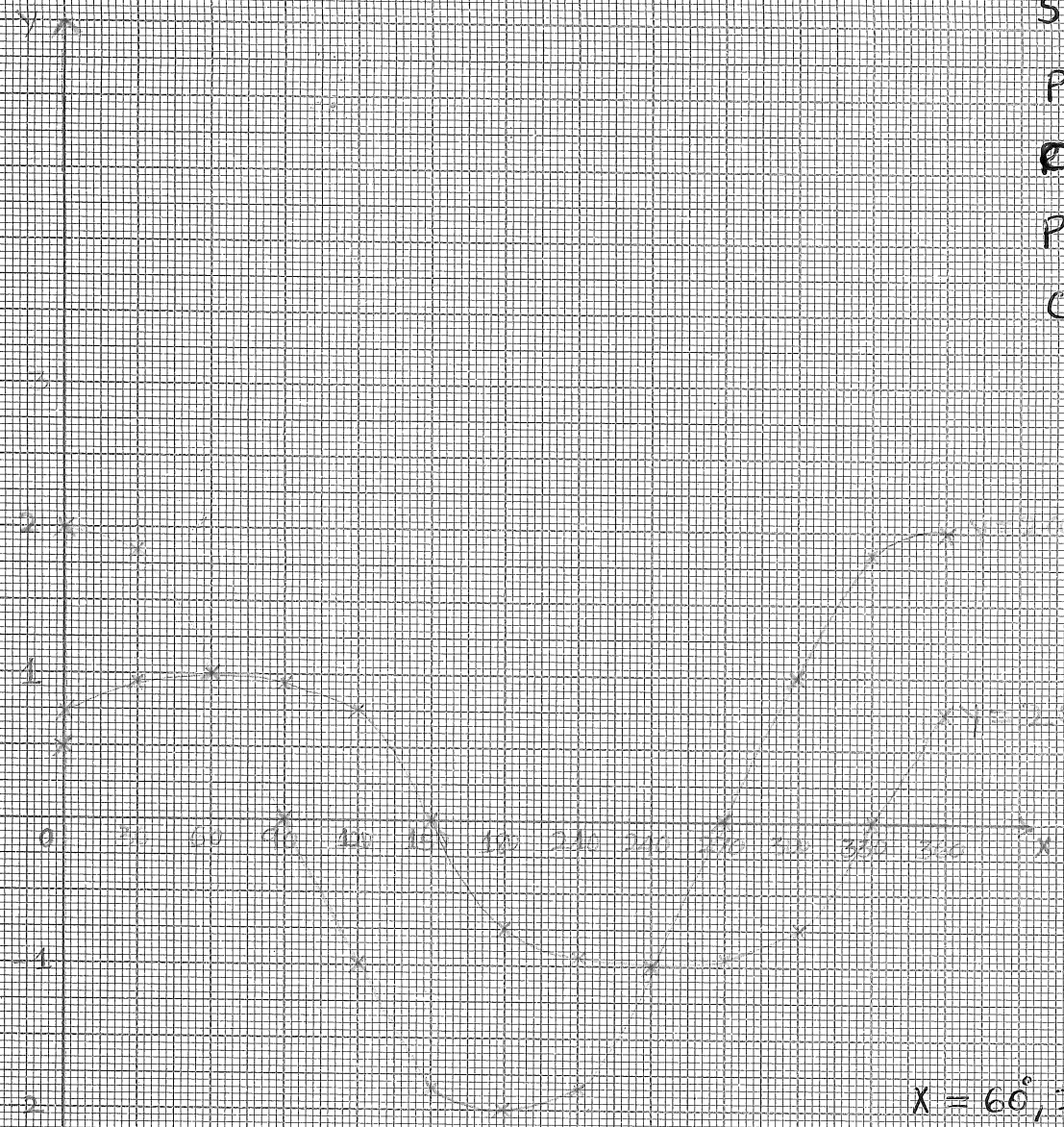
S_1 - Uniform scale

P_1 - Correct plots

C_1 - Correct curve

P_1 - Plots

C_1 - Curve



$X = 60^\circ, 240^\circ, 145^\circ$

therefore, $X + 30^\circ = 175^\circ$

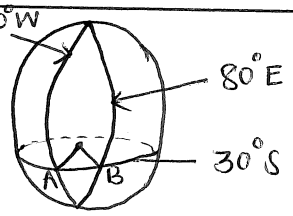
From the graph, $\sin(145^\circ + 30^\circ) = 0.08$

B_2 — values of X (three)

B_1 - if two correct

B_1 - for 0·08

10 mks

22. a) 

$= 20^\circ + 80^\circ$ ——— M1

$= 100^\circ$ ——— A1

b) i) $\frac{100}{360} \times 2 \times \frac{22}{7} \times 6370 \times \cos 30$ ——— M1

$= 9632.13 km$ ——— A1

ii) $100 \times 60 \times \cos 30$ ——— M1

$= 5196.15 nm$ ——— A1

iii) Time difference $= \frac{100 \times 4}{60} = 6\frac{2}{3} hrs$ ——— M1

$= 6 hours \ 40 minutes$ ——— A1

Time at B $= 13.45 + 6hrs \ 40mins$ ——— M1

$= 2025 hrs$ ——— A1

$= 8.25 p.m.$

10

23. a) $a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 50^\circ$ — M1

$a^2 = 36 + 64 - 96(0.6428)$

$a = \sqrt{38.29}$ — M1

$a = 6.187891402$

$a = 6.19$ — A1

b) $\frac{b}{\sin B^\circ} = \frac{a}{\sin A^\circ}$

$\sin B^\circ = \frac{6\sin 50}{6.19}$ — M1 M1

$B^\circ = 47.95^\circ$ — A1

c) $a^2 = d^2 + c^2 - 2dc\cos A$

$2.82^2 = 7^2 + 6^2 - 2 \times 7 \times 6 \times \cos A$ — M1

$7.9524 = 85 - 84\cos A$

$-77.0476 = -84\cos A$

$\cos A = 0.91723$ — m1

$A^\circ = 23.48^\circ$ — A1

d) Area $= \frac{1}{2} dc\sin A$

$= \frac{1}{2} \times 7 \times 6 \times \sin 23.48$ — m1

$= 8.37\,cm^2$ — A1

10

24 a) $V = \int (10t + 1)\, dt$

$V = 5t^2 + t + c$ ____ M_1

When $t = 0$ $V = -4$ hence $-4 = 5(0^2) + 0 + c$ ____ M_1

$c = -4$

$V = 5t^2 + t - 4$ ____ A_1

b) At $t = 3$

$v = 5(3)^2 + 3 - 4$

$= 44 m/s$ ____ B_1

c) At rest $V = 0$

$5t^2 + t - 4 = 0$ ____ M_1

$5t^2 + 5t - 4t - 4 = 0$

$5t(t+1) - 4(t+1) = 0$

$(5t - 4)(t+1) = 0$ ____ M_1

$t = \frac{4}{5}$ or $t = -1$ (ignore)

$t = \frac{4}{5}$ or 0.8 seconds ____ A_1

d) $S = \int_2^4 (5t^2 + t - 4)\, dt$

$\left[\frac{5t^3}{3} + \frac{t^2}{2} - 4t\right]_2^4$ ____ M_1

$\left[\frac{5(4)^3}{3} + \frac{(4)^2}{2} - 4(4)\right] - \left[\frac{5(2)^3}{3} + \frac{(2)^2}{2} - 4(2)\right]$ ____ M_1

$= 98.67 - 7.33$

$= 91.34 m$ ____ A_1

10