MARANDA HIGH SCHOOL

Kenya Certificate of Secondary Education

PRE-MOCK EXAMINATIONS 2022

121/2 **MATHEMATICS** Form 4

June 2022 – TIME $2\frac{1}{2}$Hours

Name: Marking Scheme.......................................Adm No: ....

Class: ....Candidate's Signature: .... Date: 29/6/2022.

Instructions to Candidates

(a) Write your name, admission number and class in the spaces provided above.

(b) Sign and write the date of examination in the spaces provided above.

*(c) This paper consists of two sections; **Section I and Section II.***

*(d) Answer **all** the questions in **Section I** and only **five** questions from **Section II***

(e) Show all the steps in your calculations, giving your answers at each stage in the spaces provided below each question

(f) Marks may be given for correct working even if the answer is wrong.

(g) Non-programmable silent electronic calculators and KNEC Mathematical tables may be used, except where stated otherwise.

(h) This paper consists of 19 printed pages.

(i) Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

For Examiner's Use Only

Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

Section II

17	18	19	20	21	22	23	24	Total

Grand Total	

SECTION I (50 marks)

Answer all questions in this section in the spaces provided

1. An empty tank of capacity 99 792 litres is to be filled with water using a cylindrical pipe of diameter 0.042 m. The rate of flow of water from the pipe is 6 m/s. Find the time in hours it would take to fill up the tank. (take $\pi = \frac{22}{7}$) (3 marks)

$99792l = 99.792m^3$ M1 - vol. per second (compatible units)

$V = \pi r^2 h$

$\frac{22}{7} \times 0.021^2 \times 6t = 99.792$ ✓ M1 exp. for time (

$t = \frac{12000s}{3600}$ ✓

$3\frac{1}{3}$hrs. ✓ A1 time in hrs (C.A.O)

2. The first term of a Geometric Progression (G.P) is 5. The common ratio of the G.P. is 4. The product of the last two terms of the G.P. is 25 600. Determine the number of terms in the G.P. (4 marks)

Last term $= ar^{n-1} = 5 \times 4^{n-1}$
2nd last term $= ar^{n-2} = 5 \times 4^{n-2}$ — B1

$5 \times 4^{n-1} \times 5 \times 4^{n-2} = 25600$ — M1

$4^{2n-3} = 4^5$ / $2^{4n-6} = 2^{10}$

$n = 4$ terms — A1

3. The expression $dx^2 - 56x + 16$ is a perfect square, where d is a constant. Find the value of d. (2 marks)

$4ac = b^2$

$4 \times d \times 16 = (-56)^2$ — M1

$d = \frac{3136}{64}$

$d = 49$ — A1

4. Make v the subject of the formula in $s = \frac{cv}{\sqrt{dv^2 - f}}$ (3 marks)

$s^2 = \frac{c^2v^2}{dv^2 - f}$ ✓

$s^2dv^2 - s^2f = c^2v^2$

$s^2dv^2 - c^2v^2 = s^2f$

$v^2(s^2d - c^2) = s^2f$ ✓

$v^2 = \frac{s^2f}{s^2d - c^2}$

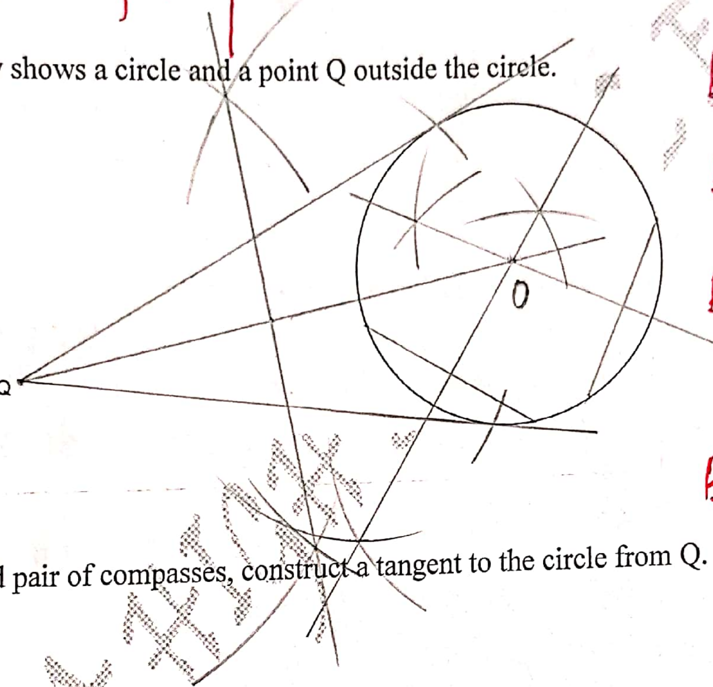
$v = \pm\sqrt{\frac{s^2f}{s^2d - c^2}}$

M_1 removal of roots correctly

M_1 – collecting terms in v and factorisation

A_1 – accuracy

5. The figure below shows a circle and a point Q outside the circle.



B_1 – Centre O.vly located

B_1 ⊥ bisector of OQ

B_1 Arc showing correct position of contact of circle and tangent

B_1 √ tangent drawn

Using a ruler and pair of compasses, construct a tangent to the circle from Q. (4 marks)

6. Four quantities W, X, Y and Z are such that W varies directly as the square root of X and inversely as the square of the sum of Y and Z. Quantity X is decreased by 19% while quantities Y and Z are each increased by 16%. Find the corresponding percentage change in W correct to 2 decimal places. (4 marks)

$W = \frac{k\sqrt{X}}{(Y+Z)^2}$

$W_{new} = \frac{k\sqrt{0.81X}}{(1.16Y + 1.16Z)^2}$

$= \frac{0.9k\sqrt{X}}{1.3456(Y+Z)^2}$

% change in W $= \left(\frac{1125}{1682} - 1\right) \times 100\%$

$= 33.12\%$ decrease

B_1 – original W

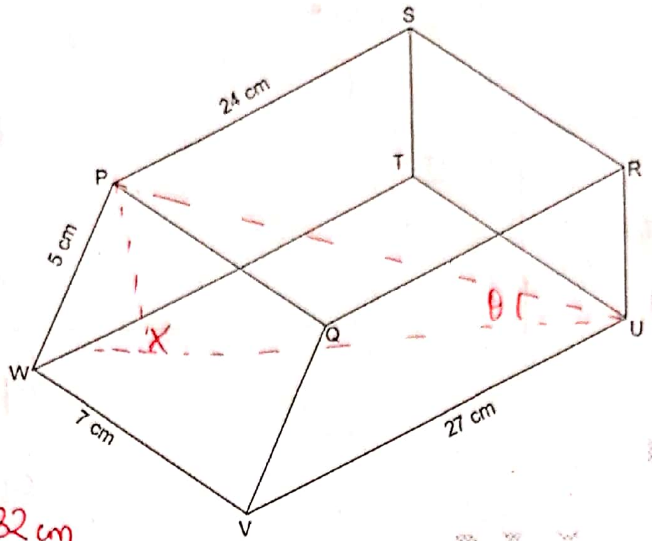
M_1 exp. for new W

M_1 exp. for % change

A_1 accuracy

33.12%

7. The figure below represents a prism PQRSTUVW of length 7 cm. The cross section QRUV of the prism is a trapezium in which VU = 27 cm, QR = 24 cm, QV = 5 cm and $\angle VUR = \angle QRU = 90^\circ$.



$RU = PX = \sqrt{5^2 - 3^2} = 4\text{ cm}$

$PR = \sqrt{24^2 + 7^2} = 25\text{ cm}$

$PU = \sqrt{25^2 + 4^2} = 25.32\text{ cm}$

Calculate correct to 1 decimal place the angle between the line UP and the plane VUTW. (3 marks)

$\sin\theta = \frac{4}{25.32\text{ cm}}$

$\theta = 9.1^\circ$

M_1 – for RU or PR

M_1 exp. for angle.

A_1 accuracy.

8. The cash price of a refrigerator is Ksh 45 000. A customer bought the refrigerator on hire purchase terms by paying a deposit of Ksh 18 000 followed by 15 equal monthly instalments of Ksh 2 300 each. Annual interest, compounded quarterly, was charged on the balance for the period of 15 months. Determine, correct to 1 decimal place, the rate of interest per annum. (4 marks)

$H.P.P = 18000 + (15 \times 2300)$
$52\,500$

$52500 - 18000 = 45000 - 18000\left(1 + \frac{r}{400}\right)^5$ ✓

$34500 = 27000\left(1 + \frac{r}{400}\right)^5$

$1.278 = \left(1 + \frac{r}{400}\right)^5$ ✓

$1.0503 = 1 + \frac{r}{400}$

$r = 20.1\%$ ✓

B_1 – amount repaid 34500/=

M_1 expression for rate.

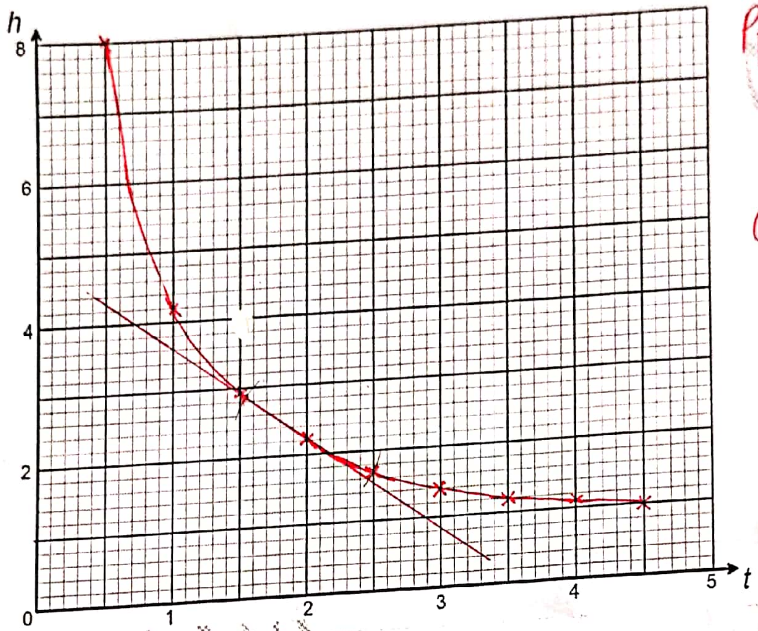
M_1 finding 5th root on both sides

A_1 – accuracy.

9. The table below shows the values of t and the corresponding values of h for a given relation.

(a) On the grid provided, draw a graph to represent the information on the table given. (2 marks)

t	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
h	8.0	4.2	2.9	2.2	1.7	1.4	1.2	1.1	1.0



P1 - all points c/ly plotted

C1 smooth curve

(b) Use the graph to determine, correct to 1 decimal place, the rate of change of h at $t = 2$ (2 marks)

(0.7, 4)
(3, 1.8)

B1 c/ly tangent drawn at t=2

Rate of change at t=2 $\frac{\Delta h}{\Delta t}$

$= \frac{1.8 - 4}{3 - 0.7}$

$= \frac{-2.2}{2.3}$

-1.0

B1 gradient

10. The equation of a trigonometric wave is $y = 2\cos(bx - 60)^{0}$. The wave has a period of 120^{0}.

(a) Determine the value of b. (1 mark)

$\frac{360}{b} = 120$

$b = 3$ ✓ B_1

(b) Deduce the phase angle of the wave. (1 mark)

Phase ange = $\underline{60^{\circ}}$ ✓ B_1

11. A point R is 1800 nm to the East of a point $T(30^{0}S, 170^{0}E)$. Find the longitude of R to the nearest degree (3 marks)

$60 \times \theta \cos 30 = 1800$ ✓ M_1

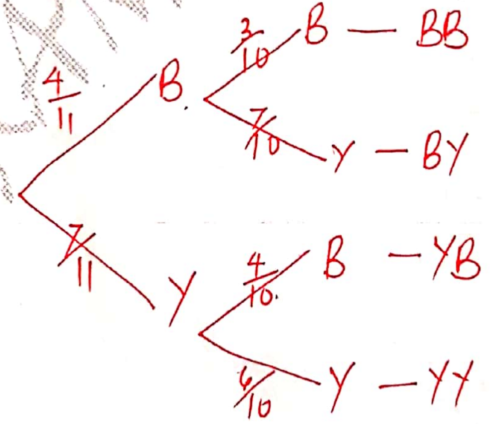
$\theta = 34.64^{\circ}$

$360 - (170 + 34.64)$ — M_1

$155^{\circ}W$ — A_1

12. A box contains 4 blue beads and 7 yellow beads. The beads are identical except for the colours. Two balls are picked at random without replacement.

(a) Draw a tree diagram to show all the possible outcomes. (1 mark)



B_1 – all branches correct.

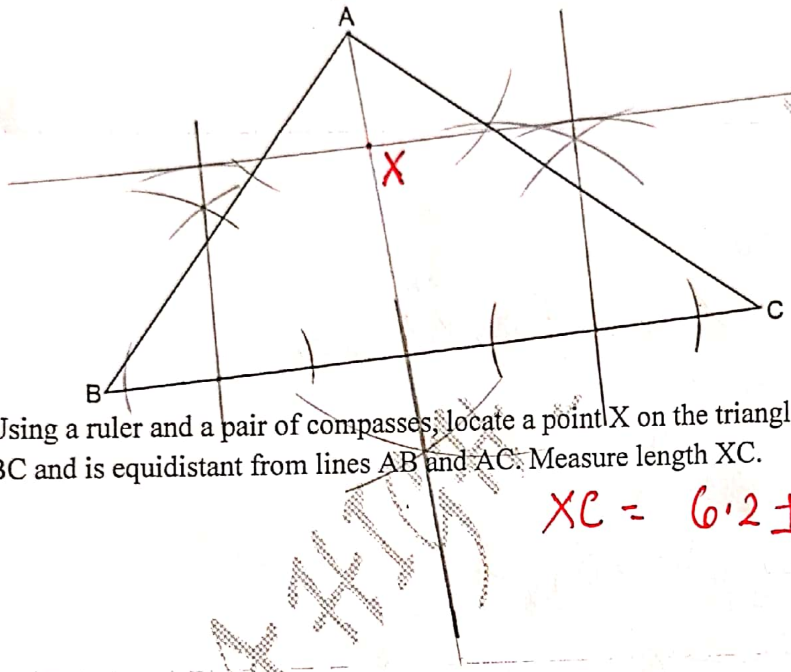
(b) Determine the probability that the balls picked are of different colours. (2 marks)

P(BY) or P(YB)

$\left(\frac{4}{11}\times\frac{7}{10}\right)+\left(\frac{7}{11}\times\frac{4}{10}\right)$ ✓ M1

$\frac{28}{55}$ ✓ A1

13. The figure below shows triangle ABC.



B1 √ construction of a straight line 3 cm from and parallel to BC.
⊥ at from BC must be seen.
B1 ∠ bisector of ∠BAC

Using a ruler and a pair of compasses, locate a point X on the triangle such that X is 3 cm from line BC and is equidistant from lines AB and AC. Measure length XC. (3 marks)

XC = 6.2 ± 1 cm B1 for XC.

14. The position vectors of points R, S and T are $\overrightarrow{OP}=3i-j+1.5k$, $\overrightarrow{OS}=6i-2.5j+3k$ and $\overrightarrow{OT}=4i-1.5j+2k$. Show that R, S and T are collinear points. (3 marks)

$\underset{\sim}{RS}=\begin{pmatrix}6\\-2.5\\3\end{pmatrix}-\begin{pmatrix}3\\-1\\1.5\end{pmatrix}=\begin{pmatrix}3\\-1.5\\1.5\end{pmatrix}$

$\underset{\sim}{ST}=\begin{pmatrix}4\\-1.5\\2\end{pmatrix}-\begin{pmatrix}6\\-2.5\\3\end{pmatrix}=\begin{pmatrix}-2\\1\\-1\end{pmatrix}$ B1

$\underset{\sim}{RS}=k\underset{\sim}{ST}$

$\begin{pmatrix}3\\-1.5\\1.5\end{pmatrix}=k\begin{pmatrix}-2\\1\\-1\end{pmatrix}$

$k=-\frac{3}{2}$

$\Rightarrow \underset{\sim}{RS}=-1.5\underset{\sim}{ST}$ ✓ B1

$\therefore \underset{\sim}{RS} \parallel \underset{\sim}{ST}$

S is a common point

Hence R, S & T are collinear B1

15. In a transformation an object of area $y\ cm^2$ is mapped on to an image whose area is $9y\ cm^2$. Given that the matrix of the transformation is $\begin{pmatrix} 2y & 1 \\ y+1 & y \end{pmatrix}$, find the possible values of y. (3 marks)

$2y^2 - 1(y+1) = \frac{9y}{y}$ ✓ M_1 equating det to a.s.f.

$2y^2 - y - 1 = 9$

$2y^2 - y - 10 = 0$

$2y^2 - 5y + 4y - 10 = 0$

$y(2y-5) + 2(2y-5) = 0$

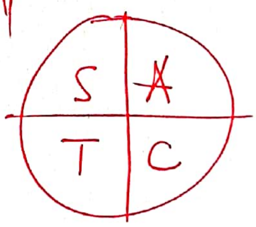
$(y+2)(2y-5) = 0$ ✓ M_1 Correct attempt to solve

$y = -2$ or $y = 2\frac{1}{2}$ ✓ A_1 both roots correct

16. Given that $\sin(y+20)^0 = -0.7660$, find y to the nearest degree, for $0^0 \le y \le 360^0$ (3 marks)

$\sin(y+20) = \sin 230°, \sin 310°$ M_1 B_1 – acute angle 50°

$\Rightarrow y = 210°, 290°$ — A_1



SECTION II (50 marks)

Answer only five questions from this section in the spaces provided

17. Pump R can fill an empty water tank in $6\frac{1}{4}$ hours while pump S can fill the same tank in $7\frac{1}{2}$ hours. On a certain day, when the tank was empty, both pumps were opened for $1\frac{7}{8}$ hours.

a) Determine the fraction of the tank that was still empty at the end of the $1\frac{7}{8}$ hours. (4 marks)

In 1h R fills $\frac{4}{25}$ & S fills $\frac{2}{15}$

In 1h both fill $\frac{4}{25} + \frac{2}{15} = \frac{22}{75}$ ✓ M1

In $1\frac{7}{8}$h both fill $\frac{15}{8} \times \frac{22}{75} = \frac{11}{20}$ ✓ M1

Fraction empty $= 1 - \frac{11}{20}$ ✓ M1

$\frac{9}{20}$ ✓ A1

b) Pump R was later opened alone to completely fill the tank. Determine the time it took pump R to fill the remaining fraction of the tank. (2 marks)

$\frac{4}{25}$ is filled in 1hr

$\frac{9}{20}$ is filled in $\frac{9}{20} \times \frac{25}{4} \times 1$ ✓ M1

2hrs 48min 45sec / $2\frac{13}{16}$h ✓ A1

c) The two pumps R and S are operated by different proprietors. Water from the full tank was sold for Ksh 31 500. The money was shared between the two proprietors in the ratio of the quantity of water supplied by each.

Determine the amount of money received by the proprietor of pump R. (4 marks)

Fraction of tank filled. S

1h $= \frac{2}{15}$

$1\frac{7}{8} = \frac{15}{8} \times \frac{2}{15} = \frac{1}{4}$

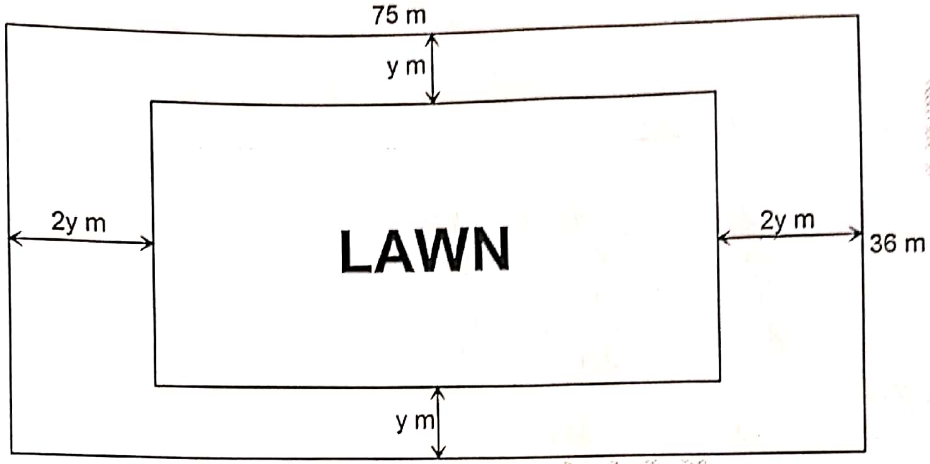
Fraction filled by R $= 1 - \frac{1}{4}$ ✓ M1

$\frac{3}{4}$ ✓ A1

Amount for R $= \frac{3}{4} \times 31500$ ✓ M1

Ksh 23,625 ✓ A1

18. A rectangular plot measures 75 m by 36 m. A lawn, rectangular in shape, is situated inside the plot with a path surrounding it as shown in the figure below.



The width of the path is $y\,m$ between the lengths of the lawn and those of the plot and $2y\,m$ between the widths of the lawn and those of the plot.

a) Form and simplify an expression in y for the area of the:

(i) Lawn; (2 marks)

Area $= (75-4y)(36-2y)$ — M_1

$2700-144y-150y+8y^2$

$2700-294y+8y^2$ — A_1

(ii) Path. (1 mark)

$(75\times 36)-(2700-294y+8y^2)$

$2700-2700+294y-8y^2$

$294y-8y^2$ — B_1

P : L
3 : 2

b) The ratio of the area of the path to the area of the lawn 3:2

(i) Form an equation in y and hence solve for y. (4 marks)

$(294y - 8y^2) = \frac{3}{2}(2700 - 294y + 8y^2)$ ✓ M1 equating v/y

$588y - 16y^2 = 8100 - 882y + 24y^2$

✓ M1 quad eqn. in 3 terms

$40y^2 - 1470y + 8100 = 0$

$4y^2 - 147y + 810 = 0$

$y = \frac{147 \pm \sqrt{(-147)^2 - 4 \times 2 \times 810}}{2 \times 4}$ ✓ M1 √ attempt to solve

$y = \frac{147 \pm 123}{8}$

$y = 33.75m$ or $y = 3m$ — A1 both values correct

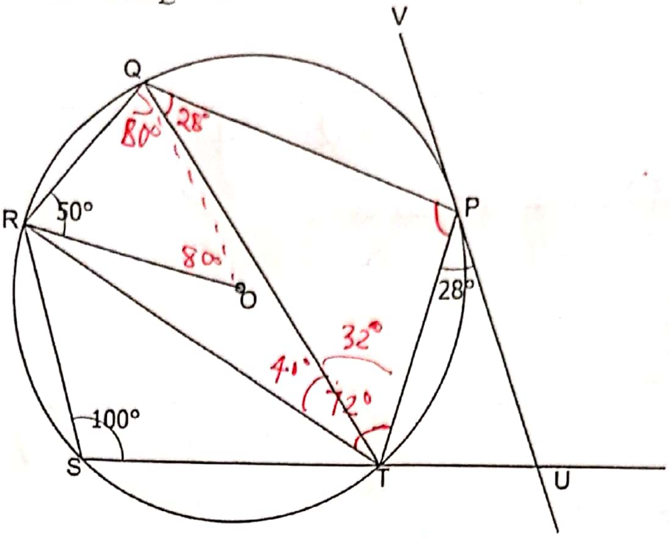
(ii) Determine the perimeter of the lawn. (3 marks)

length $= 75 - 4 \times 3 = 63m$
width $= 36 - 2 \times 3 = 30m$ } B1 for either L or w

$P = 2(63 + 30)$ ✓ M1

$= \underline{186m}$ ✓ A1

19. In the figure below, points P, Q, R, S and T lie on the circumference of a circle centre O. Line UPV is a tangent to the circle at A. Chord ST of the circle is produced to intersect with the tangent at U. Angle UPT = 28^0, $\angle RST = 100^0$ and $\angle ORQ = 50^0$.



a) Determine the size of :

(i) $\angle PTR$ (3 marks)

$\angle PQT = 28°$ (alternate segment theorem) — B_1

$\angle RQT = 180 - 100 = 80°$ (opp. ∠s of cyclic quad add to 180°) — B_1

$\angle PTR = 180 - (80 + 28)$

$= 72°$ — B_1

(ii) $\angle PTQ$ (3 marks)

$\angle ROQ = 180 - (50 \times 2) = 80°$ — B_1 / OR $\angle QPT = 120°$.

$\angle QTR = \frac{80}{2} = 40°$ — B_1

$\angle PTQ = 72 - 40 = 32°$ — B_1

b) Given that PQ = 6 cm, ST = 5.4 cm and TU = 3.5 cm. Calculate correct to 1 decimal place:

(i) The radius of the circle (2 marks)

$\frac{6}{\sin 32°} = 2R$ — M_1

$R = 5.7$ cm — A_1 — 1d.p.

(ii) The length of line PU (2 marks)

$(PU)^2 = TU \cdot SU.$

$PU = \sqrt{3.5(3.5 + 5.4)}$ — M_1

$= 5.6$ cm — A_1 — 1d.p

20. The table below shows income tax rates in a certain year.

Monthly taxable income in Kenya Shillings	Tax rates
0 – 13 458	10%
13 459 – 26 351	15%
26 352 – 39 244	20%
39 245 – 52 137	25%
52 138 and above	30%

In the year, the monthly earnings of Kaliech were as follows:

Basic salary Ksh 75 500
House allowance Ksh 13 600

Kaliech contributes 12.5% of his basic salary to a pension scheme. This contribution is exempted from taxation. He is entitled to a personal tax relief of Ksh 2 400 per month.

Calculate:

a) Kaliech's monthly taxable income (2 marks)

$\frac{87.5}{100} \times 75500 + 13600$ — M1

Ksh 79 662.50 — A1

OR $75500 + 13600 - \frac{12.5}{100} \times 75500$

Ksh 79 662.50

b) The tax payable by Kaliech that month. (6 marks)

1st slab $13458 \times \frac{10}{100} = 1345.8$ M1 (exp)

2nd slab $12893 \times \frac{15}{100} = 1933.95$

3rd slab $12893 \times \frac{20}{100} = 2578.6$ M1 (exp)

4th slab $12893 \times \frac{25}{100} = 3223.25$

5th slab $27525.5 \times \frac{30}{100} = 8257.65$ M1

17339.25 — A1

$17339.25 - 2400$ — M1

Ksh 14 939.25 — A1

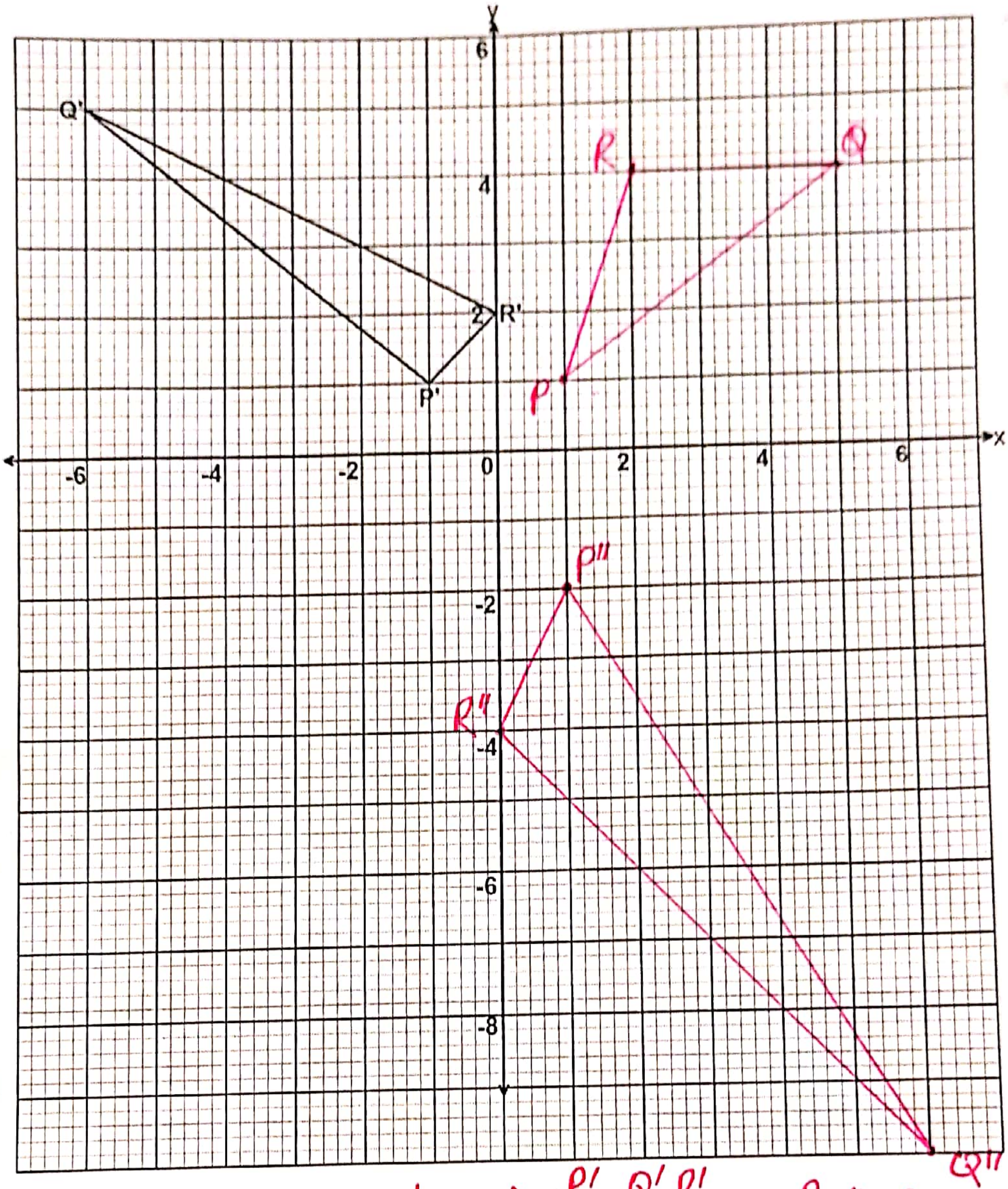
c) Kaliech's net pay that month. (2 marks)

$75500 + 13600 - \left(\frac{12.5}{100} \times 75500 + 14939.25\right)$ — M1

Ksh 64,723.25 — A1

OR $79662.50 - 14939.25$

21. The vertices of the triangle shown on the grid are $P'(-1,1), Q'(-6,5)$ *and* $R'(0,2)$. Triangle $P'Q'R'$ is the image of triangle PQR under a transformation whose matrix is $\begin{pmatrix}-2 & 1\\ 1 & 0\end{pmatrix}$.



a) Find the coordinates of triangle PQR. (4 marks)

$\det = (-2\times 0) - (1\times 1) = -1$

Inverse $= -1\begin{pmatrix}0 & -1\\ -1 & -2\end{pmatrix}$ ✓M_1

$\begin{pmatrix}0 & 1\\ 1 & 2\end{pmatrix}$ ✓A_1

$\begin{pmatrix}0 & 1\\ 1 & 2\end{pmatrix}\begin{array}{c}\begin{matrix}P' & Q' & R'\end{matrix}\\ \begin{pmatrix}-1 & -6 & 0\\ 1 & 5 & 2\end{pmatrix}\end{array} = \begin{array}{c}\begin{matrix}P & Q & R\end{matrix}\\ \begin{pmatrix}1 & 5 & 2\\ 1 & 4 & 4\end{pmatrix}\end{array}$ — M_1

$P(1,1)$ $Q(5,4)$ $R(2,4)$ — A_1

OR $\begin{pmatrix}-2 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}a & b & c\\ d & e & f\end{pmatrix} = \begin{pmatrix}-1 & -6 & 0\\ 1 & 5 & 2\end{pmatrix}$ ✓ M_1

$P(1,1)$ $Q(5,4)$ $R(2,4)$

$-2a+d=-1$, $-2b+e=-6$, $-2c+f=0$

$a=1$, $b=5$, $c=2$ —— A_1

$-2(1)+d=-1$, $-2(5)+e=-6$, $-2(2)+f=0$ — M_1

b) Triangle $P''Q''R''$ is the image of triangle $P'Q'R'$ under a transformation matrix $\begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$.

Determine the coordinates of triangle $P''Q''R''$. (2 marks)

$$\begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \begin{array}{c} \begin{array}{ccc} P' & Q' & R' \end{array} \\ \begin{pmatrix} -1 & -6 & 0 \\ 1 & 5 & 2 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} P'' & Q'' & R'' \end{array} \\ \begin{pmatrix} 1 & 6 & 0 \\ -2 & -10 & -4 \end{pmatrix} \end{array} \checkmark M_1$$

$P''(1,-2)$ $Q''(6,-10)$ $R''(0,-4)$. ✓ A_1

c) On the same grid provided, draw triangles PQR and $P''Q''R''$ (2 marks)

B_1 ΔPQR drawn

B_1 $\Delta P''Q''R''$ drawn.

d) Determine a single matrix that maps PQR onto $P''Q''R''$ (2 marks)

$$\begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix} \checkmark M_1$$

$$\begin{pmatrix} 2 & -1 \\ -2 & 0 \end{pmatrix} \checkmark A_1$$

OR

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{array}{c} \begin{array}{ccc} P & Q & R \end{array} \\ \begin{pmatrix} 1 & 5 & 2 \\ 1 & 4 & 4 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} P'' & Q'' & R'' \end{array} \\ \begin{pmatrix} 1 & 6 & 0 \\ -2 & -10 & -4 \end{pmatrix} \end{array}$$

$a+b=1$ | $c+d=-2$

$5a+4b=6$ | $5c+4d=-10$

$5a+5b=5$ | $5c+5d=-10$

$5a+4b=6$ | $5c+4d=-10$

$b=-1$ | $d=0$.

$a=1+1=2$ | $c=-2$.

M1 2 pairs of eqns and ✓ attempt to solve

$$\text{Matrix} = \begin{pmatrix} 2 & -1 \\ -2 & 0 \end{pmatrix}$$

A_1

22. Workers in an institution commute from their homes to the institution. The table below shows the distances in kilometres, covered by the workers.

Distance (km)	4–6	7–9	10–12	13–15	16–18	19–21	22–24
Number of workers	4	15	21	k 18	13	9	5
c.f	4	19	40	58	71	80	85

The mean distance covered was 13.4 km.

a) Determine the value of k and hence the standard deviation of the distances correct to 2 decimal places. (6 marks)

Class	f	x	fx	fx^2
4–6	4	5	20	100
7–9	15	8	120	960
10–12	21	11	231	2541
13–15	K	14	14K	3528
16–18	13	17	221	3757
19–21	9	20	180	3600
22–24	5	23	115	2645
	85		✓ B1 (fx column).	17131

$S.d = \sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2}$ B1 fx^2

$= \sqrt{\frac{17131}{85} - 13.4^2}$ ✓ without K or equivalent

$= 4.69$ ✓ — 2d.p. M1

A1

$\bar{x} = \frac{\sum fx}{\sum f} \Rightarrow \frac{887+14K}{67+K} = 13.4$ ✓M1

$K = \frac{10.8}{0.6} = 18$ ✓ A1

b) Calculate, correct to 2 decimal places, the interquartile range of the distances. (4 marks)

$Q_1 = \frac{1}{4} \times 85 = 21.25^{th}$

$9.5 + \left(\frac{21.25-19}{21}\right) \times 3$

9.821

$Q_3 = \frac{3}{4} \times 85 = 63.75^{th}$

$15.5 + \left(\frac{63.75-58}{13}\right) \times 3$

$= 16.827$

$Q_3 - Q_1 = 16.827 - 9.821 = 7.01$ 2d.p.

B1 Cf (may be implied in the working)

M1 any Q_1 or Q_3 expression.

M1 difference.

A1 accuracy

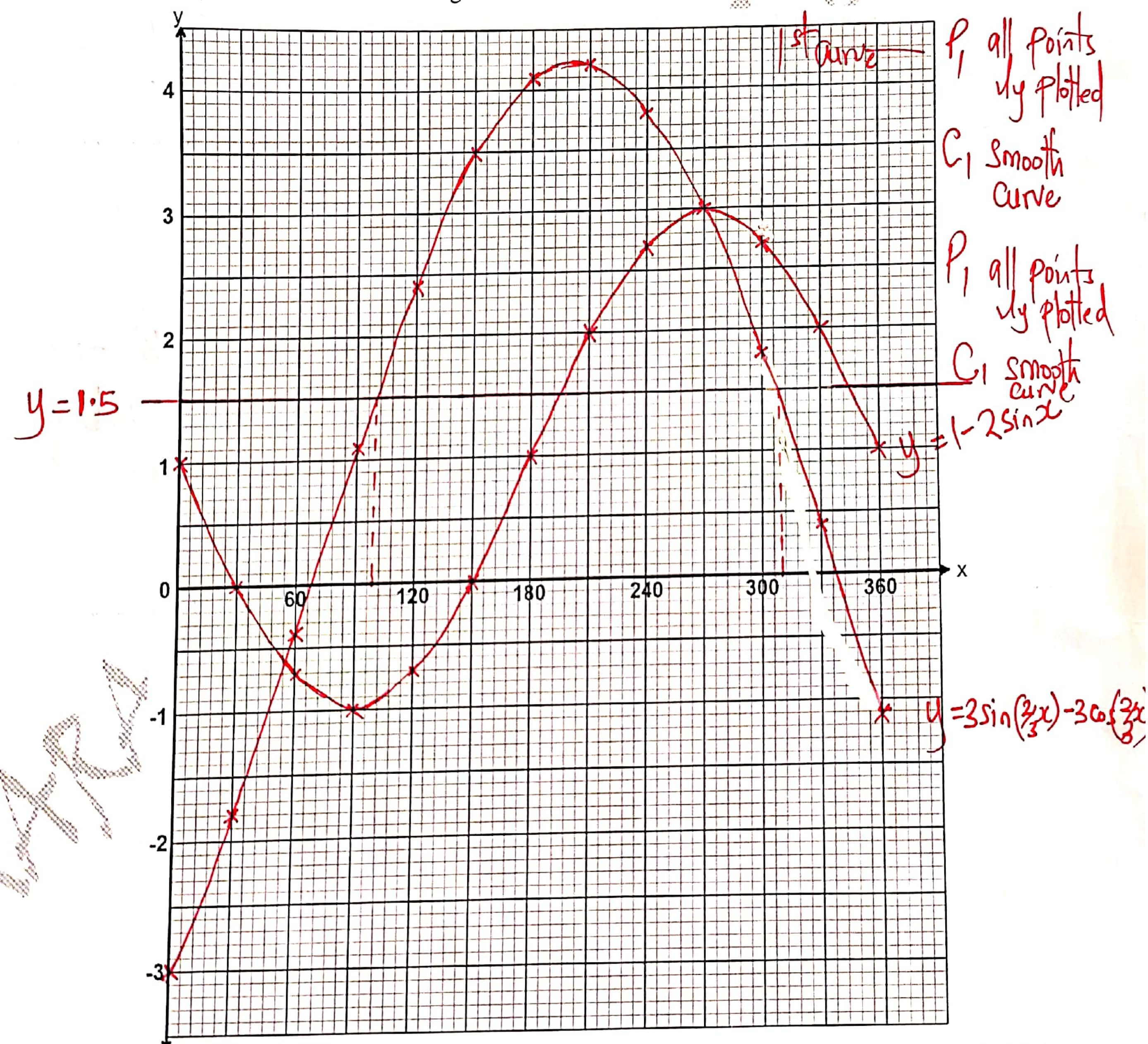
23..

a) Complete the table below giving the values correct to 1 decimal place. (2 marks)

x^0	0	30	60	90	120	150	180	210	240	270	300	330	360
$y=3\sin\left(\frac{2}{3}x\right)-3\cos\left(\frac{2}{3}x\right)$	−3	−1.8	-0.4	1.1	2.4	3.5	4.1	4.2	3.8	3	1.8	0.4	−1.1
$y=1-2\sin x$	1	0	-0.7	−1	-0.7	0	1	2	2.7	3.0	2.7	2	1

allow B_1 for any 5 B_2 - all

b) On the grid provided and using the same axis, draw the graphs of $y=3\sin\left(\frac{2}{3}x\right)-3\cos\left(\frac{2}{3}x\right)$ and $y=1-2\cos x$ for the range $0^0 \le x \le 360^0$ (4 marks)



c) Using the graphs in part (b):

(i) Find the values of x for which $\sin\left(\frac{2}{3}x\right)=\frac{1}{2}+\cos\left(\frac{2}{3}x\right)$ (2 marks)

$\sin\left(\frac{2}{3}x\right)-\cos\left(\frac{2}{3}x\right)=\frac{1}{2}$

$3\sin\left(\frac{2}{3}x\right)-3\cos\left(\frac{2}{3}x\right)=\frac{3}{2}$.

$3\sin\left(\frac{2}{3}x\right)-3\cos\left(\frac{2}{3}x\right)=y$

$y=\frac{3}{2}$

$x = 97^\circ$ to 101° or 307° to 311° — B_1

$y=\frac{3}{2}$ or 1.5 — B_1
can be implied

(ii) Determine the range of x which $3\sin\left(\frac{2}{3}x\right)-3\cos\left(\frac{2}{3}x\right)>1-2\cos x$ (2 marks)

$54^\circ < x < 270^\circ$ — B_2

allow B_1 for one inequality correct

24. A particle moves along a straight line such that its displacement s *metres* after t *seconds* is given by $s = t^3 - pt^2 + qt + 4$. Given that its velocity, v after 5 seconds was 28 m/s and its acceleration, a after 5 seconds was 20m/s^2.

a) Determine the value of p and q (5 marks)

$V = \frac{ds}{dt} = 3t^2 - 2pt + q$ M1 ✓ differentiation.

$3(5)^2 - 2p(5) + q = 28$

$75 - 10p + q = 28$ M1 Sub. 5 and equating to 28.

$-10p + q = -47$

$a = \frac{dv}{dt} = 6t - 2p$ M1 ✓ 2nd differentiation

$6(5) - 2p = 20$ M1 Sub. of 5 and equating to 20

$-2p = -10$

$p = 5$

$-10(5) + q = -47$ A1 both values of p & q

$q = 3$

b) Find the values of t when the particle is momentarily at rest. (3 marks)

$S = t^3 - 5t^2 + 3t + 4$

$V = \frac{ds}{dt} = 3t^2 - 10t + 3 = 0$. ✓ M1 — quadratic eqn. in 3 terms.

$3t^2 - 9t - t + 3 = 0$.

$3t(t-3) - 1(t-3) = 0$

$(3t-1)(t-3) = 0$ ✓ M1 correct attempt to solve.

$t = \frac{1}{3}$ or $t = 3$. ✓ A1 accuracy both values

c) Calculate the displacement of the particle at $t = 5$ *seconds* (2 marks)

$S = 5^3 - 5(5)^2 + 3(5) + 4$ ✓ M1 Correct substitution.

$S = 125 - 125 + 15 + 4$

$\underline{19m}$ ✓ A1 — accuracy.

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